The Thesis Committee for Sudesh Kumar Agrawal

certifies that this is the approved version of the following thesis:

Network Models for Battery Electric Vehicles

APPROVED BY

SUPERVISING COMMITTEE:

Supervisor: Stephen D. Boyles

Christian Claudel

Network Models for Battery Electric Vehicles

by

Sudesh Kumar Agrawal, B. Tech.

Thesis

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Master of Science in Engineering

The University of Texas at Austin

August 2015

To *pa* and *mum*.

Acknowledgments

I would like to take this opportunity to express my sincere gratitude to my advisor Steve Boyles for his exceptional guidance and for providing an excellent atmosphere for doing research. He has also been an admirable teacher, both for courses, and practical issues beyond research. Besides him, I would like to thank my reader Chris Claudel and also Doug Fearing, for their insightful comments which were helpful in improving the exposition of this thesis. Special thanks to Gopal Patil for introducing me to this field back at IIT Bombay.

I wish to thank Nan Jiang, Mehrdad Shahabi, and Avinash Unnikrishnan for their help, feedback, and support with my first paper, which forms a major part of this thesis. The research was supported by the National Science Foundation EV-STS I/UCRC and by a grant from the National Science Foundation under the project — Collaborative Research: Stochastic and Dynamic Hyperpath Equilibrium Models.

My graduate studies were enriched by a great faculty, John Hasenbein whose courses were fun and intellectually stimulating. I am also grateful to Lisa Cramer, Lisa Macias, and Lisa Smith for their exuberance with administrative assistance.

I have been blessed with an awesome and cheerful research group — Tarun, Rohan, Ehsan, Tyler, Michael, Shoupeng, Alireza, Venktesh, and Rachel. In particular, I would like to thank Tarun for being a great mentor and a friend. I would also like to acknowledge ‘the basement group’ — Prateek, Ankita, Subodh, and Rohan for not deserting the dungeon like place, and for being a part of my journey here at UT and making it enjoyable. Special thanks to Ankur, Kakkar, Ritika, Jagdale, and friends from the Bridges International group for their camaraderie.

Finally, I thank my parents and grandparents for their love and sacrifices, and I am greatly indebted to them.

Hook’em!

Abstract

Network Models for Battery Electric Vehicles

Sudesh Kumar Agrawal, M.S.E.
The University of Texas at Austin, 2015

Supervisor: Stephen D. Boyles

In this thesis a nonadditive shortest path problem to model the route choice of battery electric vehicle (BEV) drivers has been proposed. Based on this nonadditive shortest path framework several multiuser (with heterogeneous risk attitude) network models which take congestion into account have also been proposed. The proposed route choice model relaxes several assumptions of earlier literature and allows for a continuum of range limits and heterogeneous drivers who have varying risk preferences. The model also accounts for nonlinearity in travel choices — drivers value a small amount of charge more when they are close to running out of range than when the battery is close to full charge. A nonlinear nonconvex optimization problem is formulated and an approximation of the objective function leads to a convex problem which is solved using an outer approximation algorithm. A tour-based analysis, which is more appropriate for BEVs is considered; but a network transformation makes the formulation simpler. Numerical experiments on a small network demonstrate how the routes taken by BEV drivers are influenced by their risk attitudes and the uncertainty in the predicted range of the vehicle. The models developed in this thesis are applicable to networks with flows of BEVs. This work will hopefully inspire researchers to explore nonlinear travel models for BEVs and develop more general network models. These network models using survey data (extensive surveys will need to be carried out for this) will be able to predict system-wide effects of the choices made by BEV drivers and help planners and policy makers in their decision making.

Keywords: battery electric vehicles, network route choice model, nonadditive shortest path, outer approximation algorithm, traffic assignment

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Background

The past decade has experienced a growing concern about the impact of fossil-fuel powered vehicles on the environment. This growing environmental concern regarding the sustainability of the conventional fuel sources has led to policy discussions to reduce emissions. Consequently, there has been a renewed interest in developing battery electric vehicles (BEVs) *(1)*, which some believe to be an answer to clean personal transportation in the future . As BEVs gain market penetration, transportation planners would require to incorporate them in their planning models. 50 years down the line the traffic stream will possibly consist of BEVs mostly. In any case travel behavior and choices exclusively associated with BEVs will have to be accounted for in urban planning models.

BEVs have the potential to significantly reduce pollution — they do not have tailpipe pollutants, and so the green-house gas savings depend on how electricity is generated upstream. Certain BEVs have the ability of *regenerative braking* which further reduces wastage of energy. Also, electric motors are more sustainable than combustion engines. So, BEVs have the potential to (and will) possibly impact the motor industry. The most attractive benefit of a BEV for a driver is its comparatively lower operating cost *(2)*.

A distinction between plug-in hybrid electric vehicles (PHEVs) and BEVs need to be made at this point. PHEVs are vehicles which are equipped with both internal combustion engines (found in gasoline vehicles, GVs), and electric motors; it has hybrid characteristics and can store electricity from the electrical grid to reduce their gasoline consumption. This helps in extending the total driving range. Chevrolet Volt and Toyota Prius are two popular PHEVs. BEVs on the other hand, completely rely on electricity for their range. Nissan Leaf and Ford Focus Electric are two popular vehicles in this category.

BEVs are recharged by plugging its battery into the electric grid. They are marooned once the battery runs out of charge. BEVs are associated with the issue of *range anxiety* — the fear of being stranded because of battery exhaustion. On a full charge and a full tank, the 2015 Chevrolet Volt (PHEV) has range up to around 350 miles (of which only around 10% is from the battery); while the 2014 Nissan Leaf (BEV) has a range of around 80 miles. A typical BEV with current battery technology lasts for around 40 miles in real-world driving conditions. Another concern for BEVs is their long charging times and the low density of charging stations in the country. A 240 V charging outlet can charge a BEV overnight (6 to 8 hours). It can take almost a day to fully charge the vehicle with a 120 V outlet though.[1] Consequently most of the charging occurs at home or the workplace. Other probable locations for a charging infrastructure include restaurants, and shopping centers. The US government is aiding Research & Development in advancing battery technology through the Department of Energy.[2]

Currently BEVs take significantly long time to recharge and there aren't enough charging stations: for now hybrid variety is more desirable. Battery technologies are improving, but it will take time before charging speeds improve substantially. The cost of BEVs is another deterrent factor to their large scale adoption. However, innovation and substantial improvement in battery technology are likely to improve sales and fuel mass production and noteworthy reduction in cost.

1.2 Motivation

Recently, there has been innovation in battery technologies and expedited growth of charging infrastructure. Understanding the travel behavior of BEV drivers is essential to envisage the effects of large-scale adoption of BEVs on transportation infrastructure. In current planning models BEVs are assumed to be like traditional vehicles with a hard distance constraint to reflect the range limit. However, the true impact of battery charge is hard to

[1] http://www.pluginamerica.org/faq/how-long-does-it-take-charge-plug-car

[2] Currently Nickel Metal Hydride (NiMH) and Li-ion are the two major battery technologies.

quantify both technologically (for example, impact of grade and speed on the battery charge) and behaviorally (for example, risk attitude of drivers). Policy decisions and their implementation too requires capturing the travel behavior of BEV drivers. For example, travel models will need to account for charging behavior of BEV drivers for developing a better charging infrastructure and improving accessibility to charging stations. Therefore, it is necessary to study the network-wide effects of BEV adoption.

Network effects are often studied through traffic assignment models, assuming that travelers choose routes to minimize travel time (or travel cost). The routing behavior of BEV drivers may differ from GV drivers because the limited refueling opportunities and comparatively shorter range of BEVs may lead to range anxiety. A survey by Rowe et al. *(3)* found that the issue of range anxiety concerned BEV users and was amplified when travelers observed the battery charge decreasing while driving. Drivers are more anxious when close to the range limit. This nonlinearity in travel behavior of BEV drivers needs to be included in travel models for BEVs.

Literature on the range anxiety issue in network routing is limited — most of the previous works *(4, 5, 6)* have assumed a fixed range limit and formulated the network routing problem as a distance-constrained shortest path problem. However, travelers are unlikely to choose paths that use close to the full battery because of lack of complete faith in the remaining range prediction. Furthermore, the level of range anxiety varies among drivers — some are more risk-prone than others and are willing to nearly exhaust the battery before recharging, while others leave a significant margin (called the *reserved range*). Basically, there is variation of the reserved range across the population, and the risk-sensitivity of drivers determines to what extent they are willing to use the battery before charging it again. The range limit is not absolute, and as mentioned before, the actual range also varies as per driving conditions (grade profile of the road, climate control of the vehicle, etc.). Previous studies have not included these aspects of the travel behavior of BEV drivers. Therefore, a new routing behavior model is needed, which reflects the uncertainty in the actual range limit in BEV drivers' decision and accounts for nonlinearity of drivers' response to the remaining charge.

As mentioned earlier, network-wide effects are studied through traffic assignment models, and so the network route choice model can be extended to consider the effects of interaction between multiple BEV users present in the network when congestion is in play. Traffic assignment helps determine the number of travelers on each link of the network and their travel time i.e. it tells us the traffic pattern and the traffic delay experienced by users. If extensive surveys are carried out to capture the range anxiety experienced by the drivers under different scenarios and the change in route choice when driving BEVs (compared to traditional vehicles), then the network models can leverage information gained from these surveys to predict system-wide effects of large-scale BEV adoption. These new network models will eventually better reflect reality. The models described in this research also accounts for nonlinear travel preferences, and a nonadditive shortest path problem to model the route choice of BEV drivers is proposed. Though the nonadditive shortest path model developed in this thesis is quite simple, hopefully it will inspire more research into nonlinear models for BEV drivers. Also, this model can be used an input for other nonlinear models like emission models. Once surveys are carried out, the network models can be calibrated which will help get a better understanding of how people will use BEVs; this will play an important role in improving next generation network models providing better guidance to transportation planners for planning network enhancements and locating charging stations. Other potential applications include developing utility models for setting prices at charging stations, finding location to set up infrastructure for inductive charging, etc. Jafari and Boyles *(7)* tried to couple transportation network with the power grid for pricing as a tool for demand management and optimization of the electric grid usage to ensure resilience of the system's power grid. Regional power grid operators can provide electricity more effectively if they know the traffic flow pattern.

1.3 Objectives

Limited refueling opportunities for BEV drivers beg a different routing model for BEVs. This thesis develops such a model based on the shortest path problem with nonadditive costs, which generalizes the distance-constrained approach found in earlier literature and can accommodate differences in drivers' risk attitudes towards range.

The range of a typical BEV is significantly higher than the length of a typical trip. However, for BEV drivers who can only recharge overnight, the total energy consumption of all trips made in a day may be close to the battery limit. Therefore, a tour-based approach is more appropriate for the route choice model. Considering tours means considering a driver's daily activity pattern. It is assumed that drivers do not charge their vehicles during a tour. The assumption is reasonable since few charging stations exist *(5)* and charging may require a significant amount of time; Morrow et al. *(8)* and Bakker *(9)* found that most BEV drivers will need to charge their vehicles at home. The setting mentioned above describes a route choice as a constrained minimum cost tour problem. However, an equivalent trip formulation has been constructed.

It is essential to develop a network equilibrium model to analyze the effects of large scale BEV adoption in the future. This would help planners and policy makers in their decisions. For example, it is important to know the locations where charging stations should be installed to make efficient use of the electric power grid and also to provide better accessibility and convenience to BEV drivers. Within the nonadditive shortest path framework described by the network route choice in this research, a number of extensions are possible for a network equilibrium model. This work also proposes traffic assignment models for some of those extensions.

The models developed in this thesis are applicable to networks with flows of BEVs subject to uncertainty in their range limit. The contributions of this research are developing a BEV routing behavior model incorporating a more general travel behavior and a continuum of range limit for the analysis of the network-wide effects of electric vehicle adoption and proposing a non-additive shortest path problem to account for nonlinear travel preferences of BEV drivers. The network-wide analysis would provide improved guidance to policy makers. For example, it would be helpful in determining the locations of new charging infrastructure. To summarize:

- Unlike earlier models we do not assume deterministic range limits.
- We also relax two major limiting assumptions of earlier literature — all BEVs have the same range limits, and all drivers have identical risk preferences (homogeneity of BEV drivers).
- We allow for a continuum of range limit.
- Nonlinear travel preferences are accounted for through non-additivity of travel costs in the nonadditive shortest path problem.

1.4 Organization of Thesis

The remainder of this thesis is organized as follows. Chapter 2 reviews existing literature on multiobjective routing, constrained shortest path algorithms, nonadditive shortest path algorithms, and network equilibrium in the context of BEVs. Chapter 3 discusses the nonadditive shortest path problem developed in Agrawal et al. *(10)* to model routing of BEVs and presents numerical analysis on some networks [3]. Chapter 4 then builds on the network routing model to describe how different BEV drivers interact in a network when congestion comes into play. Chapter 5 concludes this thesis with a summary of the route choice and network equilibrium model and their limitations, and discusses possible directions for future research.

[3]The coauthors of this paper helped with the ideas for the paper and provided reviews for the manuscript, and Mehrdad Shahabi also helped with the implementation of the algorithm.

Chapter 2

Literature Review

2.1 Battery Electric Vehicles

Advancement in EV technologies has led to an increasing market share of EVs. Gardner et al. *(11)* draw the connection between transportation and electric power systems and discuss how the market penetration of plug-in electric vehicles (PEVs)[1] a few years down the line could adversely affect the power grid of a city. Regional power grid operators can provide electricity more efficiently if they know PEVs' travel pattern. They state that range anxiety may cause PEV drivers to take more reliable routes based on energy consumption. They assume PEVs are charged only at home, have the same range limit, and PEV drivers and non-PEV drivers are the same from a behavioral standpoint, differing only in their consumption (or emission) rate. Jiang and Xie *(2)* talk about why EVs have garnered attention with their market share prediction and about the vehicle ownership composition, and they anticipate that households will prefer to own both GVs and BEVs for a long time in the coming years. They emphasize the problem of insufficient charging infrastructure and long charging time which leads to most vehicles being charged either at home or

[1] A PEV is a vehicle which can use an external electricity source to recharge itself. PHEVs and BEVs are PEVs.

at the workplace. A similar assumption has been adopted in this thesis. Assuming a linear dependence of travel distance on operating costs, they propose a mathematical formulation of a simultaneous mode and route choice mixed equilibrium with GVs and BEVs where the travel impedance comprises of a travel time component and the operating cost, and system users comprise of people who own both GVs and BEVs. Their combined model is characterized by the equilibrium principle rather than utility theory. They suggest two solution algorithms — based on Frank-Wolfe framework with a modified label setting algorithm, and Gradient Projection framework with a pre-processing and label setting algorithm; and compare the computational advantages of both. Unlike their model, which allows for discrete choices of range limits through the incorporation of multiclass vehicles, the model proposed in this thesis allows for a continuum of range limits. Also, they use range limit only to define feasible paths for BEVs, and there has not been any focus on the behavioral change in drivers due to range anxiety. Adler et al. *(5)* propose polynomial time algorithm to find a shortest walk with minimum detour (to refuel at charging stations). More importantly, they suggest algorithms to find routes that minimize range anxiety.

An important behavioral aspect from a modeling perspective of BEV users' behavior is the nonlinearity and non-additivity in travel preferences — a small quantity of charge is valued more when one is close to exhausting the battery than when the battery is near full charge. Hensher and Truong *(12)*, and Pinjari and Bhat *(13)*, through experiments and surveys, emphasize the importance of incorporating nonlinear travel preferences. Using a non-nested likelihood ratio test on the Austin commuter stated preference survey data Pinjari and Bhat *(13)* show that not incorporating nonlinear travel preferences could lead to erroneous estimates of willingness-to-pay measures. Gabriel and Bernstein *(14)* identify issues associated with assuming additive link costs and enumerate situations (like nonadditive tolls and fares) in which the link costs are nonadditive, and they discuss the consequences of the nonadditive structure of travel costs. They prove existence and uniqueness of arc flows and the shortest travel time for the nonadditive equilibrium problem. The nonadditive aspect of travel behavior has not been captured in previous EV studies. In the formulation given in this research the disutility component associated with battery exhaustion captures the non-additivity of the range anxiety.

We are not aware of any work on the distribution of range anxiety in the population. However, Lin et al. *(15)* find that the variation in daily vehicle miles traveled can be modeled by a gamma distribution for appraisal of energy use by plug-in hybrid electric vehicles (PHEVs) and comment that this can be used to estimate range anxiety for BEVs. However, doing so will not be trivial considering that a BEV driver's range anxiety could be affected by many factors. The network route choice model we develop in chapter 3 is general and can be extended to consider any distribution once relevant data are available.

2.2 Multiobjective Routing

Chen et al. *(16)* examine how travelers with different risk attitudes respond to uncertainty in their travel cost (which they consider as risk). They stress that for most users *travel time reliability is almost as important as the travel time itself*, and it plays a significant role in the route choice decision process. Though they investigate only individual risk preferences and model the risk attitude using exponential functions, for the aggregation of these preferences at the network level they suggest segmentation of the population into groups sharing a common behavior (risk prone, risk averse, or risk neutral) or use of a random-coefficient logit model. Another thing they emphasize is that "to model risk-taking behavior, it is necessary to include network uncertainty in the route choice models".

Road users may have objectives other than reaching their destination in the shortest time. Researchers have tried to capture those objectives (minimize — monetary costs, travel time uncertainty, etc.) either by incorporating them as constraints or by including a new travel cost component reflecting that objective. Sen et al. *(17)* formulate a quadratic multiobjective model which minimizes the weighted expected travel time and variance in travel time and is efficiently solved by a series of parametric binary quadratic integer programs that take advantage of the network structure. Their model provides an efficient frontier to offer travelers with a set of alternative routes to choose from. Sivakumar and Batta *(18)* speak about finding a safe path for transporting hazarding materials. A safe path could be anticipated to have the least expected risk; however, it probably might have a high variance in risk, which is not desirable and not preferable. They introduce a hard constraint in their model to find the shortest expected risk route — the variance of risk on the optimal path should be within a certain predetermined threshold. Their formulation can be used whenever the travel costs are stochastic, and there is a possibility that there is a correlation in these costs across links. They propose an exact method based on Lagrangian relaxation followed by a duality gap closure procedure to solve the formulation. In the context of BEVs the other objective would be to minimize the probability of battery exhaustion, and so we have used expected disutility from running out of range as the new travel cost component.

2.3 Constrained Shortest Path Algorithms

Most of the research while considering the route choice for EV drivers have formulated the problem as a constrained shortest path (CSP) problem. CSP problems, which are $\mathcal{NP}$-complete *(19)*, are typically solved using either Lagrangian relaxation based frameworks *(20)*, which are efficient for a single constraint, labeling schemes *(21)*, or k-shortest path based algorithms *(22, 23)*. Pre-processing is an important aspect of labeling schemes. Dumitrescu and Boland *(24)* modify previous pre-processing methods, and their exact algorithm is based on closing the gap between the lower and upper bound using "weight scaling". Dumitrescu and Boland *(25)* through numerical experiments show how pre-processing can effectively reduce the network structure and speed up the labeling scheme.

Artmeier et al. *(26)* develop a navigation system for energy efficient routing, and they extend the general shortest path algorithm and use "prefix-bounded" shortest path trees to solve their formulation for BEVs. Given their limited battery capacity, long charging times, and the ability to recharge from braking, routes are to be energy efficient and not just least cost. Regenerative braking creates the possibility of negative costs on links, which precludes the use of many commonly used routing algorithms. Their model has a hard constraint on the energy required for a path and a soft constraint on energy regeneration, hence a path where energy gained can cause the battery charge to exceed its capacity is less likely to be taken.

2.4 Nonadditive Shortest Path Algorithms

A vast amount of literature on the shortest path problem assumes additive link costs to satisfy Bellman's principle which allows the use of efficient solution algorithms. However, in many cases (for instance, tolls) link costs are nonadditive.

The travel cost in our model has a similar structure to the cost used by Chen and Nie *(27)* who find the optimal path by solving many constrained additive shortest path sub-problems. They approximate the nonlinear cost by piecewise linear functions. The algorithm uses an efficient frontier to update the bounds of the original problem. In the worst case if the optimal path of the sub-problems are not on the efficient frontier, then path enumeration is required. They enlist a number of problems (like distance based congestion pricing schemes) where this cost structure is used. The most important facet of their work is that they assume a very general cost structure for the nonadditive shortest path problem. Tsaggouris and Zaroliagis *(28)* suggest an efficient exact algorithm for solving the nonadditive shortest path problem (NASP) which needs travel cost functions to be convex and non-decreasing. They use an extended hull algorithm (also proposed in the same paper) to reduce network size and improve gap closure.

Shahabi et al. *(29)* propose an outer approximation (OA) algorithm based on cutting plane methods for nonadditive shortest path problems with continuous and convex path cost functions. Cutting planes method iteratively improves the feasible set of the convex problem by introducing linear inequalities called cuts. The OA algorithm iteratively closes the gap between the upper bound provided by the sub-problem and the lower bound given by the master problem to get to the optimal solution. Through numerical experiments they establish the efficiency of their method in finding the exact global solution. This algorithm has been adopted in this work to find the optimal path for a BEV user.

2.5 Equilibrium

Jiang et al. *(4)* present a network user equilibrium model with distance constrained shortest path problem as its sub-problem. Their travel costs includes an out-of-range cost which ensures that flow on paths whose length exceeds the range limit is zero. Their model allows BEVs to have discrete range limits, and their solution method is based on the Frank-Wolfe algorithm. Jiang et al. *(6)* analyze network flows from a combined destination, route and parking choices subject to the distance constraint, and Jiang & Chi *(2)* evaluate mixed equilibrium flows of GVs and BEVs.

Xie et al. *(30)* extend the path-constrained traffic assignment (TA) model of Jiang et al. *(4)* and allow for stochastic distance constraints. A continuous distribution of the range limit leads to an infinite number of constraints for the TA problem; so they propose a convex program with finite constraints by presenting an alternative flow variable (called *cumulative path flow rate*). To solve it they use a k-shortest path algorithm for the linear sub-problem obtained by approximating the nonlinear objective function.

Zhang et al. *(31)* propose a stochastic user equilibrium (SUE) problem for BEV drivers to appraise how charging prices influence the route choice behavior of drivers. Their SUE model uses random utility theory and the UE principle to obtain the flow pattern, and they use a simplicial decomposition scheme to solve it.

Zhang et al. *(32)* formulate a variational inequality to model the temporal and the spatial impact of a BEV driver's behavior. They present a time-dependent network model for travel choices which takes departure time, duration of stay at charging stations and route choice into consideration for a mixed flow of GVs and BEVs. A nested Logit structure has been used for the combined choices. The choice of charging station has been changed to a route choice using an expanded transformed network with charging stations as nodes. Their model takes the variation of charging prices across stations into account (apart from the waiting costs in case of congestion at charging stations). Their multiclass VI uses an optimization-based heuristic to solve the model which produces equilibrium solution for the combined choices.

He at al. *(33)* suggest three different network equilibrium models for BEVs with different assumptions on recharging time and flow dependency of battery consumption. They examine how limited range and refueling opportunities affect route choices and the network equilibrium flow distribution. One of their equilibrium models allow the battery depletion to depend on congestion which is not there in earlier literature. However, they solve it only for a small network because of the computational intractability of their nonlinear complementarity problem.

Chapter 3

Network Route Choice Model for BEV drivers

The concern about the impact of fossil-fuel powered vehicles calls for the need of a cleaner personal transportation and BEVs are considered to be the answer to this concern. A network equilibrium model to study the effects of BEV adoption is therefore needed. Such effects are often studied through traffic assignment models, which generally assume that travelers choose routes to minimize their travel time or travel cost.

The limited driving range, and the paucity of charging stations compounded with potentially long recharging time i.e. limited refueling opportunities, lead to *range anxiety* in drivers — the concern of running out of fuel before completing a trip. The issue of range anxiety inevitably affects their route choices. Although even GV drivers refuel at gas stations, the recharging of BEVs is more frequent given their shorter range.

Literature on range anxiety has been limited to formulating the network route choice as a distance-constrained shortest path problem. What that means is that if the range limit for a vehicle is say, 50 miles, then the driver is perfectly fine taking a route which is 49.9 miles, but a path which is 50.1 miles is right out of consideration. This is not how people are likely to behave

— they are probably not willing to run right up to the limit because, the limit may not be exact and is likely to vary based on driving conditions.

Lack of complete faith in the remaining range prediction of the vehicle leads to drivers choosing routes which are comfortably within the predicted range. This reserved range varies across the population and is determined by the risk attitude of drivers. Additionally, the actual range depends on the driving conditions. Essentially, the battery consumption rate, and the heterogeneity in drivers' perception and risk preference lead to stochasticity in the range limit which determines the route choice of BEV drivers. Therefore, a new routing model has been proposed in this chapter to reflect the uncertainty in the actual range limit in a BEV driver's decision.

As mentioned earlier, a tour-based approach is more appropriate since the range of typical BEVs is significantly higher than the length of a typical trip. Also, since charging stations are scarce even in areas with high market permeation of BEVs and that charging requires substantially long time, it is reasonable to assume that drivers do not charge their vehicles during a tour (charging occurs only at the origin).

3.1 Problem Description and Model

It is reasonable to believe that people do not perceive the range limit as a hard constraint and intrinsically strive to minimize their disutility while choosing their routes under uncertainty of the range *(34)*. In the model proposed in this chapter this disutility has two components — travel time, and a function which represents the potential disutility from battery exhaustion and is based on the path distance and the perception of the range limit. Rather than an *all-or-nothing* decision where any path under the range limit is acceptable and any path above it is not allowed, the framework allows for a smoother transition in the response of a driver. As people get closer to the range limit their concern about the remaining battery charge may increase. This nonlinearity in the response of the drivers to the battery charge leads to a *nonadditive shortest path* problem i.e. the travel cost of on a path (route) is not just the sum of the sum of the travel cost on its links.

As discussed earlier, a tour based approach would be appropriate to model the route choice of BEV drivers. However, a network transformation discussed later in section 3.3 allows formulation in terms of the traditional shortest path problem and simplifies the model.

Let $G = (N, E)$ be a directed graph of the network, where N is the set of nodes with r and s denoting the origin and destination nodes respectively, and E is the set of directed links[1]. Suppose Π is the set of all simple paths and a_{ij} is the link from node i to node j. Let T_π be the travel time on a path π and D_π be the length of that path. Also let x_{ij} be a $0-1$ decision variable that determines if the optimal path uses link a_{ij} ($x_{ij} = 1$ if the optimal path uses link a_{ij}).

In the absence of evidence for a more complicated decision rule, we use

[1]Table 3.1 enlists the major mathematical notations used in this chapter.

Table 3.1: Nomenclature

G :	Directed graph of the network
N :	Set of nodes
E :	Set of directed links
r :	An origin node
s :	A destination node
Π :	Set of all simple paths
a_{ij} :	A link from node i to node j
T_π :	Travel time on a path π
D_π :	Length of a path π
t_{ij} :	Travel time on link a_{ij}
d_{ij} :	Length of link a_{ij}
t_π :	Generalized travel cost
x_{ij} :	$0-1$ decision variable that determines if the optimal path uses link a_{ij}
$\tilde{S}$:	Disutility from running out of range
$\tilde{R}$:	Random variable denoting the perceived range limit
$\mathcal{U}_D$:	Disutility incurred if one actually runs out of range
$F_{\tilde{R}}$:	Cumulative distribution function of $\tilde{R}$
$f_{\tilde{R}}$:	Probability density function of $\tilde{R}$
$g(T_\pi, D_\pi)$:	Generalized travel cost
α :	Value of time (in terms of disutility)
$\hat{g}(T_\pi, D_\pi)$:	Approximating function for g
h :	Iteration count in OA algorithm
H :	Maximum iteration for OA algorithm

expected disutility to represent the potential disutility (for instance, disutility from requiring towing) from running out of range. This disutility function is assumed to capture the range anxiety of drivers. The distribution of the range limit is built on how drivers perceive the range limit based on their driving conditions. Consider a random variable $\tilde{S}$ denoting the disutility from running out of range to be given by

$$\tilde{S} = \begin{cases} 0, & D_\pi \leq \tilde{R} \\ \mathcal{U}_D, & D_\pi > \tilde{R} \end{cases}, \tag{3.1}$$

where $\tilde{R}$ is a random variable denoting the (random) perceived range limit, and $\mathcal{U}_D$ is the disutility incurred if one actually runs out of range. Then the expected disutility from running out of range on a path would be

$$E[\tilde{S}] = \mathcal{U}_D \cdot P(\tilde{R} \leq D_\pi) = \mathcal{U}_D \cdot F_{\tilde{R}}(D_\pi), \tag{3.2}$$

where $F_{\tilde{R}}$ is the cumulative distribution function (CDF) of the perceived range limit $\tilde{R}$.

Let $g(T_\pi, D_\pi)$ denote the generalized travel cost, which we assume to be an increasing function of T_π and D_π and is given by[2]

$$g(T_\pi, D_\pi) = \alpha T_\pi + \mathcal{U}_D \cdot F_{\tilde{R}}(D_\pi), \tag{3.3}$$

where α is the value of time (in terms of disutility). The parameter $\mathcal{U}_D$ is constant for a given user, depends on the risk affinity of the user and is likely

[2]The travel cost function is assumed to be linear in time to focus on the nonlinearity in response to the range limit.

to be *higher for a risk averse user* compared to a *risk seeking* user. The use of expected disutility results in a nonadditive path disutility function. The formulation for the nonadditive shortest path problem is

$$\min \quad \sum_{\pi \in \Pi} g(T_\pi, D_\pi)$$

subject to

$$\sum_{\{j: a_{ij} \in E\}} x_{ij} - \sum_{\{j: a_{ji} \in E\}} x_{ji} = \begin{cases} 1, & i = r \\ -1, & i = s \\ 0, & \text{otherwise} \end{cases} \tag{3.4}$$

$$x_{ij} \in \{0, 1\} \quad \forall a_{ij} \in E,$$

where the first constraint of program 3.4 ensures connectivity of the shortest paths, and the second one ensures integer solutions.

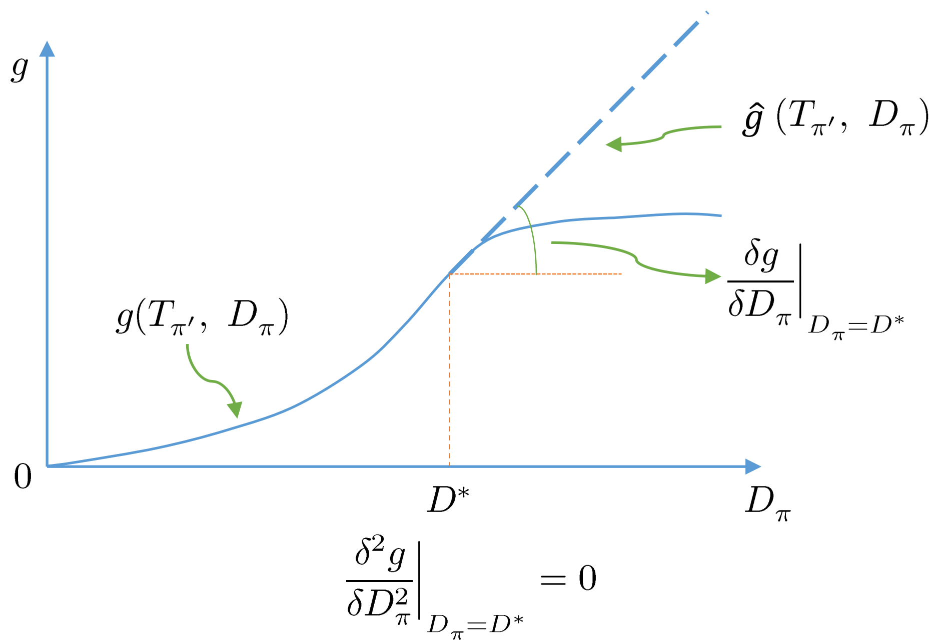


Figure 3.1: Approximating function $\hat{\boldsymbol{g}}$ (for a given $T_{\pi'}$).

However, the generalized travel cost function g may be non-convex which would make it computationally intractable to obtain the global optimal solution. For a fixed travel time $T_{\pi'}$ we believe g would be an S-shaped curve (convex at the beginning and concave later), the shape of many CDFs. To make the travel cost function convex, we have approximated this curve by a linear tangent beyond the inflection point (D^*) of the S-shaped curve (see Figure 3.1). The approximating function $\hat{g}$ can be written as:

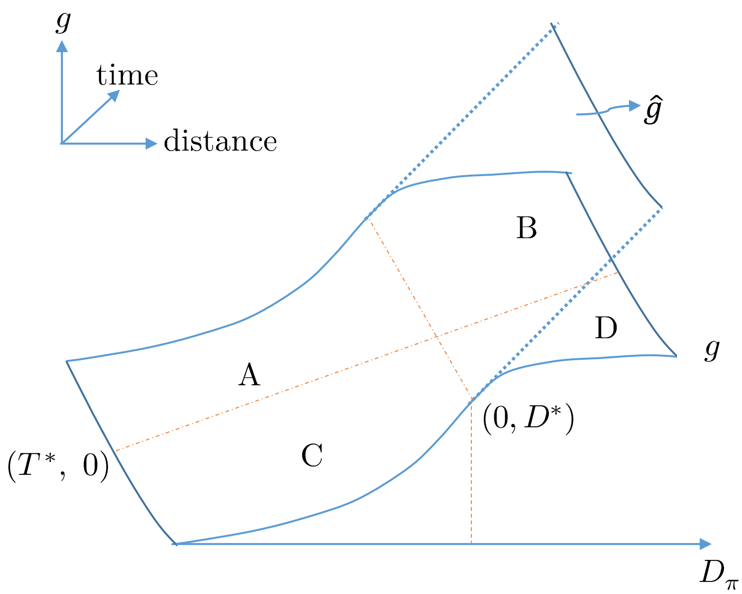
$$\hat{g}(T_\pi, D_\pi) = \begin{cases} g(T_\pi, D_\pi), & D_\pi \leq D^* \\ \mathcal{U}_D \cdot f_{\tilde{R}}(D^*)(D_\pi - D^*) + g(T_\pi, D^*), & \text{otherwise} \end{cases}, \tag{3.5}$$

where $f_{\tilde{R}}$ is the probability density function (PDF) of $\tilde{R}$.

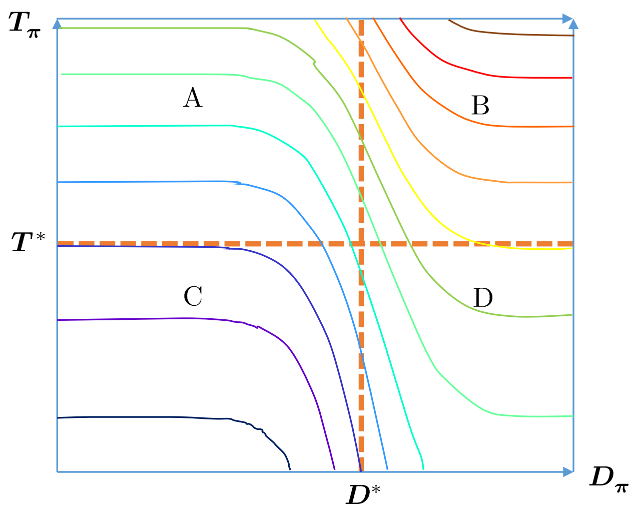
We believe it is unlikely that the approximation will affect the optimal solution since we are increasing the objective function in the concave region where the optimal solution is unlikely to lie (otherwise a person will not choose to drive a BEV in the first place). Nonetheless, we derive a condition under which this approximation will certainly not introduce any error.

3.2 Derivation of the Condition

Four distinct regions ($A, B, C,$ and D) have been marked in Figure 3.2. The plane defined by $D_\pi = D^*$ separates the graph into two regions such that region A and region C are on the same side. Let $(T^*, 0)$ be a point such that $g(0, D^*) = g(T^*, 0)$. Intuitively, region A is a region having routes with relatively high travel time but small path length as opposed to region D having routes with low travel time but large path length; and region B is a region



(a) 3D view



(b) 2D view (contours)

Figure 3.2: Division of the graph into four distinct regions ($\boldsymbol{A, B, C, D}$).

having routes with relatively high travel time and large path length in contrast to region Chaving routes with low travel time and small path length. If the optimal path (in terms of the value of g) is in A or C, then the approximation doesn't change the optimal path. The approximation doesn't alter the optimal path even when it is in B because g is increasing in T_π and D_π. The problem arises only when the optimal path lies in D. When the optimal path lies in D:

Case I: If no path in the given network lies in region A, then the approximation doesn't change the optimal path.

Case II: There exists a path in the given network which lies in region A.
Let $\pi_{\underline{a}}$ be the shortest path (in terms of the value of g) of all the paths in region A, and similarly $\pi_{\underline{d}}$ for region D. Clearly, $g(\pi_{\underline{d}}) < g(\pi_{\underline{a}})$ since the optimal path lies in D.
Now if $\hat{g}(\pi_{\underline{d}}) < \hat{g}(\pi_{\underline{a}})$, then the approximation will not change the optimal path. However, the optimal path may change if $\hat{g}(\pi_{\underline{d}}) \geq \hat{g}(\pi_{\underline{a}})$.

Therefore, the approximation finds the true optimal path when it lies in either $A, B,$ or C, and the condition required for it to work when the optimal path lies in D is given by (3.6):

$$\begin{aligned} &\hat{g}(\pi_{\underline{d}}) < \hat{g}(\pi_{\underline{a}}) \\ \Rightarrow\ &\mathcal{U}_D \cdot f_{\tilde{R}}(D^*)(D_{\pi_{\underline{d}}} - D^*) + \alpha T_{\pi_{\underline{d}}} + \mathcal{U}_D \cdot F_{\tilde{R}}(D^*) < \alpha T_{\pi_{\underline{a}}} + \mathcal{U}_D \cdot F_{\tilde{R}}(D_{\pi_{\underline{a}}}) \\ \Rightarrow\ &\alpha \left(T_{\pi_{\underline{d}}} - T_{\pi_{\underline{a}}}\right) + \mathcal{U}_D \left[F_{\tilde{R}}(D^*) - F_{\tilde{R}}(D_{\pi_{\underline{a}}})\right] + \mathcal{U}_D \cdot f_{\tilde{R}}(D^*)(D_{\pi_{\underline{d}}} - D^*) < 0. \end{aligned} \tag{3.6}$$

The condition in 3.6 requires that the generalized cost of the optimal path in region D should be smaller than the generalized cost of the optimal path in region A. Equality may be allowed in the above condition if we assume that a user would prefer a path with longer travel time over a path with greater chance of running out of range given both the paths have the same cost. It is important to note that the condition not only depends on the distribution of the range limit but also on the network structure.

The new formulation after incorporating the approximation is:

$$
\begin{aligned}
\min \quad & \sum_{\pi \in \Pi} \hat{g}(T_\pi, D_\pi) \\
\text{subject to} \quad & \\
& \sum_{\{j:a_{ij} \in E\}} x_{ij} - \sum_{\{j:a_{ji} \in E\}} x_{ji} = \begin{cases} 1, & i = r \\ -1, & i = s \\ 0, & \text{otherwise} \end{cases} \\
& x_{ij} \in \{0,1\} \quad \forall a_{ij} \in E.
\end{aligned} \tag{3.7}
$$

Now the problem formulation has the same structure to the one in Shahabi et al. *(29)* and satisfies the convexity and differentiability conditions. Therefore, their OA algorithm can be applied. OA algorithm can minimize a convex function over a convex feasible region by sequentially defining linear cuts instead of the nonlinear terms of the program (see Figure 3.3). Theoretically, OA is capable of delivering the global optimal for convex mixed integer programs.

One can refer to Shahabi et al. *(29)* for the actual algorithm and to Shahabi et al. *(35, 36)* for specific cases of the nonadditive shortest path problem. The formulation corresponding to the algorithm is presented in section 3.4.

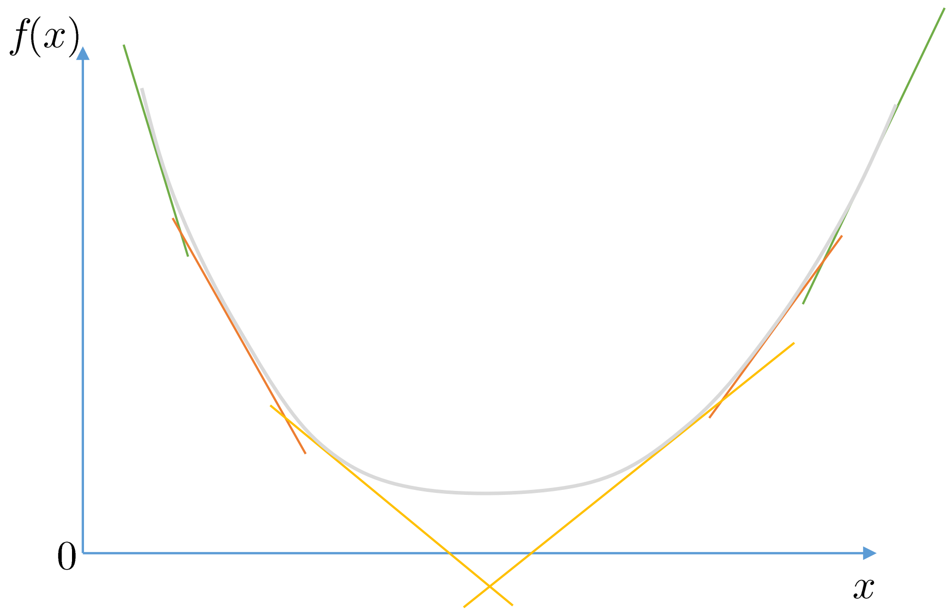


Figure 3.3: Intuition for Outer Approximation algorithm.

A flowchart of the OA framework has been reproduced from the paper in Figure 3.4.

3.3 Network Transformation

This section describes a network transformation which allows analysis of tours in a trip based formulation. For example, if we know that a BEV driver tours through nodes 1, 3, 5, and 7 in a given network, then the transformed network ensures that any trip from node 1 to node 7 in the transformed network passes through nodes 3 and 5 in the original network. In a constrained

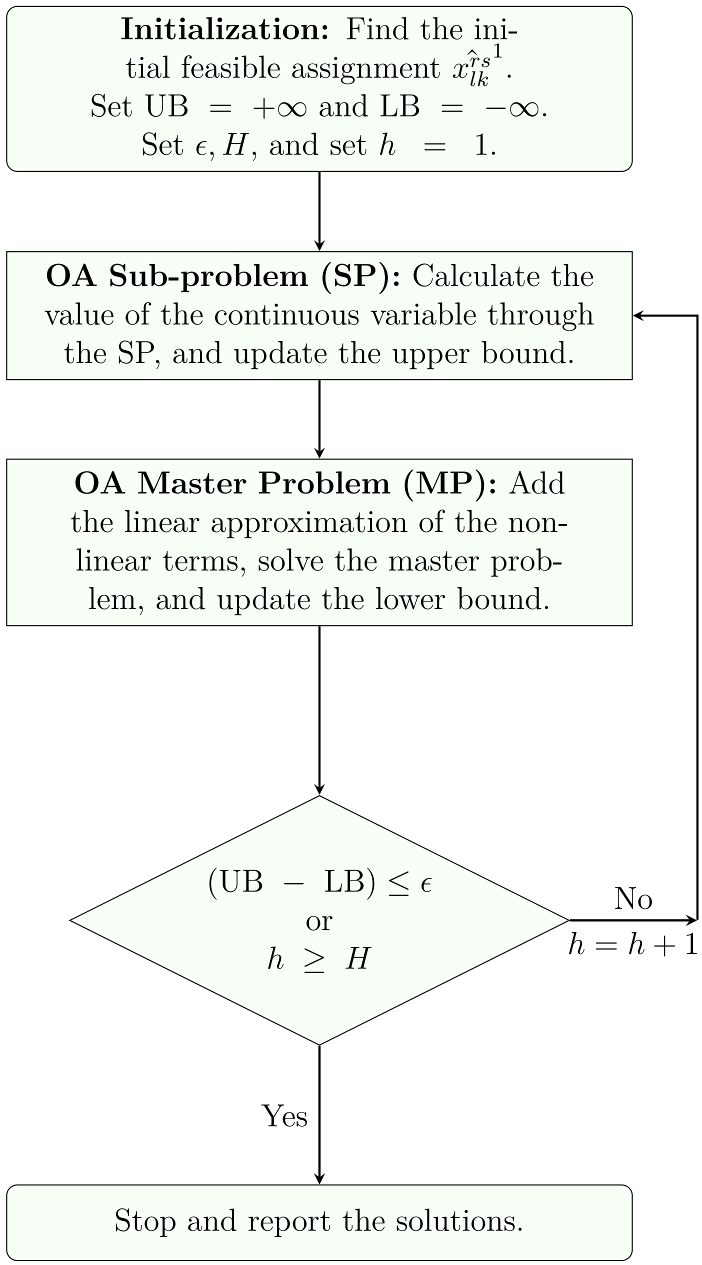
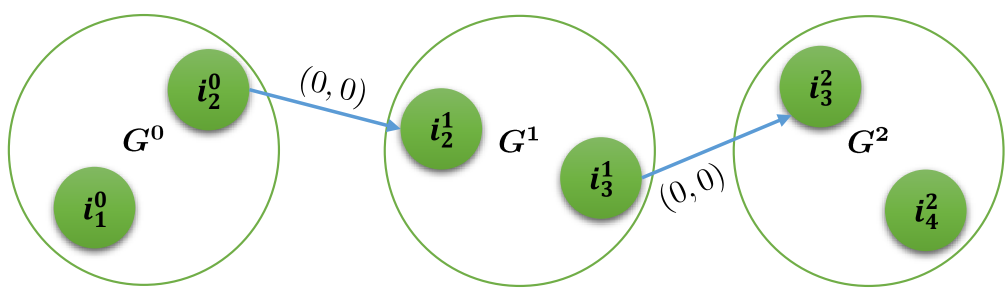


Figure 3.4: OA framework.

minimum cost tour problem corresponding to the problem described in this paper, the origin, the destination, and an ordered set of locations (nodes) to be visited in the tour, are given. Assuming that BEVs are fully charged at the start of the tour and cannot recharge during the tour, the objective is to find the optimal path which passes through all the given nodes while observing the range limit. In a basic constrained problem it is simple to find the shortest path, but with a constrained tour, the distribution of battery charge has more possibilities and is more complex to optimize.



Network G: $G^0 \equiv G^1 \equiv G^2$; superscripts (on nodes) indicate the index of the network copy; Tour: $[r \equiv i_1, i_2, i_3, i_4 \equiv s]$

Figure 3.5: Network transformation.

To find the optimal tour connecting the nodes $[i_1, i_2, \ldots, i_k]$, where i_1 and i_k are the origin and the destination nodes respectively, the following network transformation (steps) is applied (see Figure 3.5 for a demonstration):

1. The original network G is copied $k-2$ times so that the final expanded network has $k-1$ original networks.

2. A link is added between i_{p+1} of G_p and i_{p+1} of G_{p+1} $\forall p \in \{1, 2, \ldots, k-2\}$.

3. All the newly created links have zero length and zero travel time.

To find the minimum cost tour, a constrained shortest path algorithm can be used in this new expanded network. Once the focus is on trips we no longer need to consider a hard distance constraint. It is easy to see that if only individual trips are considered for analysis, then we'd essentially be solving the regular shortest time path problem (assuming the battery is fully charged at the beginning of each trip) since the disutility component for each trip due to charge exhaustion will be insignificant in comparison to travel time for most of the trips.

3.4 Formulation

This section provides the OA framework given in Shahabi et al. *(29)* in the context of the model developed in earlier section.

The problem in its general form is

$$
\begin{aligned}
& Z = \min \quad \sum_{\pi\in\Pi} \hat{g}\left(\sum_{a_{ij}\in E} t_{ij}x_{ij}, \sum_{a_{ij}\in E} d_{ij}x_{ij}\right) \\
& \text{subject to} \\
& \qquad \sum_{\{j:a_{ij}\in E\}} x_{ij} - \sum_{\{j:a_{ji}\in E\}} x_{ji} = \begin{cases} 1, & i = r \\ -1, & i = s \\ 0, & \text{otherwise} \end{cases} \\
& \qquad x_{ij} \in \{0,1\} \quad \forall a_{ij} \in E,
\end{aligned}
\tag{3.8}
$$

where t_{ij} is the travel time on link a_{ij} and d_{ij} is the length of that link. The formulation above (3.8) is nonlinear and convex in the objective function and linear in the constraints.

(**Note**[3]**:** $\boldsymbol{T_\pi}$ is the travel time on a path π, $\boldsymbol{t_\pi}$ is the *generalized travel cost* — the sum of travel time and disutility from running out of range, on a path π, and $\boldsymbol{t_{ij}}$ is the travel time on a link a_{ij}.)

The mixed integer nonlinear program (MINLP) corresponding to the formulation above is

$$
\begin{aligned}
& Z = \min \quad \sum_{\pi\in\Pi} t_\pi \\
& \text{subject to} \\
& \qquad \hat{g}\left(\sum_{a_{ij}\in E} t_{ij}x_{ij}, \sum_{a_{ij}\in E} d_{ij}x_{ij}\right) \le t_\pi \quad \forall \pi \\
& \qquad \sum_{\{j:a_{ij}\in E\}} x_{ij} - \sum_{\{j:a_{ji}\in E\}} x_{ji} = \begin{cases} 1, & i = r \\ -1, & i = s \\ 0, & \text{otherwise} \end{cases} \\
& \qquad x_{ij} \in \{0,1\}, t_\pi \ge 0 \quad \forall a_{ij} \in E, \forall \pi.
\end{aligned}
\tag{3.9}
$$

[3] To prevent confusion

The formulation above (3.9) is linear in the objective function but nonlinear and convex in the constraints — it is a nonlinear program (NLP). The sub-problem is formed by removing the last constraint from this NLP. The first constraint in 3.9 enables the solution of the SP to be expressed through closed form equations once integer assignments (x_{ij}) are fixed by the master problem, thereby reducing the OA algorithm to solving only the master problem. The objective function of the sub-problem provides the upper bound (UB) of the algorithm for every iteration h and is given by $Z^h = \min \sum\limits_{\pi \in \Pi} t_\pi^h$. The master problem of the algorithm, whose solution provides the lower bound (LB) is given by the following set of equations (H is the maximum iterations):

$$\mathrm{LB} = \min \sum_{\pi} t_\pi \tag{3.10}$$

$$\left[\sum_{a_{ij} \in E} \left\{ t_{ij} \frac{\partial \hat{g}(T_\pi, D_\pi)}{\partial T_\pi} \times (x_{ij} - x_{ij}^h) \right\} + \sum_{a_{ij} \in E} \left\{ d_{ij} \frac{\partial \hat{g}(T_\pi, D_\pi)}{\partial D_\pi} \times (x_{ij} - x_{ij}^h) \right\} \right] \\ - (t_\pi - t_\pi^h) \leq 0 \quad \forall \pi \in \Pi, \forall h = 1 \dots H \tag{3.11}$$

$$\sum_{\{j:\ a_{ij} \in E\}} x_{ij} - \sum_{\{j:\ a_{ji} \in E\}} x_{ji} = \begin{cases} 1, & i = r \\ -1, & i = s \\ 0, & \text{otherwise} \end{cases} \tag{3.12}$$

$$\mathrm{LB}^{h-1} \leq \sum_{\pi} t_\pi \quad \forall h = 1 \dots H \tag{3.13}$$

$$\mathrm{LB}^h \leq \mathrm{UB} \quad \forall h = 1 \dots H \tag{3.14}$$

$$x_{ij} \in \{0, 1\} \quad \forall a_{ij} \in E \tag{3.15}$$

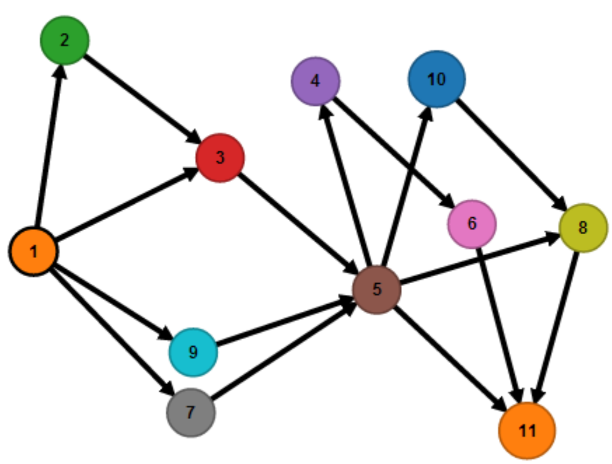
$$t_\pi \geq 0 \quad \forall \pi \in \Pi. \tag{3.16}$$

The program formed by the set of conditions above (3.10–3.16) is a mixed integer linear program (MILP). The inequality in 3.11 is basically a linear approximation of the nonlinear counterpart in the MINLP (supporting hyperplane of the first constraint in program 3.9) and adds linear cuts (or OA cuts) at every iteration of the algorithm. The condition in 3.13 ensures that the lower bounds are produced in a non-decreasing sequence by the algorithm, and the condition in 3.14 guarantees that the solution of the MP is less than the upper bound. To summarize:

- MP is obtained by a Taylor series approximation of the MINLP.
- MP is a MILP and gives a lower bound.
- SP is obtained by fixing integer variables in the MINLP through the solution of MP.
- SP is a NLP and gives an upper bound.
- The continuous variable ($\boldsymbol{t_\pi}$) is optimized through the SP.

3.5 Numerical Experiments

In this section we first demonstrate how route choice changes with a change in the risk attitude of drivers (as measure by $\mathcal{U}_D$), using a small artificial network. Risk averse users are likely to choose routes with a lesser chance of exhausting the battery, and so they will have a higher value of $\mathcal{U}_D$. It is expected that as drivers become more risk averse, length of the optimal path should decrease. Next, we run some experiments to demonstrate the effects of uncertainty in range prediction on route choice. And finally, we present some running time statistics of the OA algorithm on some large networks *(37)* to give an idea of the amount of time it would take to solve the formulation for real life scenarios. The model was implemented in GAMS and CPLEX was used as the solver. All the experiments have been done on a server with Intel(R) Xeon(R) CPU X5680 @ 3.33GHz with 23.45 GB RAM. All instances in the experiments have been solved to *zero optimality gap*.



(a) Network graph

Link	Time	Length
(1, 2)	0.3	0.5
(1, 3)	2.0	0.5
(1, 7)	0.6	1.3
(1, 9)	2.0	1.0
(2, 3)	0.7	1.5
(3, 5)	1.0	0.5
(4, 6)	1.0	1.0
(5, 4)	1.0	2.0
(5, 8)	2.0	3.0
(5, 10)	4.0	1.5
(5, 11)	5.0	1.0
(6, 11)	2.0	2.0
(7, 5)	0.4	0.7
(8, 11)	5.0	2.0
(9, 5)	3.0	1.0
(10, 8)	2.0	1.5

(b) Link attributes

Figure 3.6: Artificial network Z (length is in km; time is in minute).

The aim of the first experiment is to illustrate the change in the optimal path with a varying risk attitude of the user (given by $\mathcal{U}_D$). Drivers who are more risk averse choose routes to minimize their probability of running out range, and so it is expected that as we increase $\mathcal{U}_D$ the length of the optimal path should decrease. For this experiment we choose network Z shown in Figure 3.6 along with the link attributes. A truncated normal distribution with the following parameters have been used to model the perceived range limit: $\alpha = 1; \mu = 10, \sigma = 10, a = 0, b = 20$ where, μ and σ are the mean and the standard deviation of the parent normal distribution, and a and b are the truncation parameters. It should be noted that the formulation is not restricted to any distribution and a truncated normal distribution was used for lack of any data on the distribution of the range limit. The tour chosen for the experiment is [1 5 11] i.e. a trip from node 1 to node 11 via node 5. Figure 3.7 shows the network transformation for this tour. Table 3.2 shows the results of the experiments. When $\mathcal{U}_D = 0$ the optimal path is the regular shortest time path which is the optimal path for a driver who is not risk averse at all and does not care about the battery getting exhausted. The optimal path remains the same for drivers who are relatively less risk averse or are risk seeking (low value of $\mathcal{U}_D$). As drivers get more risk averse ($\mathcal{U}_D$ increases) optimal path changes such that the path length decreases at the cost of an increased travel time. From the table we can see that a driver who is less averse to risk ($\mathcal{U}_D = 1$) will choose a route with a travel time of 5 minutes and a length of 7 km while another user who is fifty times more averse to risk will choose a route with a travel time of 8 minutes and a length of only 2 km.

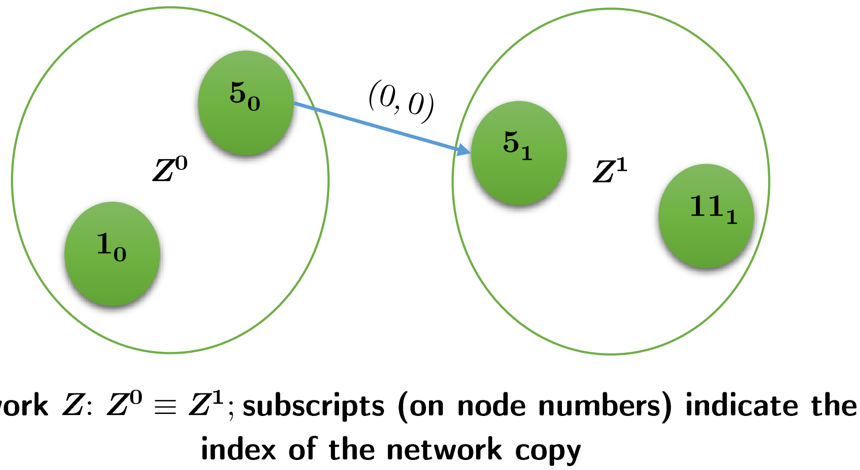


Figure 3.7: Schematic of the transformation of network Z for tour [1, 5, 11].

The distribution and the value of the parameters used for this experiment are not based on real data as no survey data were available, and so they may seem unrealistic. Nevertheless, such trade-offs between travel time and distance as demonstrated in the experiment can be found when one of the available routes comprises mainly of freeways and highways while the other routes consist of arterials with shorter distance but longer travel time due to congestion. Similar trade-offs (between faster routes with high costs and slower routes with low costs) have been found on some routes as mentioned by Gabriel and Bernstein *(14)*.

The aim of the second experiment is to demonstrate how the routes

Table 3.2: Results of the first experiment on network Z

$\mathcal{U}_D$	**Optimal Path**	$\hat{g}$	T_π	D_π	**Probability of running out of range (%)**
0	[1, 7, 5, 4, 6, 11]	5.000	5	7	32.7
1	[1, 7, 5, 4, 6, 11]	5.327	5	7	32.7
5	[1, 7, 5, 11]	6.610	6	3	12.2
10	[1, 7, 5, 11]	7.220	6	3	12.2
20	[1, 7, 5, 11]	8.441	6	3	12.2
30	[1, 7, 5, 11]	9.661	6	3	12.2
40	[1, 7, 5, 11]	10.88	6	3	12.2
50	[1, 3, 5, 11]	11.90	8	2	7.8

taken by a given driver change with the uncertainty of predicted range of the vehicle. The standard deviation of the distribution describes the uncertainty in the prediction. We expect that as the standard deviation converges to zero, the result converges to the constrained shortest path ([1, 7, 5, 4, 6, 11]). In this experiment $\mathcal{U}_D$ was kept constant and the standard deviation was varied, and this process was followed for different values of $\mathcal{U}_D$. Table 3.3 reports the results of the experiment for three values of $\mathcal{U}_D$. It can be seen from the table that for a given BEV driver (i.e. for a fixed $\mathcal{U}_D$) as the uncertainty increases (i.e. σ increases) the disutility of the optimal path increases, and when the uncertainty increases to a particular value (for instance, $\sigma = 2$ for $\mathcal{U}_D = 20$) optimal path changes to a path with a decreased probability of running out of range.

Let Π_1 denote the path [1, 7, 5, 4, 6, 11], Π_2 denote the path [1, 7, 5, 11], and Π_3 denote the path [1, 3, 5, 11]. The standard deviation corresponding to a change in the optimal path from Π_1 to Π_2 was found for a few values of

Table 3.3: Results of second experiment

$\boldsymbol{\sigma}$	**Optimal Path**	$\hat{\boldsymbol{g}}$	$\boldsymbol{T_\pi}$	$\boldsymbol{D_\pi}$	**Probability of running out of range (%)**
$\boldsymbol{\mathcal{U}_D = 20}$					
0.1	[1, 7, 5, 4, 6, 11]	5.000	5	7	32.7
1	[1, 7, 5, 4, 6, 11]	5.027	5	7	32.7
2	[1, 7, 5, 11]	6.005	6	3	12.2
3	[1, 7, 5, 11]	6.188	6	3	12.2
4	[1, 7, 5, 11]	6.686	6	3	12.2
5	[1, 7, 5, 11]	7.215	6	3	12.2
10	[1, 7, 5, 11]	8.441	6	3	12.2
$\boldsymbol{\mathcal{U}_D = 40}$					
0.1	[1, 7, 5, 4, 6, 11]	5.000	5	7	32.7
1	[1, 7, 5, 4, 6, 11]	5.054	5	7	32.7
2	[1, 7, 5, 11]	6.009	6	3	12.2
3	[1, 7, 5, 11]	6.376	6	3	12.2
4	[1, 7, 5, 11]	7.371	6	3	12.2
5	[1, 7, 5, 11]	8.431	6	3	12.2
10	[1, 7, 5, 11]	10.88	6	3	12.2
$\boldsymbol{\mathcal{U}_D = 50}$					
0.1	[1, 7, 5, 4, 6, 11]	5.000	5	7	32.7
1	[1, 7, 5, 4, 6, 11]	5.067	5	7	32.7
2	[1, 7, 5, 11]	6.012	6	3	12.2
3	[1, 7, 5, 11]	6.470	6	3	12.2
4	[1, 7, 5, 11]	7.714	6	3	12.2
5	[1, 7, 5, 11]	9.039	6	3	12.2
10	[1, 3, 5, 11]	11.90	8	2	7.8

Table 3.4: Standard deviation corresponding to change in optimal path from Π_1 to Π_2

$\mathcal{U}_D$	σ
1	Very Large
5	4.7
10	2.4
20	1.9
30	1.7
40	1.6
50	1.5

$\mathcal{U}_D$ and the same has been shown in Table 3.4. It can be seen that as the risk averse nature of the driver increases the range prediction of the vehicle needs to get better for the optimal path to have lesser travel time. For, a change from Π_2 to Π_3 the required standard deviation was very large for most of the cases, and so the results have not been presented here.

Finally, the running time statistics of the algorithm on some large networks is presented in Table 3.5. The statistics are for implementation on a server with Intel(R) Xeon(R) CPU X5680 @ 3.33GHz with 23.45 GB RAM[4]. The running time is for a transformed network (for a tour with 4 nodes) obtained via the network transformation discussed in section 3.3. Given that the formulation is nonlinear and the network transformation increases the network size (more than 100 thousand links in the transformed Chicago Regional Network) the running times of the algorithm for the networks seem reasonable.

[4]With the transformation described in section 3.3, the number of nodes is linear in the number of stops in the tour. So, 'classical' Dijkstra's algorithm — which is $\mathcal{O}(n^2)$ for an origin, will take $\mathcal{O}(n^2p^2)$ for a tour with n nodes (in the original network) and p stops.

Table 3.5: Running time statistics (in seconds) for OA algorithm

	Original Network		**Transformed Network**		**Running time**
Network	**Nodes**	**Links**	**Nodes**	**Links**	(*seconds*)
Anaheim	416	914	1248	2744	0.23
Winnipeg	1052	2836	3156	8510	0.66
Austin	7388	18954	22164	56864	5.25
Chicago Regional	12982	39018	38946	117056	27.1

3.6 Discussion

In this chapter a network route choice model based on range anxiety has been proposed. A constrained minimum cost tour problem which is more appropriate is considered and is converted to an equivalent shortest path problem on a transformed network. The non-additivity over links of the disutility from running out of range leads to a nonadditive shortest path formulation which is solved using OA algorithm. Numerical experiments have been conducted to demonstrate the effect of the risk attitude and the uncertainty in predicted range on the route choice, and also to check the efficiency of the OA algorithm in solving the model formulation.

The proposed model is very general because it does not assume any particular distribution for the disutility from running out of range i.e. it can be extended to distributions other than the truncated normal which has been used for conducting numerical experiments in this chapter. Lack of survey data is the reason for using the distribution. Extensive surveys need to be carried out to know the form of the distribution and calibrate the model. However, some distributions like the exponential distribution $\left(f(x) = \lambda e^{-\lambda x} \cdot [x > 0]\right)$

are unlikely to correctly describe disutility from running out of range because that would mean that an increase in unit path length would cause more increase in disutility when a driver has a fully charged battery than when the driver is near running out of range. We expect the distribution to be close to an S-shaped curve.

Understanding the decision regarding the choice of routes is indispensable to develop a network model for contemplating the effects of BEV adoption on transportation infrastructure. The proposed model is the key step in developing a network equilibrium model. The next chapter explores how the route choices of different drivers are affected when congestion effects come into play and develops network models for different possible scenarios based on the model developed in this chapter.

Chapter 4

Traffic Assignment

Chapter 3 formulated a route choice model for BEV drivers, which is essential to develop a network model for traffic assignment. This chapter builds on this route choice model and discusses possible extensions.

Traffic assignment is the process of accruing trips on links for all trip interchanges between each origin-destination (OD) pair *(38)*. It is the fourth step in the conventional *four-step forecasting model* (the first three are *trip generation*, *trip distribution*, and *mode choice*) and gives us the pattern of the traffic delay. It accounts for the fact that the cost of trips depends on route choices, and vice-versa.

User equilibrium (UE) and *system optimal* (SO) are the two traditional ways of traffic assignment. While the UE principle tries to predict the expected traffic pattern by assuming that drivers have perfect knowledge of the transportation network and try to minimize travel cost while making travel decisions, the SO traffic assignment produces flows which planners should aim for when designing the network or building infrastructure on existing network. User equilibrium assignment follows *Wardrop's first principle*, which says drivers cannot unilaterally reduce their travel costs by choosing a different route. Beckmann proposed an optimization formulation (popularly known as the *Beckmann formulation*) in the 1950s for solving the UE assignment problem. Optimization formulations are desirable because there exist a large literature on efficient ways to solve an optimization problem. Equation 4.1 presents the Beckmann formulation for the UE assignment. Finding the optimality conditions after Lagrangianizing (equation 4.2) gives us a set of conditions (equation 4.5) which defines the UE principle — every used path between each OD pair has equal and minimal travel cost.

$$
\begin{aligned}
\min_{x,\,h} \quad & \sum_{(i,j)\in A} \int_0^{x_{ij}} t_{ij}(\omega) d\omega \\
\text{subject to} \quad & x_{ij} = \sum_{\pi\in\Pi} \delta_{ij}^{\pi} h^{\pi}, \quad \forall (i,j) \in A \\
& \sum_{\pi\in\Pi^{rs}} h^{\pi} = q^{rs}, \quad \forall (r,s) \in Z^2 \\
& h^{\pi} \geq 0, \quad \forall \pi \in \Pi,
\end{aligned}
\tag{4.1}
$$

where

δ_{ij}^{π} : 0–1 variable to determine if path π use link a_{ij}

h^{π} : flow on path π

Π^{rs} : set of all paths between OD pair (r, s)

q^{rs} : demand between r and s

$Z \subseteq N$: set of all origins and destinations in the network.

Lagrangianizing the second constraint (‘no vehicle left behind’ constraint) in program 4.1 we get

$$\begin{aligned}\min_{h} \quad & \sum_{(i,j)\in A} \int_{0}^{\sum\limits_{\pi\in\Pi}\delta_{ij}^{\pi}h^{\pi}} t_{ij}(\omega)d\omega + \sum_{(r,s)\in Z^2}\kappa_{rs}\left(q^{rs} - \sum_{\pi\in\Pi^{rs}} h^{\pi}\right) \\ \text{subject to} \quad & h^{\pi} \geq 0, \quad \forall \pi \in \Pi,\end{aligned} \tag{4.2}$$

where $\boldsymbol{\kappa}$ is the Lagrange multiplier for the 'no vehicle left behind' constraint.

For a general optimization formulation with nonnegativity constraints, the optimality conditions are given by

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial h^{\pi}} &\geq 0 \quad \forall h^{\pi} \\ h^{\pi} &\geq 0 \quad \forall \pi \in \Pi \\ h^{\pi}\frac{\partial \mathcal{L}}{\partial h^{\pi}} &= 0 \quad \forall \pi \in \Pi,\end{aligned} \tag{4.3}$$

where $\mathcal{L}$ is the Lagrangian of the objective function. For the formulation in program 4.1,

$$\mathcal{L}(\mathbf{h}, \boldsymbol{\kappa}) = \sum_{(i,j)\in A} \int_{0}^{\sum\limits_{\pi\in\Pi}\delta_{ij}^{\pi}h^{\pi}} t_{ij}(\omega)d\omega + \sum_{(r,s)\in Z^2}\kappa_{rs}\left(q^{rs} - \sum_{\pi\in\Pi^{rs}} h^{\pi}\right).$$

Differentiating[1] the Lagrangian $\mathcal{L}(\mathbf{h}, \boldsymbol{\kappa})$ w.r.t. flow h^* on path π^* be-

[1] $\frac{d}{dx}\left(\int_{f_1(x)}^{f_2(x)} g(t)\,dt\right) = g(f_2(x))f_2'(x) - g(f_1(x))f_1'(x)$

tween OD pair (r^*, s^*):

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial h^*} &= \sum_{(i,j)\in A} \frac{\partial}{\partial h^*} \left(\int\limits_0^{\sum\limits_{\pi\in\Pi} \delta_{ij}^{\pi} h^{\pi}} t_{ij}(\omega) d\omega \right) - \kappa_{r^* s^*} \\
&= \sum_{(i,j)\in A} t_{ij} \left(\sum_{\pi\in\Pi} \delta_{ij}^{\pi} h^{\pi} \right) \cdot \delta_{ij}^{\pi^*} - \kappa_{r^* s^*} \\
&= C_{\pi^*} - \kappa_{r^* s^*},
\end{aligned} \tag{4.4}$$

where C_{π^*} is the cost of using path π^*, and $\kappa_{r^* s^*}$ can be interpreted as the cost of using the least cost path of OD pair (r^*, s^*).

Finally we have (for each OD pair (r, s)),

$$\begin{aligned}
C_{\pi} - \kappa_{rs} &\geq 0 \quad \forall \pi \\
h^{\pi} &\geq 0 \quad \forall \pi \\
h^{\pi} \left(C_{\pi} - \kappa_{rs} \right) &= 0 \quad \forall \pi
\end{aligned} \tag{4.5}$$

as the optimization conditions for the UE assignment. The UE assignment principle has been demonstrated using travel time as the travel cost. However, it may be possible to formulate an optimization formulation for a generalized travel cost function — one can work backward from the optimality conditions to get an optimization formulation.

We can see that when link travel costs are fixed, a route choice model like the one in chapter 3 performs this user equilibrium assignment. However with many BEV drivers using the networks, congestion effects are likely to come into play. Due to interactions between network users, travel time and battery charge (and consequently the travel cost) will depend on the traffic state of the network. These drivers will have different risk preferences too, so $\mathcal{U}_D$ will vary across the population. We also need to consider the possibility of different distribution of the range limit — different BEV drivers would have different types of BEVs (for example, BEVs manufactured by different companies) and will face different traffic conditions; and so the distribution of the range limit could be different. Essentially, we need to include the possibility of travel time and battery charge depending on the flow, and variation of risk preferences of BEV drivers and distribution of range limit across the population. Table 4.1 shows some of the possible extensions to the model developed in Chapter 3. The sections that follow discuss some of these extensions based on how the distribution of the range limit and the risk attitude (as measured by $\mathcal{U}_D$) vary across the population of BEV drivers and on the flow dependency of travel time and battery charge (which affects distance traveled). The case where travel time and battery charge is independent of flow and the distribution of the range limit is same across the population has already been dealt with in Chapter 3.

4.1 Models

We could model the variation of risk preferences of BEV drivers and distribution of range limit across the population either as a continuous variation or as discrete set of user classes where each user class in the set has a different behavior in terms of risk preferences and the underlying distribution of range limit. (User classes can be defined by a combination of the distribu-

Table 4.1: Possible extensions (t is link travel time and d is a proxy for battery charge.)

	Same $\mathcal{U}_D$ & distribution	Same $\mathcal{U}_D$, different distribution	Different $\mathcal{U}_D$, same distribution	Different $\mathcal{U}_D$ & distribution
Constant t, d	Chapter 3	section 4.1.1	section 4.1.2	section 4.1.3
Flow dependent t	section 4.1.4	section 4.1.5	section 4.1.5	section 4.1.5
Flow dependent t, d	Future Research	Future Research	Future Research	Future Research

tion of range limit — defined by a location parameter and a scale parameter, and the risk preference of BEV drivers.) Continuous variations will allow us to develop models which are not very sensitive to scaling of the network. It may seem that developing a model which allows all these variations would be the best thing to do, but it may not be useful and worth the extra complexity in certain scenarios — "Essentially, all models are wrong, but some are useful."[2] The following subsections describe optimization formulations for some cases, and comments on the possible ways to model others.

[2]A quote by George Edward Pelham Box from his book *Response surface methodology* with Norman R. Draper.

4.1.1 Constant t and d; same $\mathcal{U}_D$; discrete sets of distribution of range limit

As described earlier, let the generalized travel cost be given by

$$\hat{g}(T_\pi, D_\pi) = \begin{cases} \alpha T_\pi + \mathcal{U}_D \cdot F_{\tilde{R}}(D_\pi) & D_\pi \leq D^* \\ \alpha T_\pi + \mathcal{U}_D \cdot F_{\tilde{R}}(D^*) + \mathcal{U}_D \cdot f_{\tilde{R}}(D^*)(D_\pi - D^*) & \text{otherwise} \end{cases}. \tag{4.6}$$

Let the variation of the distribution of the range limit across the population be described by a finite set L whose elements are an ordered pair of a location parameter and a scale parameter. Each element (or user class) $l \in L$ then describes a different distribution. Let $F^l_{\tilde{R}}$ and $f^l_{\tilde{R}}$ denote the CDF and the PDF of the distribution for user class l, respectively. Since t and d are constant, we can treat each user class of BEV drivers (following a particular distribution) differently and solve the nonadditive shortest path problem for each class independently to get the traffic pattern of the network. For each $l \in L$ we have

$$\begin{aligned} \min \quad & \sum_{\pi \in \Pi} \hat{g}^l(T_\pi, D_\pi) \\ \text{subject to} \quad & \sum_{\{j: a_{ij} \in E\}} x^l_{ij} - \sum_{\{j: a_{ji} \in E\}} x^l_{ji} = \begin{cases} 1 & i = r \\ -1 & i = s \\ 0 & \text{otherwise} \end{cases} \\ & x^l_{ij} \in \{0, 1\}, \quad \forall a_{ij} \in E, \end{aligned} \tag{4.7}$$

with,

$$\hat{g}^l(T_\pi, D_\pi) = \begin{cases} \alpha T_\pi + \mathcal{U}_D \cdot F^l_{\tilde{R}}(D_\pi) & D_\pi \leq D^* \\ \alpha T_\pi + \mathcal{U}_D \cdot F^l_{\tilde{R}}(D^*) + \mathcal{U}_D \cdot f^l_{\tilde{R}}(D^*)(D_\pi - D^*) & \text{otherwise} \end{cases}. \tag{4.8}$$

The link flows can then be obtained by

$$x_{ij} = \sum_{l \in L} q^l x_{ij}^l, \tag{4.9}$$

where q^l denotes the number of BEV drivers that belong to user class l. So, we need to just solve the nonadditive shortest path problem for each user class and sum the 'weighted' link flows to obtain the link flow solution.

4.1.2 Constant t and d; discrete sets of $\mathcal{U}_D$; same distribution of range limit

This case is similar to the one above (section 4.1.1). Let the generalized travel cost be given by

$$\hat{g}(T_\pi, D_\pi) = \begin{cases} \alpha T_\pi + \mathcal{U}_D \cdot F_{\tilde{R}}(D_\pi) & D_\pi \leq D^* \\ \alpha T_\pi + \mathcal{U}_D \cdot F_{\tilde{R}}(D^*) + \mathcal{U}_D \cdot f_{\tilde{R}}(D^*)(D_\pi - D^*) & \text{otherwise} \end{cases}. \tag{4.10}$$

Let the variation of the risk attitudes $\mathcal{U}_D$ across the population be described by a finite set H. Each user class $h \in H$ then describes a group of BEV drivers with a different risk preference. Let $\mathcal{U}_D^h$ denote the risk preference of user class h. Since t and d are constant, we can treat each user class of BEV drivers differently and solve the nonadditive shortest path problem for each class independently to get the flow pattern. For each $h \in H$

$$\begin{aligned} \min \quad & \sum_{\pi \in \Pi} \hat{g}^h(T_\pi, D_\pi) \\ \text{subject to} \quad & \sum_{\{j: a_{ij} \in E\}} x_{ij}^h - \sum_{\{j: a_{ji} \in E\}} x_{ji}^h = \begin{cases} 1 & i = r \\ -1 & i = s \\ 0 & \text{otherwise} \end{cases} \\ & x_{ij}^h \in \{0, 1\}, \quad \forall a_{ij} \in E, \end{aligned} \tag{4.11}$$

with,

$$\hat{g}^h(T_\pi, D_\pi) = \begin{cases} \alpha T_\pi + \mathcal{U}_D^h \cdot F_{\tilde{R}}(D_\pi) & D_\pi \le D^* \\ \alpha T_\pi + \mathcal{U}_D^h \cdot F_{\tilde{R}}(D^*) + \mathcal{U}_D^h \cdot f_{\tilde{R}}(D^*)(D_\pi - D^*) & \text{otherwise} \end{cases}. \tag{4.12}$$

The link flows can then be obtained by

$$x_{ij} = \sum_{h \in H} q^h x_{ij}^h, \tag{4.13}$$

where q^h denotes the number of BEV drivers that belong to user class h. So, we need to just find the nonadditive shortest path for each user class and sum the 'weighted' link flows to obtain the link flow solution.

4.1.3 Constant t and d; discrete sets of $\mathcal{U}_D$; discrete sets of distribution of range limit

This case is again similar to the two above (section 4.1.1 and section 4.1.2). To formulate the optimization problem for this case we just need to create a new set of user classes $W = L \times H$. This set encompasses all possible combinations of risk attitude $\mathcal{U}_D$, the location parameter, and the scale parameter of the distribution. Let $F_{\tilde{R}}^w$ and $f_{\tilde{R}}^w$ denote the CDF and the PDF of the distribution for user class w, respectively, and let $\mathcal{U}_D^w$ denote the risk preference of user class w. The formulation then for each user class $w \in W$ is

$$\begin{aligned} \min \quad & \sum_{\pi \in \Pi} \hat{g}^w(T_\pi, D_\pi) \\ \text{subject to} \quad & \sum_{\{j: a_{ij} \in E\}} x_{ij}^w - \sum_{\{j: a_{ji} \in E\}} x_{ji}^w = \begin{cases} 1 & i = r \\ -1 & i = s \\ 0 & \text{otherwise} \end{cases} \\ & x_{ij}^w \in \{0, 1\}, \quad \forall a_{ij} \in E, \end{aligned} \tag{4.14}$$

with,

$$\hat{g}^w(T_\pi, D_\pi) = \begin{cases} \alpha T_\pi + \mathcal{U}_D^w \cdot F_{\tilde{R}}^w(D_\pi) & D_\pi \leq D^* \\ \alpha T_\pi + \mathcal{U}_D^w \cdot F_{\tilde{R}}^w(D^*) + \mathcal{U}_D^w \cdot f_{\tilde{R}}^w(D^*)(D_\pi - D^*) & \text{otherwise} \end{cases}. \tag{4.15}$$

The link flows can then be obtained by

$$x_{ij} = \sum_{w \in W} q^w x_{ij}^w, \tag{4.16}$$

where q^w denotes the number of BEV drivers that belong to user class w. The final link flows can therefore be obtained by just solving the nonadditive shortest path problem for each user class and doing a weighted sum of the link flow solutions obtained from each problem.

4.1.4 t is flow dependent; d is constant; same $\mathcal{U}_D$ and same distribution

Let the generalized cost be given by (for simplicity we assume $\alpha = 1$)

$$\hat{g}(T_\pi, D_\pi) = \begin{cases} T_\pi + \mathcal{U}_D \cdot F_{\tilde{R}}(D_\pi) & D_\pi \leq D^* \\ T_\pi + \mathcal{U}_D \cdot F_{\tilde{R}}(D^*) + \mathcal{U}_D \cdot f_{\tilde{R}}(D^*)(D_\pi - D^*) & \text{otherwise} \end{cases}. \tag{4.17}$$

Let

$$G(D_\pi) = \begin{cases} F_{\tilde{R}}(D_\pi) & D_\pi \leq D^* \\ F_{\tilde{R}}(D^*) + f_{\tilde{R}}(D^*)(D_\pi - D^*) & \text{otherwise} \end{cases}. \tag{4.18}$$

Then the generalized cost can be written as

$$\hat{g}(T_\pi, D_\pi) = \sum_{(i,j) \in A} t_{ij}(x_{ij}) \cdot \delta_{ij}^\pi + \mathcal{U}_D \cdot G(D_\pi). \tag{4.19}$$

The UE formulation for this case can be written as:

$$\min_{h} \sum_{(i,j)\in A} \int_{0}^{\sum_{\pi\in\Pi} \delta_{ij}^{\pi} h^{\pi}} t_{ij}(\omega) d\omega + \sum_{\pi\in\Pi} h^{\pi} \left(\mathfrak{U}_D \cdot G(D_{\pi})\right) + \sum_{(r,s)\in Z^2} \kappa_{rs} \left(q^{rs} - \sum_{\pi\in\Pi^{rs}} h^{\pi} \right) \quad (4.20)$$

subject to $h^{\pi} \geq 0, \quad \forall \pi \in \Pi.$

For a given path π^*

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial h^{\pi^*}} &= \sum_{(i,j)\in A} t_{ij} \left(\sum_{\pi\in\Pi} \delta_{ij}^{\pi} h^{\pi} \right) \cdot \delta_{ij}^{\pi^*} + \mathfrak{U}_D \cdot G(D_{\pi^*}) - \kappa_{r^*s^*} \\ &= C_{T_{\pi^*}} + C_{D_{\pi^*}} - \kappa_{r^*s^*} \\ &= C_{\pi^*} - \kappa_{r^*s^*}, \end{aligned} \quad (4.21)$$

where

$$\mathcal{L}(\mathbf{h}, \boldsymbol{\kappa}) = \sum_{(i,j)\in A} \int_{0}^{\sum_{\pi\in\Pi} \delta_{ij}^{\pi} h^{\pi}} t_{ij}(\omega) d\omega + \sum_{\pi\in\Pi} h^{\pi} \left(\mathfrak{U}_D \cdot G(D_{\pi})\right) + \sum_{(r,s)\in Z^2} \kappa_{rs} \left(q^{rs} - \sum_{\pi\in\Pi^{rs}} h^{\pi} \right) \quad (4.22)$$

and $C_{\pi^*} = C_{T_{\pi^*}} + C_{D_{\pi^*}}$.

Equation 4.21 shows that the nonadditive shortest path problem can be used as a sub-problem for the TA problem. Any path-based algorithm (like gradient projection) can be used to solve for the equilibrium flows.

4.1.5 Flow dependent t; d is constant; discrete sets of $\mathcal{U}_D$; same distribution of range limit

Let the variation of the risk attitudes $\mathcal{U}_D$ across the population be described by a finite set W so that $\mathcal{U}_D^w$ represents the risk preference of a driver of user class $w \in W$. Let the generalized cost for this user class be given by (assuming $\alpha = 1$)

$$\hat{g}^w(T_\pi, D_\pi) = \begin{cases} T_\pi + \mathcal{U}_D^w \cdot F_{\tilde{R}}(D_\pi) & D_\pi \leq D^* \\ T_\pi + \mathcal{U}_D^w \cdot F_{\tilde{R}}(D^*) + \mathcal{U}_D^w \cdot f_{\tilde{R}}(D^*)(D_\pi - D^*) & \text{otherwise} \end{cases}. \tag{4.23}$$

Let

$$G(D_\pi) = \begin{cases} F_{\tilde{R}}(D_\pi) & D_\pi \leq D^* \\ F_{\tilde{R}}(D^*) + f_{\tilde{R}}(D^*)(D_\pi - D^*) & \text{otherwise} \end{cases}. \tag{4.24}$$

Then we can write,

$$\hat{g}^w(T_\pi, D_\pi) = \sum_{(i,j)\in A} t_{ij}(x_{ij}) \cdot \delta_{ij}^\pi + \mathcal{U}_D^w \cdot G(D_\pi). \tag{4.25}$$

Let $h^\pi = \sum\limits_{w\in W} h^{\pi,w}$, where $h^{\pi,w}$ is no. of users with risk preference $\mathcal{U}_D^w$ on path π.

For formulating a UE for this case we need to virtually separate flows on a given path by their user class. The UE formulation for this case can be formulated as:

$$\min_h \sum_{(i,j)\in A} \int\limits_0^{\sum\limits_{\pi\in\Pi}\sum\limits_{w\in W}\delta_{ij}^\pi h^{\pi,w}} t_{ij}(\omega)d\omega + \sum_{\pi\in\Pi}\sum_{w\in W} h^{\pi,w}\left(\mathcal{U}_D^w \cdot G(D_\pi)\right) + \sum_{(r,s)\in Z^2} \kappa_{rs}\left(q^{rs} - \sum_{\pi\in\Pi^{rs}}\sum_{w\in W} h^{\pi,w}\right) \tag{4.26}$$

subject to $h^\pi \geq 0, \quad \forall \pi \in \Pi.$

For a given path π^* and user class w

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial h^{\pi^*,w}} &= \sum_{(i,j)\in A} t_{ij}\left(\sum_{\pi\in\Pi} \delta_{ij}^{\pi} h^{\pi}\right) \cdot \delta_{ij}^{\pi^*} + \mathcal{U}_D^w \cdot G(D_{\pi^*}) - \kappa_{r^*s^*} \\ &= C_{T_{\pi^*,w}} + C_{D_{\pi^*,w}} - \kappa_{r^*s^*} \end{aligned} \tag{4.27}$$

where

$$\begin{aligned} \mathcal{L}(\mathbf{h}, \boldsymbol{\kappa}) = \sum_{(i,j)\in A} \int_0^{\sum\limits_{\pi\in\Pi} \delta_{ij}^{\pi} h^{\pi}} t_{ij}(\omega) d\omega + \sum_{\pi\in\Pi} h^\pi \left(\mathcal{U}_D \cdot G(D_\pi)\right) \\ + \sum_{(r,s)\in Z^2} \kappa_{rs} \left(q^{rs} - \sum_{\pi\in\Pi^{rs}} h^\pi\right). \end{aligned} \tag{4.28}$$

The case where we have discrete sets of distribution of range limit while the risk attitude is same throughout the population, and the case where we have discrete sets of risk preferences and discrete sets of distribution of range limit can be handled similarly.

4.2 Discussion

The battery consumption rate of BEVs also depends on the speed of the vehicle, and consequently the equilibrium flows. Therefore, it is essential to develop an equilibrium model which accounts for the flow dependency of the rate of battery consumption. He et al. *(33)* developed an equilibrium model with flow-dependent energy consumption by relating energy consumption of the battery to the travel time. However, their model was a *nonlinear complementarity problem* (NCP). It may be possible to develop a NCP for the framework discussed in this research, but an optimization problem[3] would be a better option and will be explored in future research.

The only other case for which a model has not been developed in this work is one which allows continuous variation of risk preferences and the parameters of distribution of range limit across the population. Preliminary ideas based on the work of Dial *(38)* is in works and will be explored in future research. He developed a bicriterion equilibrium traffic assignment model that allows the value of time parameter to vary continuously across the population. The problem that he solves is similar to the case which allows continuous variation of risk preferences while the distribution of range limit stays the same; the only difference is that he assumes a linear generalized cost, while we have a nonlinear travel cost. The idea hinges on the fact that the OA algorithm does linear approximation to the nonlinear objective function to find the optimal solution, and hence it may be possible to formulate an optimization problem based on Dial's idea.

[3]He et al. *(33)* mentions that the NCP can be converted to a nonlinear optimization problem through a gap function.

Chapter 5

Conclusions

5.1 Summary

In this thesis, a nonadditive shortest path problem was proposed to model the route choice behavior of BEV drivers. The model relaxed some major assumptions of earlier literature — all BEVs have the same range limit and all BEV drivers have the same reserve range. It also allows for a continuum of range limits and incorporates nonlinear travel preferences of drivers to allow a smoother transition in the response of drivers. Since it is hard to quantify the impact of battery charge and the heterogeneity of drivers' preferences, the range limit was modeled to be a random variable. Justified approximations were made to make the objective function convex, and OA algorithm was used to solve the problem. Numerical experiments were performed which demonstrated that drivers who are more risk averse choose routes to minimize probability of running out of range, and as drivers get more risk averse the optimal path changes so that the path length decreases at the cost of an increased travel time. Also, as uncertainty increases the disutility of the optimal path increases. Efficiency of the OA algorithm to solve the model was also demonstrated. The proposed model is very general and can be extended to distributions other than the one considered in the numerical experiments.

Extensive surveys will be required to calibrate the model. Optimization formulations were proposed for extension of the route choice model to model the congestion effects and the effects of interaction of multiple BEV drivers with different travel preferences.

Some of the possible extensions mentioned in Chapter 4 were not discussed and will be investigated in future research . Distance was used as a proxy for battery charge for network equilibrium models when the charge consumed was taken to be dependent on traffic flow. It might be practical and easier to use battery charge instead and make it dependent on the travel time. Survey data will also be required to quantify this relation. The research in this thesis is the first step to develop a more general network equilibrium model. It is a critical next step towards improved guidance to policy makers. It will help planners in their decision making process. Network equilibrium model for example can help determine optimal location for charging stations. Utility models can also be developed for setting prices at charging stations. While many route choice models have been proposed in literature for BEVs, the non-additivity of link costs in this research distinguishes it from most earlier research literature. The nonadditive shortest path problem has important applications in transportation models especially nonlinear models like emission models. While we emphasized user equilibrium flows, system optimal models will also need to be developed for congestion pricing and other pricing related policies.

5.2 Future Scope

One of the major assumptions for the route choice model was that BEV drivers charge their vehicles at the start of their tour. This assumption may be reasonable for now because of the current battery technology and limited refueling opportunities (long charging times and low density of charging stations). But in future, battery technology will improve and more charging stations are going to be installed. Travel models should then allow for relay locations. Duration of stay at charging stations will also become important. Quick battery exchange stations have been established, which allow drivers to quickly replace almost drained batteries with fully charged ones *(5)*. However, standardization issues may arise and it may take a long time before this option becomes practical and reliable *(31)*. Zhang et al. *(32)* also considered departure time choice in their variational inequality network formulation. They change the choice of charging station to a route choice using an expanded network with charging station as nodes, and their models takes the variation of charging prices and waiting costs at charging stations into account.

The models developed in this thesis is for BEV drivers only. Large scale adoption of BEVs is a future scenario. Until then, households will own both GVs and BEVs (or probably a hybrid). So, it is essential to explore appropriate models for mixed equilibrium flows of BEVs and GVs.

We know that the shortest path problem is a subproblem for the Beckmann formulation. However, the assumption that drivers have perfect knowledge of the travel times may not hold and there might be stochasticity in the perception of travel times. So, a stochastic user equilibrium, which is generally considered to be more suitable to reflect drivers' behavior might need to be explored as well. That involves developing a k-shortest path version of the problem.

A nonadditive shortest path problem was proposed in this thesis to model the route choice of BEV drivers. While all the possible extensions to it were not discussed, this research laid the foundations for nonlinear network models for BEVs. It will hopefully inspire and initiate more research to develop nonlinear travel models for BEVs.

Bibliography

[1] Zhenhong Lin and David L Greene. Promoting the market for plug-in hybrid and battery electric vehicles. *Transportation Research Record: Journal of the Transportation Research Board*, 2252(1):49–56, 2011.

[2] Nan Jiang and Chi Xie. Computing and analyzing mixed equilibrium network flows with gasoline and electric vehicles. *Computer-Aided Civil and Infrastructure Engineering*, 29(8):626–641, 2014.

[3] Ella Graham-Rowe, Benjamin Gardner, Charles Abraham, Stephen Skippon, Helga Dittmar, Rebecca Hutchins, and Jenny Stannard. Mainstream consumers driving plug-in battery-electric and plug-in hybrid electric cars: A qualitative analysis of responses and evaluations. *Transportation Research Part A: Policy and Practice*, 46(1):140–153, 2012.

[4] Nan Jiang, Chi Xie, and S Travis Waller. Path-constrained traffic assignment: model and algorithm. *Transportation Research Record: Journal of the Transportation Research Board*, 2283(1):25–33, 2012.

[5] Jonathan D Adler, Pitu B Mirchandani, Guoliang Xue, and Minjun Xia. The electric vehicle shortest-walk problem with battery exchanges. *Networks and Spatial Economics*, pages 1–19, 2012.

[6] Nan Jiang, Chi Xie, Jennifer C Duthie, and S Travis Waller. A network equilibrium analysis on destination, route and parking choices with mixed gasoline and electric vehicular flows. *EURO Journal on Transportation and Logistics*, pages 1–38, 2013.

[7] Ehsan Jafari and Stephen Boyles. Network contraction methods for dynamic pricing at charging stations. Presented at the Annual Meeting of the Institute for Operations Research and Management Sciences, San Francisco, CA, 2014.

[8] Kevin Morrow, Donald Karner, and James Francfort. Plug-in hybrid electric vehicle charging infrastructure review. *Battelle Energy Alliance*, 2008.

[9] J Bakker. Contesting range anxiety: The role of electric vehicle charging infrastructure in the transportation transition. *Eindhoven University of Technology*, 2011.

[10] Sudesh K Agrawal, Stephen D Boyles, Nan Jiang, Mehrdad Shahabi, and Avinash Unnikrishnan. Network route choice model for battery electric vehicle drivers with different risk attitudes. *Transportation Research Record: Journal of the Transportation Research Board*, (2498):75–83, 2015.

[11] Lauren M Gardner, Melissa Duell, and S Travis Waller. A framework for evaluating the role of electric vehicles in transportation network infrastructure under travel demand variability. *Transportation Research Part A: Policy and Practice*, 49:76–90, 2013.

[12] David A Hensher and Truong P Truong. Valuation of travel time savings: a direct experimental approach. *Journal of Transport Economics and Policy*, pages 237–261, 1985.

[13] Abdul Rawoof Pinjari and Chandra Bhat. Nonlinearity of response to level-of-service variables in travel mode choice models. *Transportation Research Record: Journal of the Transportation Research Board*, 1977(1): 67–74, 2006.

[14] Steven A Gabriel and David Bernstein. The traffic equilibrium problem with nonadditive path costs. *Transportation Science*, 31(4):337–348, 1997.

[15] Zhenhong Lin, Jing Dong, Changzheng Liu, and David Greene. Estimation of energy use by plug-in hybrid electric vehicles. *Transportation Research Record: Journal of the Transportation Research Board*, 2287(1): 37–43, 2012.

[16] Anthony Chen, Zhaowang Ji, and Will Recker. Travel time reliability with risk-sensitive travelers. *Transportation Research Record: Journal of the Transportation Research Board*, 1783(1):27–33, 2002.

[17] Suvrajeet Sen, Rekha Pillai, Shirish Joshi, and Ajay K Rathi. A mean-variance model for route guidance in advanced traveler information systems. *Transportation Science*, 35(1):37–49, 2001.

[18] Raj A Sivakumar and Rajan Batta. The variance-constrained shortest path problem. *Transportation Science*, 28(4):309–316, 1994.

[19] Mark Ziegelmann. *Constrained shortest paths and related problems*. PhD thesis, Universitätsbibliothek, 2001.

[20] Gabriel Y Handler and Israel Zang. A dual algorithm for the constrained shortest path problem. *Networks*, 10(4):293–309, 1980.

[21] Martin Desrochers and François Soumis. A generalized permanent labeling algorithm for the shortest-path problem with time windows. *Infor*, 26 (3):191–212, 1988.

[22] Jin Y Yen. Finding the k shortest loopless paths in a network. *management Science*, 17(11):712–716, 1971.

[23] Ernesto de Queiros Vieira Martins. An algorithm for ranking paths that may contain cycles. *European Journal of Operational Research*, 18(1): 123–130, 1984.

[24] Irina Dumitrescu and Natashia Boland. Algorithms for the weight constrained shortest path problem. *International Transactions in Operational Research*, 8(1):15–29, 2001.

[25] Irina Dumitrescu and Natashia Boland. Improved preprocessing, labeling and scaling algorithms for the weight-constrained shortest path problem. *Networks*, 42(3):135–153, 2003.

[26] Andreas Artmeier, Julian Haselmayr, Martin Leucker, and Martin Sachenbacher. The shortest path problem revisited: Optimal routing for

electric vehicles. In *KI 2010: Advances in Artificial Intelligence: 33rd Annual German Conference on AI, Karlsruhe, Germany, September 21-24, 2010, Proceedings*, volume 6359, page 309. Springer Science & Business Media, 2010.

[27] Peng Will Chen and Yu Marco Nie. Bicriterion shortest path problem with a general nonadditive cost. *Procedia-Social and Behavioral Sciences*, 80:553–575, 2013.

[28] George Tsaggouris and Christos Zaroliagis. Non-additive shortest paths. In *Algorithms–ESA 2004*, pages 822–834. Springer, 2004.

[29] Mehrdad Shahabi, Avinash Unnikrishnan, and Stephen D Boyles. An algorithm for non-additive shortest path problem. In *Transportation Research Board 93rd Annual Meeting*, number 14-5178, 2014.

[30] Chi Xie, Xing Wu, and Stephen Boyles. Network equilibrium of electric vehicles with stochastic range anxiety. In *Intelligent Transportation Systems (ITSC), 2014 IEEE 17th International Conference on*, pages 2505–2510. IEEE, 2014.

[31] Ti Zhang, Chi Xie, and S Travis Waller. Network flows of plug-in electric vehicles: Impacts of electricity-charging price. In *Transportation Research Board 92nd Annual Meeting*, number 13-2611, 2013.

[32] Ti Zhang, Stephen Boyles, and S Travis Waller. Modeling combined travel choices of electric vehicle drivers with a variational inequality network

formulation. In *Transportation Research Board 92nd Annual Meeting*, number 13-2619, 2013.

[33] Fang He, Yafeng Yin, and Siriphong Lawphongpanich. Network equilibrium models with battery electric vehicles. *Transportation Research Part B: Methodological*, 67:306–319, 2014.

[34] Ankita Chaudhary, Chandra R Bhat, Jennifer Duthie, and Subodh Dubey. Impact of range anxiety on driver route choices using a panel-integrated choice latent variable model. Working paper supported by EV-TEC Center (now known as EV-STS Center).

[35] Mehrdad Shahabi, Avinash Unnikrishnan, and Stephen D Boyles. An outer approximation algorithm for the robust shortest path problem. *Transportation Research Part E: Logistics and Transportation Review*, 58: 52–66, 2013.

[36] Mehrdad Shahabi, Avinash Unnikrishnan, and Stephen D Boyles. Robust optimization strategy for the shortest path problem under uncertain link travel cost distribution. *Computer-Aided Civil and Infrastructure Engineering*, in press 2014.

[37] Hillel Bar-Gera. Transportation test problems. `http://www.bgu.ac.il/~bargera/tntp/`, 2001. Accessed July 2014.

[38] Robert B Dial. Bicriterion traffic assignment: basic theory and elementary algorithms. *Transportation Science*, 30(2):93–111, 1996.