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Getting to Work on Time:
A Proposed Time-Equitable Tolling Scheme

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Getting to Work on Time:
A Proposed Time-Equitable Tolling Scheme

by

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Dedication

This thesis is dedicated to my parents, Leta and Mark, who daily show me what it is to love.

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Abstract

Getting to Work on Time:
A Proposed Time-Equitable Tolling Scheme

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Dynamic tolls present an opportunity for municipalities to eliminate congested roadways and fund infrastructure. Known variously as congestion pricing or value pricing and implemented through dynamically priced toll lanes and cordon charges, planners have tried to create free flowing conditions by charging higher prices for travelers who wish to travel in the middle of the peak period. Because these tolls vary in response to demand, travelers are forced to more explicitly consider the cost of travel and so may choose to forgo it entirely or to shift the time of travel for trips that are less important.

The imposition of tolls that regulate travel along a public highway through the use of a monetary fee raises worries of inequity. This thesis is thematically divided into two projects. The first is a qualitative investigation into equity. The objective of this discussion is to provide a framework for *why* we value equity in order to explain whether new policies (without established legal guidance) are inequitable. This explicitly normative discussion draws on work by the philosopher John Rawls to argue that equity

concerns in transportation are primarily rooted in a desire to respect all travelers equally and that time poverty ought to be considered in policy-making in the same ways that income poverty already is. I then argue that tolling schemes (by which I mean any structured description of a time-varying toll) that produce time-poverty among poorer travelers ought to be examined as a potential equity concern.

The second project is the application of the qualitative equity investigation to a particular implementation of a time-varying toll in the Vickrey bottleneck model to examine whether it raises equity concerns. To achieve this end, I selected an analytically tractable example of the Vickrey bottleneck that eliminates congestion through targeting each traveler's value of time. I compare and contrast the cost burdens of a no-toll, system optimal toll, and what I will call a "time-equitable" toll on homogeneous and heterogeneous traveler groups. I will show that the time-equitable toll is able to eliminate congestion while creating equitable travel patterns amongst traveler groups.

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“Transportation doesn’t hire you. But it makes getting to work possible.
It doesn’t treat illness, but it makes getting healthcare possible.
It doesn’t teach, but it makes learning possible.
It’s not a house, but try getting to yours without it.”

Anthony Foxx, U.S. Secretary of Transportation
Address to the 2016 Transportation Research Board
(Foxx, 2016)

Chapter 1: Introduction

The US road network is not overwhelmed because too many people want to travel. It is overwhelmed because too many people want to travel at the same time. The congestion that arises from too many travelers using the same roads to reach similar destinations within a constrained time period, e.g. morning rush hour, imposes severe costs on all commuters. As roads become more congested, the marginal congestion created by an additional vehicle increases and is primarily experienced by other drivers (rather than the individual joining the corridor). In 2014, the Texas Transportation Institute estimated that congestion delayed American travelers by 6.9 billion hours, wasted 3.1 billion gallons of fuel, and cost the country $160 billion – $960 per auto commuter (Schrank, Eisele, Lomax, & Bak, 2015). Declining receipts from traditional funding sources have restricted the growth and rehabilitation of infrastructure and so transportation planners and engineers are increasingly turning to innovative approaches in order to control congestion and raise revenues.

An increasingly popular option being considered around the world is to charge commuters for traveling at peak periods. Known variously as congestion pricing or value pricing and implemented through dynamically priced toll lanes and cordon charges, planners have tried to create free flowing conditions by charging higher prices for travelers who wish to travel in the middle of the peak period. Because these tolls vary in response to demand, travelers are forced to more explicitly consider the cost of travel and so may choose to forgo it entirely or to shift the time of travel for trips that are less important. Although most implementations of these tolling schemes are solely interested in their ability to efficiently raise revenue to pay for infrastructure, congestion relief has been an enduring theme of the time-varying toll literature.

The imposition of tolls that regulate travel along a public highway through the use of a monetary fee intuitively raises worries of inequity. The policy and theoretical background of equity will be discussed in later sections, but for now it is enough to note that it seems likely that predicating access to roads at peak periods on one's ability to pay will decrease the likelihood that poorer travelers will choose to travel at peak periods. If nondiscriminatory access to public roads is a public good, then this observation is a prima facie, though defeasible, case against such policies. In particular, research on time poverty by sociologists raises worries that there are significant harms to low-income travelers associated with being segregated to off peak travel over and above any concerns raised about the monetary costs of the tolls.

This thesis is thematically divided into two projects, and hence has two argumentative chapters. The first project, contained in Chapter 2, is a qualitative investigation into equity. The objective of this discussion is to provide a framework for why we value equity in order to explain whether new policies (without established legal guidance) are inequitable. This explicitly normative discussion draws on work by the philosopher John Rawls to argue that equity concerns in transportation are primarily rooted in a desire to respect all travelers equally and that time poverty ought to be considered in policy-making in the same ways that income poverty already is. I then argue that tolling schemes (by which I mean any structured description of a time-varying toll) that produce time-poverty among poorer travelers ought to be examined as a potential equity concern.

The second project, found in Chapter 3, is the application of the qualitative equity investigation to a particular implementation of a time-varying toll to examine whether it raises equity concerns. To achieve this end, I selected an analytically tractable example of the Vickrey bottleneck that eliminates congestion through targeting each traveler's value of time. I compare and contrast the cost burdens of a no-toll, system optimal toll, and what I will call a "time-equitable" toll on homogeneous and heterogeneous traveler groups. I will show that the time-equitable is able to eliminate congestion while creating equitable travel patterns amongst traveler groups.

As will become clear in Chapter 3, the dynamic tolling scheme derived from the Vickrey bottleneck is highly constrained by the assumptions governing traveler preferences and the complete knowledge of preferences of all travelers by both the toll agency and the individual travelers. I nonetheless believe there is significant value to be gained from working closely with this deliberately constrained model for three reasons.

First, its deterministic view of traveler behavior and preferences allows for the formulation of closed form solutions and simple replication of results. Second, a tolling scheme is an expression of values. Whether a tolling scheme is optimized to reduce congestion, pollution, or traveling costs for the poor it expresses a value judgment for how the goods of travel should be allocated and who should bear the costs. Using a deliberately simple traveler model allows us to bring this value judgment to the fore and to better understand how the results are affected by changes in the parameters. I hope that this thesis will help to spark a debate on what objectives we should prioritize in a tolling scheme that can then later inform empirical research into how those objectives may be fulfilled. Third, big data analytical techniques bring us ever closer to a world where the full information assumptions are actually realized. It will be possible to micro-target individuals to gain a fine-grained estimate of their willingness to pay and so we should have an informed debate over tolling policy objectives before these schemes are implemented by industry.

1.1 A Brief History of Toll Roads in the United States

Congestion tolls represent a significant change in the way we determine access to public roads. Beginning in 1795 with the Philadelphia and Lancaster Turnpike and continuing to today, almost all roads in the United States are either free or require the payment of a fixed toll. There are 6,088 miles of toll roads in the United States (Federal Highway Administration Office of Highway Policy Information, 2016) that are tolled for three main reasons (Giuliano, 1992). The first kind of toll is the fee charged to cross a bridge or travel along a road that is intended only to recoup the costs of construction and maintenance. The Golden Gate Bridge, Brooklyn Bridge and roads like State Route 400 in Georgia and the Pennsylvania Turnpike were originally tolled only to pay back construction costs though tolls were continued to pay for maintenance or the construction of other road facilities. The tolls along these routes are insensitive to demand; they were not optimized to achieve either profit maximization or congestion relief. A second kind of toll, the cordon toll, is designed to either decrease emissions, raise revenue, or to generally discourage traffic through a fixed rate that is not sensitive to real time information about network congestion. Although New York City considered and rejected a cordon toll, current implementations of this kind of toll are found around the world, in

Milan's Area C, London, and Singapore (FHWA, 2008). Finally, there are dynamically priced tolls that explicitly respond to changes in demand either to maximize profits, or to minimize congestion, or both. The LBJ TEXpress Lanes in Dallas, TX and Interstate Highway 495 in Washington, D.C. are both examples of this this latter structure.

1.2 A SURVEY OF CURRENT TOLL ROAD RESEARCH

Contemporary research on the dynamic pricing of toll roads has largely been concentrated on their ability to effectively relieve congestion, improve environmental pollution measures, and maximize profits for the road operator. Results have been dramatic even when toll rates are static. Cordon tolls in Sweden that increase over the course of the peak period have reduced congestion by 22% and enjoy widespread public support (Sørensen, Isaksson, Macmillen, Åkerman, & Kressler, 2014). A survey of European congestion tolls saw 14%-23% decreases in vehicle counts in Milan and Bologna (respectively); 30% and 33% decreases in delay times in London and Stockholm; CO2 emissions were cut by 13% in Stockholm and 21% in Rome; and collision rates in Milan (the only city in the study to properly track accident rates) declined by 14% (May, Koh, Blackledge, Humphrey, & Fioretto, 2009). These effects are predictable and reproducible across a wide variety of contexts, because consumer elasticity towards tolls has been found to stably vary between -0.2 and -0.8 across a wide variety of research (Wuesterfeld & Regan, 1981; White, 1984; Ribas, Raymond, & Matas, 1988; Jones & Hervik, 1992; Hirschman, Mcknight, Pucher, Paaswell, & Berechman, 1995; Gifford & Talkington, 1996; Odeck & Bråthen, 2008; Odeck & Kjerkreit, 2011).

The effectiveness of tolls at reducing congestion has led to research in a number of new areas, including setting optimal tolls to maximize revenue (Verhoef, Nijkamp, & Rietveld, 1996; Joksimovic, Bliemer, & Bovy, 2005; Bertsimas & Perakis, 2006), relieve congestion at minimal cost (Dial, 1999; de Palma, Kilani, & Lindsey, 2005), or reduce emissions (Harrington, Krupnick, & Alberini, 2001; Hensher, 2008; Rotaris, Danielis, Marcucci, & Massiani, 2010). Some authors have even introduced equitable tolling requirements as a constraint in the bi-level network optimization problem (Paleti, He, & Peeta, 2016).

Real world practitioners are keenly aware of the equity pitfalls congestion tolls present. Public support for value pricing in a number of projects has been highly responsive to the claim that the toll roads benefit wealthy travelers at the expense of the poor. The popular moniker "Lexus Lanes" for high occupancy toll lanes bears witness to this fact. Adequately addressing equity concerns has propelled the success of a number of projects while ignoring equity has led to significant public protest (Weinstein & Sciara, 2006). For the most part though the consensus seems to be that equity concerns are a purely public relations (Wang, Yang, Zhu, & Li, 2012) or education (Giuliano, 1992) problem. Giuliano found that much of the opposition to value pricing results from public focus on the immediate user cost of a toll rather than its effects on delay or travel times across the system, skepticism that the toll would be set at an optimal rate, a conviction that tolls would lead to significant spillover onto no-toll local links, and a belief that funds collected would not be used to improve the network. If users could only understand the way tolls would be determined, she argues, their opposition would disappear.

Authors who see resistance to congestion tolls from equity concerns as purely a messaging problem have tended to recommend that the problem be managed by allocating toll revenues to projects that visibly increase equity. These authors argue that tolls lack a natural constituency. Every individual traveler feels the cost when they must pay as they travel but the benefits of lower congestion are hard to see and so there are no champions willing to bear a political price for the toll. One proposed solution to solve this problem is to ensure that funds collected by road tolls are then used to build other new infrastructure. Public support tends to be greater if funds are used for similar projects rather than assigned to a general fund with no intuitive relationship to the source of funding (Gomez-Ibanez, 1992). Researchers have also found that public support is greater if the funds are used on new infrastructure along the tolled corridor (Bay Area Economic Forum, 1990). Burtraw (1991) suggests that the most equitable, and also politically viable solution, is to link funding for indirect, non-monetary compensation to the harms of a policy decision. Improving the road network around busy exit ramps, installing sound walls, or even building parks and open spaces help local communities to feel that their concerns are valued by decision makers and can blunt opposition to tolling plans that seem to harm residents for the benefit of other travelers. Others have proposed that revenues (rather than physical improvements) ought to be assigned to local municipalities along the route of the toll road (King, Manville, & Shoup, 2007). Although the tolls may end up in a general fund, these authors argue that it is more equitable for regional travelers to pay for the cost of the infrastructure through a toll than for local residents to pay for it through property or sales taxes when they may rarely use the road. As a side benefit, making toll revenues a reliable funding stream may also convert many cash-strapped cities into motivated advocates.

A second popular option to increase support and assuage worries over equity, which could be seen as a variant of the first, is to use the revenues for public transit

(Small, 1992). The toll revenues are used to directly make the poorest travelers better off by improving service connections and transportation options that will further improve congestion. Transit funding can be a powerful tool to promote equity because it improves travel opportunities both for travelers with vehicles as well as those without. One might compare this to funding for sidewalks and curb cuts which are absolutely vital for some travelers but assist every member of the community. A third suggestion that addresses equity as a public relations problem is that revenues be evenly distributed amongst travelers as a substitution for gasoline or other sales taxes (Small, 1992). This option lacks the clear benefit to a core constituency that could cheerlead the measure, but it also provides a clear reduction in taxes and fees that may create a broad base of support.

Chapter 2: Equity

Given the complexities of real world policies, their unpredictable interactions, and the way support for policies in one arena motivate our preferences in others, judgments about the fairness of a particular policy are often unclear and uncertain. Nonetheless, the goal of creating and administering fair policies is deeply rooted in the political culture of the United States. The most common way this concern is articulated in policy is as a concern for *equity*. Fairness, and by extension equity, are fuzzy concepts. Much like Justice Potter Stewart's famous dictum "I know it when I see it" to define whether a film was an example of pornography[1] or an inability to come up with a clear, concise definition of music that captures the concept fully (Kania, 2011; Godt, 2005; Nettl, 1977), political concepts are often the subject of intense debate. These definitions can be central to our political understanding and so are contentious. In order to facilitate this debate and avoid just building my preferred theory into the definition of equity, let us begin with a highly general and widely accepted definition and proceed to refine the concept from there. I will expand on this concept as the argument develops, but let us begin with a tentative description that an equitable policy is one that fairly allocates its costs and benefits (Litman, 2016). Then, once this definition is settled, we will ask whether time is the kind of good that raises equity concerns.

There is a widespread demand for equitable policies in a wide variety of public services. In transportation, equity is implicated in decisions regarding the provision of

[1] *Nico Jacobellis v. Ohio* 378 U.S. 184 (1964)

facilities and services, usage charges and regulations for same, quality and frequency of transit services, mitigation and compensation for externalities and other negative impacts, and enforcement efforts related to transportation facilities and services. Take, for instance, a municipal bus service. The best strategy for creating a profitable bus authority may involve buying new touring buses with built-in Wi-Fi and other amenities for rapid commuter routes serving wealthier travelers (in order to increase the attractiveness of transit in relation to driving alone) while shunting older buses to lines serving poorer neighborhoods (where ridership is less elastic since other modes are cost prohibitive). Such a strategy strikes many people as unfair, because conveys the judgment that the comfort of some riders as worthy of more concern by the authority. These same concerns arise in non-transportation services as well. One of the most damning criticisms of 'stop and frisk' policing strategies is that they expose innocents in a minority group to significantly more state intrusion based on nothing other than their inclusion in the group (Maynard-Moody & Mushenso, 2012; Glaser, 2005; Brown Jr., 2013). Although there may be some benefit to targeting a group (the evidence is at best mixed), singling people out for immutable characteristics violates norms that police must have specific evidence of wrongdoing to rightfully detain persons.

Todd Litman, executive director of the Victoria Transport Policy Institute, argues that there are three diverse (and sometimes conflicting) conceptions of equity in the transportation sector: horizontal equity, vertical equity with regard to income and social class, and vertical equity with regard to mobility need and ability (Litman, 2016).

1) Horizontal equity

A policy exhibits horizontal equity when its costs and benefits are distributed strictly by cost or need. The goal is perfectly equal treatment; individuals subject to the policy should receive equal benefits, pay equal costs, and be treated in a procedurally identical manner. When allocations of goods are determined by payments, everyone should "get what they pay for and pay for what they get." Horizontally equitable policies include the triage system of determining the order patients are seen at an emergency room, the allocation of one person one vote, and movements to ensure that local property taxes ought to be spent on local projects.

2) Vertical equity with regard to demography or income

Vertically equitable policies distribute their costs and benefits with a sensitivity to the distribution of impacts between groups that differ with respect to income, social class, race, or some other identifiable distinction. Transport policies may be vertically equitable if they favor disadvantaged groups in a manner that compensates for structural inequalities in the larger society (Rawls, 1999). Policies that favor disadvantaged groups and remove inequalities can be termed progressive, and policies that disfavor or exacerbate inequalities may be called regressive. Vertically equitable transit policies may call for higher levels of service to poorer neighborhoods in recognition of the fact that many residents lack alternative transport modes.

3) Vertical equity with regard to mobility need and ability

A second variety of vertically equitable policies is focused on the different ways differing mobility needs and abilities change access to social goods. Policies under this aspect of equity include universal design, e.g. curb cuts for wheelchairs, audible beacons for crosswalks, or wheelchair ramps on kneeling buses. The central point here is that urban design geared solely for persons without disabilities would render the city inaccessible to persons with mobility related disabilities and so the equitable response is to design all infrastructure to be compatible with as wide a variety of travelers as is feasible.

Other forms of equity have been discussed in the literature, including geographic equity (i.e. services and social goods available to citizens living in different regions), modal equity (i.e. the attractiveness of transit services vs. drive alone), industry (e.g. heavy manufacturing may be the lifeblood of a region's economy and so the trucking industry may be taxed in a way that avoids paying the full cost of the damage their trucks do to the road network), or trip type (e.g. a transit agency may focus on providing access or fare subsidies to medical facilities, schools, and grocery stores, but not to theme parks or the nightlife district) but these have limited application to our current questions (Ungemah, 2007). With respect to variable road tolling schemes, we are most concerned with whether such a scheme ought to be implemented in a horizontally equitable manner (i.e. the only determinant of access is one's willingness and ability to pay the toll) or whether there are identifiable socioeconomic groups that, for a reason we will need to articulate, ought to receive a subsidy on their travel relative to the general public. While there are many interesting research questions into how we might devise a tolling scheme that addresses Litman's third kind of equity or even racial inequities, I will put them to the side in this thesis because the parameters of the Vickrey bottleneck are most clearly applied to income and socio-economic status.[2]

Let us set aside the question of whether a subsidy is appropriate in the time-varying toll case identified by the thesis for the moment in order to briefly explore the general evaluation of a vertically equitable transportation policy. The policy debate begins with an assumption that a horizontally equitable policy is correct. In the absence of compelling evidence, citizens are to be treated in a facially equal manner. The onus is on anyone who proposes a subsidy to achieve vertical equity. If a subsidy is appropriate, then it is appropriate because 1) there is some difference in the travel behavior of persons in different classes and 2) there is some moral, political, or legal significance in that difference which calls for state intervention.[3]

[2] However, I would direct the interested reader to papers which examine this category of equity as a variation of the network design problem with equity constraints. Ferguson et al. (2012) incorporates equity into a solution of the transit frequency-setting problem. Duthie and Waller (2008) examines a solution to the bi-level network design problem that adopts equity as part of the objective function. And Boyles (2015) looks at the problem of distributing funding for roadway maintenance in a geographically equitable manner.

[3] Ideally political and legal motivations for equity interventions will be grounded in a moral theory which justifies them, but that will not always be the case.

To identify a disparity, planners decide on the relevant analysis group, and then decide whether they will examine travel statistics, costs imposed, or both. So, for instance, if a planner allocating funds for safety improvements is interested in whether minority neighborhoods are receiving an equitable investment, they may be interested in whether the pedestrian accident rates are significantly higher in traffic analysis zones (TAZs) with higher minority populations than elsewhere in the city. Or, if a planner is considering a city transit subsidy for low-income households, they may examine whether those households have significantly fewer total trips, experience longer travel times or delays, and trip elasticity to user fee changes. If a gap is found and a small decrease in fare prices for low-income would bring minority trips in line with the rest of the city, then the subsidy may be adopted. The exact features that must be examined will be unique to the population being considered and the proposed remedy, but Table 1 contains some of the most common analysis units. They can be mixed, matched, and combined as needed to most accurately understand a given situation.

Table 1: Examples of measures of effectiveness for evaluating equity policies in transportation.

Unit Size	Travel Units	Costs
# of Persons in HH # of Adults in HH # of Commuters in HH Traffic Analysis Zone	VMT PMT # of Trips Trip Travel Time Minutes of Delay Accessibility Indices	Tax Rates User Fees Direct and Indirect Subsidies Measured Pollutants Accident Rates Gini Coefficient[4]

This process has been formalized by Ng (2005) into a five-step process for evaluating the equity of a policy.

1) Identify disadvantaged demographic groups.
2) Identify disadvantaged geographic areas using census data.
3) Identify degrees of disadvantage. (Ng suggests using five degrees of disadvantage. This allows the population to be meaningfully separated, but prevents segregation into so many groups that no effective policy can be formulated.)
4) Identify locations of important public services and destinations.
5) Evaluate transportation plans by their effect on disadvantaged communities.

[4] The Gini coefficient measures the inequality of the distribution of wealth or income in a country by examining its concentration. A country where everyone has the same wealth or income would have a Gini coefficient of 0, while a society where one person owns all of the wealth and everyone else has nothing would have a Gini coefficient of nearly 1. Amartya Sen (1977) has an enlightening discussion on the use of Gini coefficients to measure poverty and inequality.

2.1 THE POLICY FOUNDATIONS OF EQUITY IN THE US LEGAL CODE

The discussion in this thesis is framed around two normative questions: is equity a value that should be pursued in our public policy and, if so, how should it be best achieved with respect to time-varying tolls? It is important to note, though, that equity considerations are not merely philosophical. Consideration of equity and the distributional effects of transportation projects is enshrined in United States law and by presidential executive orders. The most important legal protections come from Title VI of the Civil Rights Act of 1964 (*42 USC § 2000d et seq.*), which generally prohibits discrimination in federal programs: "No person in the United States shall, on the ground of race, color, or national origin, be excluded from participation in, be denied the benefits of, or be subjected to discrimination under any program or activity receiving Federal financial assistance." This broad statement on discrimination was extended to programs receiving funding from the US Department of Transportation by regulations in the FHWA (*23 CFR part 200*) and FTA (*49 CFR part 21*).

Legal protections against discrimination in transportation funding are also found in:

1) Section 109 (h) of the Federal-Aid Highway Act of 1970 (*23 USC 109*), which requires projects to be approved "in the best overall public interest" and that effort should be made to eliminate or minimize the effect on community cohesion, employment, and the displacement of people;

2) Section 162 (a) of the Federal-Aid Highway Act of 1973 (*23 USC 324*), which addresses discrimination based on sex;

3) Section 504 of the Rehabilitation Act of 1973 (*29 USC § 701 et seq.*), which addresses disability discrimination;

4) The Age Discrimination Act of 1975 (*29 USC §6101*);

5) The Civil Rights Restoration Act of 1987, which identified the extent to which Title VI applied to include all federal-aid recipients, sub-recipients, and contractors, rather than only when the specific activity receives federal funding;

6) The Americans with Disabilities Act of 1990 (*42 USC § 12101 et seq.*).

The National Environmental Policy Act of 1969 (*42 USC 4321-4347*)[5] required that proposed major transportation facilities receive an analysis of environmental impacts that went beyond the infrastructure itself to include a broader geographic area, where the environment included both the physical landscape as well as the communities potentially affected by the facility. This helps to explain why President Clinton's Executive Orders on equity in transportation use the term "environmental justice" instead of equity. Executive Order 12898 (*59 FR 7629*), *Federal Actions to Address Environmental Justice in Minority Populations and Low-Income Populations*, directs that "each Federal agency shall make achieving environmental justice part of its mission by identifying and addressing, as appropriate, disproportionately high and adverse human health or

[5] And updates to the law: Public Law 91-190 (1970), Public Law 94-52 (1975), Public Law 94-83 (1975), and Public Law 97-258 (1982).

environmental effects of its programs, policies, and activities on minority populations and low-income populations." We see here that, although the language is slightly different, the spirit of equity, i.e. that the differential effects of policies upon groups should be minimized, is at the heart of the order. Updates to the Executive Order further require that the U.S. DOT ensure the full, fair, and meaningful participation of potentially affected communities and that it prevent the denial of, reduction in, or significant delay in the receipt of benefits by minority and low-income populations (FHWA, 2015). Building on this legacy, the U.S. DOT has issued a number of important orders and regulations that continue to demonstrate the Federal government's interest in creating an equitable transportation system.

2.2 Grounding Equity in Contemporary Moral Theories

Concern for equitable policies is enshrined in US law and is an accepted part of transportation planning. However, this background state of affairs does not determine the question of whether a time-varying toll in particular creates an equity concern. The effects of time-varying tolls are unclear prior to an investigation, but we can sketch out the conditions under which a toll scheme might prompt equity concerns. The argument that a toll scheme raises equity concerns has two parts.

First, I propose a moral judgment that grounds our beliefs about equity. Our concern for equity is rooted in a desire to show equal respect for all persons. The desire to show equal respect for all persons is more fundamental than judgments about specific policies and can serve as a guide to help us form judgments in new, difficult, or contentious cases.[6] Second, given that a policy which does not show equal respect for persons is inequitable, I will argue that a tolling scheme which prioritizes travelers solely by the absolute amount they are able to pay shows insufficient respect for poorer travelers. Low-income households already face significant burdens in managing their time and shifting these travelers to the margins of the peak period (even if unintentionally) will create more strain on their ability to lead their lives. The next step of showing that a particular toll scheme in the Vickrey bottleneck model shows insufficient respect for the time of poorer travelers is a major argument of the next chapter.

This thesis is an inappropriate forum to describe all of the debates that have taken place in the philosophical literature regarding equity and justice, but I believe a brief description of work by John Rawls, a central author in the egalitarian justice literature, can shed light on the issue without requiring an extended foray into the minutiae of the particular debates. His argument provides compelling reasons that we should care about equity and we will see that respect for persons plays an outsized role in explaining its importance.

Rawls' primary project is to understand the question: what are fair terms of social cooperation for free and equal citizens? Rawls' answer ran through a thought experiment that he termed the Original Position. The Original Position provides some guidelines to help us reach generally acceptable terms of cooperation by asking us to abstract away

[6] The method here is akin to a case where a biologist wants to know whether a new specimen qualifies as a member of a species. There are certain features that members of the species possess and checking the features helps to guide the biologist's considered judgment of whether she has a new species. So too, when we want to know if a policy is fair or equitable, we must first know the moral features that make any policy in general equitable and can then move on to judge the particular case.

from many of the things that we know about ourselves (that might tempt us to set the rules of cooperation in ways that favor us) and to focus on creating a society that would be acceptable to all rational persons.

To understand the Original Position, picture ourselves as policy makers interested in constructing a just state from the ground up (and who eventually will be interested in devising a just time-varying tolling scheme). There are no laws, no distributions of rights or resources, and no political or social structure. We begin legislating *ab nihilo*. That is, imagine that we have gathered to legislate in complete ignorance of our own physical and mental characteristics. We don't know our gender, race, sexual orientation, or age. We are ignorant of our mental and physical strengths, limitations, and infirmities. We do know that the citizens of our society will have diverse beliefs about the best way to live their lives and that these plans will require economic goods. We know that our society is constrained by a moderate scarcity where there are not enough resources for everyone to get everything they want. And we also know general facts about human social life, including uncontroversial conclusions of science and human social life.[7]

With this limited knowledge about ourselves and humankind, we are in a position to answer Rawls' question. Because we do not have any information in the Original Position that would allow us to govern society in our favor, the terms of cooperation we devise will be fair. With our limited knowledge, if we choose rules that permit unlimited

[7] Rawls' project is to understand the terms of cooperation that would be reached by *human* citizens, and this would be impossible if, for instance, the Original Position hid from us the reality that human beings will starve to death without food or that human beings generally form bonds of attachment to their offspring.

accumulation of capital into the hands of, say, the top 1% of IQ test scorers and disenfranchise the remaining 99%, then we would have to accept that we would have a 99% chance of having no future voice in the direction of our society. If we choose rules that permit racial slavery, subjugation of a sex, or other discriminatory policies, then we do so with the risk that we might end up on the short end of the stick.

Rawls uses the Original Position as a heuristic to help us understand what rules we would be rational to endorse, and is under no illusion that we could undergo such a debate or create such a consensus. (After all, the first step is to abstract away from all of the unique characteristics that make us who we are and inform our views about the good life.) Under such uncertainty, Rawls argues that a rational deliberative body just described will choose basic principles of justice that will ensure, whatever role those legislators eventually have in society, that they will be able to participate in society and that their economic and social circumstances will not fall below a minimum floor. These conclusions are formally stated as the **Equal Liberty Principle** and the **Difference Principle** (Rawls, 2001)**.**

- **Equal Liberty Principle:** Each person has the same indefeasible claim to a fully adequate scheme of equal basic liberties and this scheme is compatible with the same scheme of liberties for all;
- **Difference Principle:** Social and economic inequalities are to satisfy two conditions:

a. They are to be attached to offices and positions open to all under conditions of *fair equality of opportunity*;
b. They are to be to the greatest benefit of the least-advantaged members of society.[8]

Rawls' first principle enshrines a respect for both *de jure* and *de facto* equality (not equity) of political power and participation. Citizens would be guaranteed the basic rights that have long been a part of the Western liberal tradition: freedom of association, speech, and religious belief, the right to vote, hold public office, and the be treated equally by the rule of law. These rights are inalienable by the citizens.

The difference principle arises from two arguments. First, as legislators in the Original Position who have no knowledge of the resources or talents that we will possess once our social rules are realized, we will create conservative rules for the distribution of certain social and economic goods.[9] If we have an equal chance of receiving any of the specified amounts of resources, then we will choose to minimize our risk of losing out by allocating equal shares to each person in society. The idea that everyone would be so risk averse that rationality itself demands an equal partitioning of social goods has been

[8] Different social roles and occupations may generate inequalities. For instance, it might (and I emphasize might because the details would be crucial) be permissible for doctors to earn more than manual laborers because the work they do helps the least well off and their income may be necessary to incentivize enough people to spend years mastering an incredibly difficult craft. This should help alleviate concerns that differences are permissible because of desert; people who take those risks will deserve the rewards they receive, but it will also be true that the least advantaged will not be undeservingly confined to that position.
[9] Note that these primary social goods are broadly construed – "These goods normally have a use whatever a person's rational plan of life. For simplicity, assume that the chief primary goods at the disposition of society are rights and liberties, powers and opportunities, income and wealth" – and stand in contrast to what Rawls calls primary natural goods, e.g. health and vigor, intelligence and imagination, which are certainly important to a happy life but cannot be directly controlled by a social structure (Rawls, 1999).

extensively criticized[10], but it seems plausible that persons truly ignorant of their station in life would be highly concerned with maximizing their well-being in their worst-case scenario.

A second, better, argument to motivate this position is that the social class into which a citizen is born and the resources that their parents have to raise them are morally arbitrary facts. Individuals have no control over or responsibility for their parents and so there could be no sense in which a particular individual deserves the advantages or disadvantages that they may experience as a result of their innate intelligence, the wealth of parents, or the stability and prosperity of the country in which they are born. And, because we do not morally deserve to receive more or less than other people at the moment of our birth, there is at least a *prima facie* reason for society to be structured so that everyone has an equal share of economic resources.

However, a society where everyone lacks access to food could still be equal and allowing for some limited inequalities with respect to wealth and resources may incentivize citizens to take risks or to invest in education that may be difficult and take a long time to pay off. Therefore, Rawls allows, the rational legislators in the Original Position may permit those inequalities that improve the situation of the worst-off citizens. Since the legislators are in complete ignorance of their station in society, they will consent to rules that make their worst-case scenario as bearable as possible.

[10] Cf. Harsanyi (1975).

There are again a number of caveats that should be expressed, most notably that Rawls denies that the legislators might be willing to make everyone worse off as a result of envy for the highest earners, but let us set those specifics to the side.[11] Furthermore, we should be clear that Rawls does not endorse using his theory of justice to judge particular applications of policy. It is not a theory designed to help pass judgement on a law or administrative rule because he intended to discuss how the entire structure of society ought to be organized. So, we should resist any easy attempt to pass judgment on a particular tolling policy by subjecting it to a direct test of the difference principle. However, it is appropriate to use Rawls' thought experiment of the legislators in the original position to understand the contours and demands of a large social goal, e.g. equity.

The thought experiment of the Original Position can help us to explain our interest in equity and the reason why it has even been enshrined in US law. After all, equity may require large sacrifices and make markets less efficient, so it is rather remarkable that it has such widespread support. Rawls' equal liberty principle is derived from the belief that all citizens are fundamentally equal and so deserve to be shown equal respect by the state. As legislators, we would not accept policies that exclude us from the goods of social cooperation. Furthermore, those inequalities that we might accept out of the Original Position will be tied to making everyone better off.

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[11] Rawls' theory of justice has inspired a large and varied literature. A small sampling of this further discussion can be found in Rawls (1999; 2001; 2005), Sandel (1998) and Sen (2009).

Equity is and ought to be a social policy priority precisely because being shown sufficient respect by society and the state is a central human good. By engaging in Rawls' exercise, we come to understand that one reason to endorse equity is that many of the economic differences we encounter in society are morally arbitrary. We endorse policies geared to ensure equity or to rectify inequalities because we reflectively endorse policies that minimize economic and social marginalization due to morally arbitrary factors. This discussion provides us with something like a litmus test to evaluate potential policies. When we consider a potential policy, we can ask of it: "Does this policy show equal respect to all citizens?" We now have a theoretical grounding for Litman's distinctions amongst various kinds of equity. In cases where horizontal equity is the primary social good, this is because we judge that the citizens are on an equal playing field and so we show them equal respect by treating everyone in the same manner. In cases where vertical equity is important, this is because we judge that citizens are not on an equal playing field and so we show them equal respect by ensuring that the least advantaged receive compensatory benefits that allow them to access or compete for social and economic goods from which they would otherwise be shut out.

2.3 Historic Equity Concerns and Toll Roads

As discussed in Section 2.2, the allocation of the primary social goods is determined by adherence to Rawls' Equal Liberty Principle and Difference Principle. Access to the transportation system is clearly one of those goods for two reasons. First, the transportation system plays such a central role in the lives of all citizens that it is foundational to the pursuit of virtually any conception of a good life. And second, the transportation system requires such large up-front capital investments that monopolies are likely to develop and so citizens deserve a voice in the governance of those systems.

Anthony Foxx's 2016 address to the Transportation Research Board in Washington, DC, which prefaces this thesis, makes precisely this point. The US economy is growing ever more complex and citizens rely upon the transportation network to connect them to their jobs, to food, to healthcare, and to their friends and family. If citizens are isolated from the transportation network, they would be unable to pursue many of the activities that are most important to them. Because the transportation network is so important, equal respect for fellow citizens demands that they have some voice in its delivery. A market-norm driven transportation system would allow people to choose not to travel, but would not provide opportunities for them to weigh in on the future goals or priorities of the system.

The transportation system is subject to monopolies and high barriers to entry. The American Society of Civil Engineers estimates that the US would need to spend $101 billion every year just to maintain the current road and bridge networks at their current states (2013). Laying down new flexible pavements costs $120,000-$250,000 per mile even when no special terrain features increase the complexity of the project (Federal Highway Administration, 2006). Furthermore, roadway projects have expected lifecycles of 30-40 years and heavily influence the built environment around them resulting in significant changes to the areas in which they are placed. These large impacts point to a need for citizens to have a voice in the way the network changes their lives.

2.3.1 Toll Road Related Equity Concerns- Literature Review

This brings us to toll roads, which have long been accused of increasing inequity in a number of ways. The most important burden that tolls impose is financial. Tolls are typically imposed across bridges or other bottlenecks precisely because it is difficult to choose a route that avoids the toll. Harvey (1991) and Giuliano (1992) make this point, noting that tolls create winners and losers and do not lead to a Pareto improvement as workers in areas with poor transit are highly likely to pay bridge tolls at peak hours because they lack alternatives. A 2009 study by the Washington State Department of Transportation found that the impacts of tolls are highly dependent upon geography, rates of car ownership, level of employment, ability to switch modes, and the opportunity to choose routes that avoid the toll. Under the scenarios that were examined, a $2 toll across bridges in the Seattle area would impose an annual cost of $2600 per family, an amount equal to 15% of the federal poverty income level, as they traveled for work (Plotnick, Romich, & Thacker, 2009). These are significant costs for low-income travelers, many of whom must travel to downtown Seattle for work but are not paid enough to afford to live closer to their workplace. Higher income travelers also possess more flexibility to change their residence in order to respond to such costs.

Tolls are also opposed because they can create environmental costs and congestion in poorer neighborhoods when travelers choose to use local roads in order to avoid the toll. Even when tolls are accompanied by promises that the revenue will be reinvested in the community, e.g. through increased transit funding, many local stakeholders are mistrustful. Transit authority promises of increased mass transit service or new lines are often not credible because the places that have to pay the fee are often in areas that are not traditionally well-served by transit. Two examples are the San Gabriel

Valley reaction to high occupancy toll (HOT) lanes, where a free lane is available but there is an immediately parallel tolled lane that can be used to escape congestion, on Interstates 110 and 10 in 2009 and the outer borough reaction to Mayor Michael Bloomberg's 2007 plan to institute a congestion charge in lower Manhattan (Taylor & Kalauskas, 2010).

Resistance to Bloomberg's congestion scheme (one quite similar to the London cordon charge) demonstrates both of these phenomena. Opposition to the plan centered around data that showed the funds raised would largely come from lower-middle class families and skepticism that raised funds would in fact be used on transit. Assemblyman Richard Brodsky laid out these arguments in a report on the proposal (2007). Data collected from the city showed that residents of Queens, the Bronx, Brooklyn, and Staten Island, who had a collected average income of $46,004, would account for 47% of the total fees while making 24% of tolled trips. Residents of Manhattan, with an average income of $74,676, would pay 42% of fees but make 72% of trips in the affected area. The toll, if paid daily by a commuter, would come to about $2000 per annum, which would be ~4.5% of the annual income of a driver from Bronx, Brooklyn, or Queens, but only ~2.7% of the income of a driver from Manhattan.

These tolls also faced equity-oriented resistance because of concerns that the tolling scheme would divert traffic onto overburdened local streets and that this would worsen environmental pollution along alternate routes. Many of the neighborhoods most concerned by the potential for diversion onto their local streets had long been neglected by the city and felt they had little to gain from the promised benefits. Assembly Speaker Sheldon Silver argued that the toll plan could not adequately handle traffic diverted onto local streets and that this would worsen environmental pollution along the route and in some of the poorest neighborhoods that lay just outside the congestion fee zone (Hakim, 2007).

Time-varying tolls aimed at reducing congestion have also been heavily criticized on equity grounds. First, these systems work best with cashless transactions, but poorer travelers are less likely than affluent travelers to have credit cards or bank accounts, which makes it both more difficult and more expensive to access facilities with electronic tolling systems (Plotnick, Romich, & Thacker, 2009). A second worry is that many poor travelers face constrained schedules that effectively force them to travel at peak periods in the same way that bridges across rivers would effectively force poorer travelers in Washington to use tolled bridges. For instance, single mothers must pick up and drop their kids off in narrow windows and so avoid nonstandard working hours if at all possible (Presser, 1995).

2.3.2 Equitable Tolling

Although many groups opposed to both time-varying and constant tolls have opposed them on equity grounds, others have argued that tolls do not have a uniform impact on equity because their effect on the road network is highly sensitive to the specifics of the proposed plan, particularly its price structure, quality of alternatives, use of raised revenues, and whether driving is a luxury or a necessity, i.e. whether transit or routes that avoid the toll are available (Litman, 1996; Litman, 2016; Rajé, 2003; Golub, 2010; Schweitzer, 2009; Santos & Rojey, 2004). Thus, these authors concede that though there are particular implementations of time-varying tolls that may have regressive income effects, tolls can also be fine-tuned to have progressive outcomes.

Other researchers have argued that the equity implications of a time-varying toll are entirely dependent on the alternative methods available for funding transportation projects. In the US today, roadway projects are largely financed through either through general sales taxes or through a specific tax on gasoline products.[12] This funding structure is highly regressive for a number of reasons. First, user fees are regressive by their nature. Poorer travelers have less money and so $1 in tax to them almost by definition represents more of their income than a $1 tax would represent for wealthier travelers. An examination State Route 91 in Orange County, CA found that funding the road through a sales tax heavily burdened the poorest families. Table 2, adapted directly from Schweitzer and Taylor (2008), shows that the poorest travelers, with a median income of only $7,000 pay roughly $3.4M per year in sales taxes that would be earmarked for the road, but almost none of them would use the road if it were tolled. Middle income

Table 2: Estimated contributions by each of five income groups to SR91 Express Lanes costs (Schweitzer & Taylor, 2008).

Group	Median Income	Sales taxes	Tolls	Gain or Loss	Loss/Gain per family
1 Low	$7,126	$3,353,241	$0.00	-$3,353,242	-$66.60
2 Low-Mid	22,221	1,789,375	3,906,577	2,117,202	36.72
3 Mid	40,902	3,977,632	7,345,369	3,367,737	42.47
4 Mid-high	67,427	10,798,820	12,731,744	1,932,924	14.60
5 High	180,830	14,080,930	10,006,040	-4,074,890	-27.46

[12] The Federal tax rate for gasoline products is currently 18.3 cents per gallon and the state of Texas imposes a tax rate of an additional 20.0 cents per gallon (Heleman & Wright, Texas' Motor Fuels Taxes: Essential Levies Support Roads, Education, 2016). Municipalities have turned to local sales taxes to fund specific initiatives that go beyond the assistance they receive from either their Federal or State governments. One such example is the city of San Francisco, which levies a 0.25% sales tax on all retail sales to finance transit operations and paratransit services for disabled passengers and a 0.5% sales tax in Alameda, Contra Costa, and San Francisco counties to fund the Bay Area Rapid Transit Authority and the Metropolitan Transit Commission (Metropolitan Transport Commission, 2017). In Texas, 10 transit authorities levy local sales taxes to help fund their operations and 218 municipalities levy local sales taxes to fund street maintenance and repair (Heleman, 2016).

travelers, by contrast, receive an annual subsidy of $42.47 when the road is funded by a sales tax rather than a toll.

Second, the poor are also the most likely to own older and less fuel-efficient vehicles. They spend more money (which is a larger percentage of their income) to go fewer miles and that disparity holds for all travel, not just for the most congested travel. In Gauteng, South Africa, there was a choice between tolls for freeways and area wide fuel taxes. The fuel tax was the most regressive means of funding the road because poorer travelers lived close to their workplaces and so mostly traveled on heavily congested local and arterial streets that made older vehicles even less fuel efficient. The revenues raised by the fuel taxes were then spent on improving the highway network that benefitted richer travelers (Venter & Joubert, 2014).

Third, poorer travelers tend to live in the urban core and drive less frequently, so area-wide sales taxes tend to force them to subsidize infrastructure that they do not use. Thus, non-discriminating sales taxes pose problems for horizontal equity among travelers. If we adopt the principle that travelers should pay for the infrastructure that they use, then usage based fees are philosophically preferable to sales fees, because they charge people for the miles the congestion they cause as well as the miles they travel. That low-income households subsidize higher income households when infrastructure is funded by sales taxes is unavoidable. In general, low-income households make fewer trips than high income households, which means that these households would pay more on a per trip basis if funds are raised using sales taxes (Litman, 2016). Furthermore, low-income households often choose to live in the urban core, because that gives them access to public transportation. Thus, financing roads through tolls removes this tax burden on the poor and possibly frees up household income to support public transit (Glaeser, Kahn,

& Rappaport, 2008). Finally, a Florida study examining strategies to decrease vehicle miles traveled found that a tradeable credit scheme would benefit low-income households because they would be able to trade away their credits and so receive compensation for traveling less (Mamun, Michalaka, Yin, & Lawphongpanich, 2016). A version of the tradeable credit scheme was implemented in Minnesota with some success. The state toll road authority, MnPass, credited every account with a positive balance that could be used and supplemented with a user's funds. If a user chose not to travel on toll lanes, however, the funds could be applied towards registration fees, a cost savings that would benefit all drivers (Taylor & Kalauskas, 2010).

2.4: Time Poverty

Section 2.2 laid out an argumentative framework to help us evaluate the equity implications of any dynamic tolling scheme. If the tolling plan leads to an impoverished choice set for poorer travelers that substantially affects their ability to access primary social goods, then it is incumbent upon us to implement an equity based remedy to alleviate that harm. In this section, I raise a related body of literature on time poverty to add an additional dimension to the equity analysis. To understand the full implication of a dynamic tolling scheme we ought to look both at the economic costs borne by travelers as well as their departure time decisions.

Time poverty should be included in an equity analysis because it has the same importance to travelers as unfairly distributed economic costs. First, low-income travelers experience poverty both with respect to money but also with respect to their time. Second, time poverty creates concerns that low-income travelers will be excluded from primary social goods. Third, the socially optimal dynamic tolling plan will exacerbate these problems. Therefore, the dynamic tolling plan creates equity concerns and we should examine alternatives that mitigate this issue.

2.4.1 Time Poverty Defined

Academic research into time poverty owes its genesis to Clair Vickery's 1977 article, "The Time-Poor: A New Look at Poverty". Vickery argued that measurements of poverty based solely upon household income miss an important aspect of poverty, namely that the time to fulfill basic household needs increases as income decreases. Her introduction to the thesis is classic:

> Since the official poverty index was developed in the mid-1960s by the Social Security Administration (SSA), their categorization by income has been accepted as an equitable criterion with which to compare different types of households. As a result, policy-makers have thought that adjusting the benefit structure of an income-transfer program for money differentials across house-holds corrected for the resource differences of these households. But households differ in their time resources as well as their money income. This paper argues that to base the benefit schedule of an income-support program on an index that defines poverty in terms of money income alone is to create gross inequities across households that vary in their number of adult hours. The equity problem, important in itself, takes on added significance when it creates incentives for individuals to adjust their living arrangements, and the problem becomes aggravated if the household structure appears to be in a transitional phase as in the 1970s. (Vickery, 1977)

All households are constrained by a certain number of needs, including food purchase and preparation, cleaning, maintenance, and paying bills among many others. Completing these tasks requires household member time. Someone must plan dinner, go to the grocery store, pick out ingredients, wait in the checkout line, and then finally prepare the meal. It is possible to streamline this process somewhat, e.g. by buying in bulk, shopping for the entire week, and if household income is high enough it can be outsourced entirely by eating at restaurants, or hiring a maid, nanny, or personal shopper.

Although the literature covers a wide variety of perspectives, the discussion of time poverty has often been framed as a problem facing female managers as they struggle to balance long and demanding workdays with the traditional responsibilities of a homemaker (Rutherford, 2001; Simpsons, 1998; Rosin, 1990). But this focus on the upper-middle class and corporate executives misses the crucial experience of millions of low-income families. Anti-poverty assistance programs in the US, such as Temporary Assistance for Needy Families (TANF) and the Supplemental Nutrition Assistance Program (SNAP), build an assumption that poor families will have ample time to perform unpaid household tasks into their support models. These programs were created at a time when the average American household had a very different structure. When President Lyndon Johnson began the War on Poverty with his Great Society Programs in the 1960s 74.3% of households included married couples, by 2009 that number had shrunk to 50.5%. In 1975, only 45.5% of women with children under the age of 18 were in the labor force; that number grew to 71.4% by 2008 (Albeda, 2011). The most recent Current Population Survey conducted by the US Census bureau in 2016 found that single mothers with no partner present formed fully 24.4% of all households in which a mother was present and that 16.1% of all US households with children are headed by a parent with no partner present (United States Census Bureau, 2016). The increasing number of employed women, even women heading single parent households, is largely attributable to the insistence that all assistance should be tied to efforts by the poor to find employment that would move them out of poverty.

As the American family structure has changed, the time required for low-income households to keep the home running has put these families in a bind. As families move out of monetary poverty, they can find themselves suddenly time poor. For instance, 50-

60% of unemployed single parents are below the monetary poverty threshold, but only 3% are time poor (Harvey & Mukhopadhyay, 2007). When single parents are employed only 26-31% are monetarily poor, but 98% are time poor. Only 5.3% of single parents are neither time nor monetarily poor. Even a partner to share the load is not a panacea, since 20-30% of employed two-parent families are still time poor (i.e. they work above the threshold of 11.5 hours per day per parent).

The most pressing problem facing these families is often the problem of finding affordable childcare during working hours. Employed mothers, as one might expect spend significantly less time with their children and on housework (Bianchi, 2000; Sandberg & Hoffreth, 2001; Bianchi, Robinson, & Milkie, 2006) and this difference can largely be explained by household income level (Kendig & Bianchi, 2008). Childcare is also problematic for working parents. When children are sick or school is not in session, it can be difficult to find a safe and affordable option short of calling out of work (which comes with its own set of problems). Interviews with poor and low-income employed women have consistently found that the high cost of childcare meant that many women felt they had to choose between clandestinely bringing their children to work, leaving them in unsafe situations, or being fired for missing work as a result of staying home with them (Scott, Edin, London, & Kissane, 2004; Dodson, 2007).

Assumptions regarding the appropriate allocation of household time and money also stretch low-income household time budgets. Vickery (1977) found that the average household needed to spend about two hours per day household chores. But TANF assumes that low-income households stretch their food dollar by preparing all meals from scratch, suggesting recipes that take anywhere from 80 minutes to 2.5 hours per day to prepare on top of time required to purchase ingredients (Mancino & Newman, 2007).

Furthermore, TANF forbids households from saving time by purchasing premade foodstuffs.

Families that do not fall into the income definition of poverty can still experience time poverty. This is because, even when extreme want is out of the equation, time poverty is a problem driven by the number of hours worked, when they are worked, and the physical intensity of that work (Warren, 2003). Persons score well with respect to time poverty when they have enough time (i.e. they are able to work the number of hours they desire but not so many as to have involuntary overtime), they have time at the right time of their day, week or season, they have control over their personal time, and their time fits into the rhythms of family and friends (Reisch, 2001). Higher paid workers can control when they work, where they work, and have more flexibility to take time off, which creates greater job satisfaction, lowers stress levels, and creates work-life balance (Industrial Society, 2001). And, as noted above, moving up the socio-economic scale allows people to "buy back" their time by paying others to do domestic chores (Gregson & Lowe, 1994; Roberts, 1998; Stephens, 1999). For other families, shift work and schedules that are driven by the employer's need for flexibility rather than the employee's have made it difficult for low-income families to have all these benefits. Shift work, even when parents are not employed in multiple jobs, leads to desynchronized family and social circle schedules (Presser, 1995; Roberts, 1998; Wheelock, 1990; Glucksmann, Cottons and Casuals: The Gender Organisation of Labour in Time and Space, 2000). Finding enough time at the right time can be challenging for all families, but an acute equity concern should be raised by the especially challenging landscape facing low-income families.

2.4.2 Exclusion from Fundamental Social Goods

Time poverty ought to be a central focus of our equity evaluation because time is one of the most precious goods humans possess. Free time and to spend with friends and family and the flexibility to pursue varied projects is a fundamental social good for both intrinsic and extrinsic reasons. Intrinsically, free time provides an opportunity for rest, social interaction, leisure participation, and self-realization, which makes it an important non-monetary welfare resource (Chatzitheochari & Arber, 2012). Philosophers, economists, and social theorists have consistently conceptualized free time as a primary good for individual well-being (Marx & Engels, 1968; Rawls, 1999; Putnam, 2000; Vickery, 1977; Fraser, 1994; Douthitt, 2000; Blackden & Wodon, 2008; Dodson, 2007; Hochschild, 1997a). Empirical research has shown that free time activities bring numerous benefits to people's health, and to subjective and family well-being (Iso-Ahola & Mannell, 2004; Coleman & Iso-Ahola, 1993). This right is so fundamental that it is recognized in a number of international treaties, including the Universal Declaration of Human Rights (UN, 1948), the International Covenant on Economic, Social and Cultural Rights (UN, International Covenant on Economic, Social, and Cultural Rights, 1966), and the Convention on the Rights of the Child (UN, 1989). These political documents recognize the fact that human beings work in order to participate in those activities and to be with those people who make life interesting and fun. The principles of equal respect articulated by Rawls help us to understand why these goods should be shared by all citizens and not just the wealthy. Possessing time that is not controlled by an obligation to work for others or to fulfill biological needs is also crucial to the pursuit of many other projects. Most importantly, it is crucial to fulfilling our roles as citizens. Anderson (1993) notes that participatory democracy requires both spaces for us to encounter fellow citizens as well as the time to learn about public issues, participate in civil actions, and decide issues of social importance.

In the next chapter, we will examine a toll scheme whose goal is the minimization of the total cost to all travelers by setting the toll in such a way that travelers will choose to travel at exactly the capacity of a system bottleneck. We will carefully examine whether a tolling scheme meeting this objective leads to either disproportionate or inequitable monetary or time burdens on low-income travelers.

Chapter 3: The Single Link Economic Model- Formulation and Discussion

3.1 Background

The first proposals for dynamic tolling can be found in the economics literature of the early 1920s. Congestion arises because of an externality, viz. my decision to queue causes a disutility paid for by others and not captured by the costs I must pay. Therefore, every traveler has an incentive to discount the true cost of their decision to travel. Arthur Pigou in his classic *The Economics of Welfare* (1920) defined "incidental uncharged disservices", what are now referred to as 'externalities', as costs that affect parties which are not party to a contract agreement. The following passage is typical.

> Thus, incidental uncharged disservices are rendered to third parties when the game-preserving activities of one occupier involve the overrunning of a neighbouring occupier's land by rabbits—unless, indeed, the two occupiers stand in the relation of landlord and tenant, so that compensation is given in an adjustment of the rent. They are rendered, again, when the owner of a site in a residential quarter of a city builds a factory there and so destroys a great part of the amenities of the neighbouring sites; or, in a less degree, when he uses his site in such a way as to spoil the lighting of the houses opposite: or when he invests resources in erecting buildings in a crowded centre, which, by contracting the air space and the playing-room of the neighbourhood, tend to injure the health and efficiency of the families living there. (Pigou, 1920)

Because externalities by definition lead actors to misestimate the true costs and benefits of their actions they cause those actors to over- or under-produce. The most economically efficient remedy is for the state to respectively tax or subsidize these behaviors, where the actor's cost is adjusted to reflect their marginal impact on social welfare.

In traffic congestion, the excess costs imposed on other travelers call for a tax or fee. William Vickrey wrote the first seminal papers on congestion pricing theory to solve the problem of underpriced transportation infrastructure beginning in the 1950s. Vickrey set out the conceptual framework for peak hour charges in his recommendations for pricing the New York City Subway. Passengers should be charged, he argued, according to the marginal cost that they impose on all other travelers. (For instance, 20 additional passengers may so crowd the subway car that the other 200 passengers already in the car would pay 3 cents to get rid of them. This means that they have imposed a cost of 30 cents each on everyone, but they themselves only feel 3 cents worth of discomfort.) He also advocated distance based pricing for transit, similar to the one used for the Tokyo subway, where users would pay the highest fare upon entering the system and be reimbursed at the end of their trip in order to prevent fare jumping (Vickrey, 1955).

In addition to articulating a need for value pricing, Vickrey also contributed to the literature by creating what is now known as the "Vickrey bottleneck" model, which is an idealization of a road network that allows for the analytical inspection of various solutions to congestion (Vickrey, 1969). The simplest version of the bottleneck is a single link between the origin and destination. The road has unlimited capacity before and after the bottleneck, but at the bottleneck has a limited throughput capacity. The result is similar to the concept of a point queue model in dynamic traffic assignment. The network can then be modified to add additional links or bottlenecks as necessary to discuss new scenarios. The genius of this approach is that the problem to be analyzed stays in sharp focus while also remaining analytically tractable with a solution space that has a geometric and intuitive explanation.

While I will discuss the relevant conclusions of the bottleneck model below in Section 3.2, I want to now briefly survey some of the important work that has been done using this model as a basis. The direct inspiration for this thesis, a two-decade corpus of work by Richard Arnott, Robin Lindsey, and André de Palma explores a wide variety of scenarios. Their earliest papers described the Vickrey bottleneck to derive pricing schemes and estimates of welfare (1987a)[13], optimal capacity for an optimally tolled roadway (1987b), and the welfare effects of a coarse stepped toll (1990a). They also derived welfare costs and departure time decisions for travelers choosing between tolled and no-toll facilities (1990b).

Since then, they have examined a version of the Braess paradox on freeways that can be alleviated through ramp metering (1993a) and other alternatives to road pricing (Arnott, 1994). Their work has since attempted to increase the realism of their model through incorporating stochastic demand and bottleneck capacity (1999), better estimating the externalities imposed on non-travelers (Arnott, 2007), and revisiting stepped tolls with a more complex model of traveler behavior (Lindsey, van den Berg, & Verhoef, 2012). Some of their most recent work has steps outside of the strict analytic Vickrey bottleneck but stays within the spirit of that project by examining the congestion behavior of married couples that wish to coordinate their departure times and they show that marriage can increase the costs of peak hour congestion (de Palma, Lindsey, & Picard, 2015). They have also published high-level summaries of their work on the Vickrey bottleneck that serve as helpful reference texts (1993b; de Palma & Lindsey, 2004).

3.2 Definitions of Key Terms

Trips along the Vickrey bottleneck model have five stages. A departure, a free flow period to the bottleneck, a congested phase while waiting for prior vehicles to pass

[13] Some of their work on heterogeneous traveler groups was first written into working papers but only published later, see (Arnott, Palma, & Lindsey, 1994).

the bottleneck, passing the bottleneck, and an arrival. The bottleneck is assumed to function perfectly, allowing travelers to pass through at a rate D vehicles per minute. Travelers are assumed to belong to a homogeneous group of similar travelers of size N who have identical characteristics and preferences.

Their primary characteristics relate to their perceived utility of travel. All travelers in the group wish to arrive at the destination at a single point in time, τ^*. Think of the bottleneck as modeling the commuting decisions of workers arriving to start a standard morning work shift. Either picture the group of travelers as workers at a single factory on the far edge of (a very narrow) bridge or, at a more abstract level, all of the workers in a city choosing to use a much larger bottleneck. All travelers possess a general value of time, α (e.g. dollars per minute). This represents the perceived cost of a minute of time spent traveling or waiting in a queue. These costs are typically high, as time spent in traffic is essentially "wasted". Although travelers may now be able to conduct some business or catch up with a friend using a cell phone, travelers are not able to engage in many preferred leisure activities or business that requires extended visual concentration.

Travelers also have schedule delay penalties for early and late arrival, β and γ, respectively. These factors represent the cost of having to wait at one's destination rather than arriving exactly on time. In the commuting case, these penalties are intuitively understood to be influenced by whether the worker has a flexible schedule that permits them to be productive before the desired start time, the magnitude of penalties the worker faces for arriving late, and other schedule constraints that may make it costly for the worker to arrive before or after their start time (e.g. convenience charges by childcare services to drop a child off early).

Here is an example of how the bottleneck can be interpreted. Traveler i departs at t_i, experiences a free flow travel time, FF, waits in a queue of length $q(t)$, passes the bottleneck, and arrives at her destination at τ_i. Because there is more than one traveler in each group of travelers, let us generalize the individual case to say that $n(t)$ travelers depart over an infinitesimal period at t. While the physical interpretation of such a model requires that vehicles be discrete entities, a choice to model the travelers as an infinitely divisible fluid creates a smooth and predictable distribution of departures and arrivals that is derivable and so time is considered to be continuous (although of course in a numerical solution it must be treated in a discrete fashion). Travelers in the model transition instantly from free flow to waiting in a queue to arriving at their destination. This is analogous to point queue models used in dynamic traffic assignment, where there is no backward shockwave and vehicles are able to depart as soon as their turn arrives (Zhang, Nie, & Qian, 2013; Szeto & Lo, 2006; Kuwahara & Akamatsu, 1997).

The utility of each traveler is captured in a cost function, $C(t)$, which is composed of four distinct costs: the cost (determined by α) of the time required to travel from the origin to the destination in free flow conditions, $FF(t)$; the cost of the time required to wait in any queue that exceeds the bottleneck's discharge rate, $TT(t)$; the schedule delay cost (determined by β and γ) created by arriving at the destination at a time other than the preferred arrival time, $SD(t)$; the direct cost of a toll that is imposed, $TR(t)$. Each these costs can vary over the peak hour and it is possible for a subset of the costs to be equal to 0. For instance, it is possible to vary the toll rate in order to encourage travelers to choose a departure rate that is less than or equal to the discharge rate of the bottleneck, $n(t) \leq D$ and then if the length of the queue is zero, $TT(t_i) = 0$.

Table 3: Key terms and variables in the Vickrey bottleneck model with a homogeneous traveling population.

t_i	Traveler departure time for traveler i. (If the number of the traveler has no special importance, this is shortened to t.)
τ_i	Traveler arrival time for traveler i. (If the number of the traveler has no special importance, this is shortened to τ.)
t_0	Departure time of the first traveler.
t_f	Departure time of the final traveler.
t_{ij}	Transitional departure time when travel switches between groups i and j in a scenario with heterogeneous traveling groups.
τ_0	Arrival time of the first traveler.
τ_f	Arrival time of the final traveler.
τ^*	Desired arrival time of all travelers.
t^*	Departure time of the traveler who arrives at τ^*.
α	Value of time during travel.
β	Schedule delay penalty for early arrival.
γ	Schedule delay penalty for late arrival.
D	Discharge rate of the bottleneck.
N	The total number of travelers in the network over the peak hour.
$n(t)$	Number of travelers choosing to depart at t.
FF	Free Flow Travel Time along corridor.
$q(t)$	Queue length experienced by travelers departing at t.
$C(t)$	The sum of costs facing an individual traveler who departs at t.
$FF(t)$	The free flow costs facing an individual traveler who departs at t.
$TT(t)$	The travel time costs facing an individual traveler who departs at t.
$SD(t)$	The schedule delay costs facing an individual traveler who departs at t.
$TR(t)$	The toll rate paid by an individual traveler who departs at t.
TC	The total travel costs incurred by all travelers over the peak hour.
FFC	Fixed costs of traveling at free flow when no other travelers are present.
SDC	The sum of the schedule delay costs over all travelers.
TTC	The sum of the waiting time costs encountered by all travelers.
TRC	The sum of the toll costs paid by all travelers over the peak hour.

3.3 Assumptions and General Cost Expressions

To determine the travel departure choice behavior of a homogeneous group of travelers, we make three assumptions about their choices and the structure of the bottleneck.

1) All travelers are purely rational. They will travel only at times that minimize their generalized costs. There is a great deal of interesting research to be done on the estimation of stochastic parameters of traveler choices under uncertainty or non-rationality, but the rationality assumption allows us to retain analytical tractability.
2) Because travelers are purely rational, Wardrop's (1952) principles tell us that the generalized cost of travel will be the same at every time travelers choose to travel at all. If this were not the case, then some travelers could decrease their own costs by choosing travel earlier or later. That is, for all travelers in a homogeneous group,
$$C(t_i) = C(t_j), \qquad t_0 \leq t_i \leq t_j \leq t_f$$
3) The free flow travel time of the bottleneck is zero. A constant free flow term merely adds a constant term to all of the cases in the proof, but the general lessons of the model are most clear when this constant term is left to the side. Thus,
$$FF(t) = 0, \qquad t_0 \leq t \leq t_f$$

Given these assumptions we can make the following identity and definitional statements for the problem.

i. $C(t) = FF(t) + TT(t) + SD(t) + TR(t),\ \forall t\ t_0 \leq t \leq t_f$

ii. $SD(t) = MAX\left(\beta * \left(\tau^* - t - FF - \frac{q(t)}{D}\right), \gamma * \left(t + FF + \frac{q(t)}{D} - \tau^*\right)\right)$

iii. $TT(t) = \alpha * \frac{q(t)}{D}$

iv. $q(t+\delta t) = q(t) + \int_t^{\delta t} (n(t) - D)\, dt$

v. $\tau_0 = t_0$

vi. $\tau_f = \tau_0 + \frac{N}{D}$ and in general $\tau = t + \frac{q(t)}{D}$

Statement (i) is definitional and captures all potential cost components facing the traveler. Depending on the model and the time a particular traveler departs, one or more of these terms may be equal to zero (e.g. travel time costs are zero for the first traveler at the beginning of the peak hour, and schedule delay costs are zero when a traveler arrives exactly at their desired arrival time, and $FF = 0$ at all times for this discussion). The costs of actively traveling, $FF(t)$ and $TT(t)$, are equal to the time spent traveling multiplied by the traveler's value of time, α. Schedule delay costs are determined by the different in actual arrival time and the desired arrival time multiplied by the early or late arrival penalty, β or γ respectively. Because $FF = 0$ we can assert that the first vehicle, facing no queue from prior vehicles, will arrive at its destination at the moment it enters the bottleneck. Furthermore, because the bottleneck will be used at its full capacity for as long as any vehicles travel, N cars can be discharged at a rate D in $\tau_f - \tau_0$ minutes. We know that the bottleneck will be fully utilized to discharge vehicles because if, *arguendo*, it was not, then a vehicle could arrive closer to τ^* without incurring additional costs by traveling at that underutilized moment.

3.4 Analytical Solution and Discussion for Departure Rates, User Costs at t for a Single Traveler Class in User Equilibrium with No Toll

Given Wardrop's principles it is possible to analytically determine rates of departure that will satisfy the equal and minimal cost conditions. The departure rates of travelers in a homogeneous group and the costs experienced by the group are exhaustively examined in an unpublished working paper by Arnott, de Palma, and

Lindsey (1987a). In the following section, I will briefly highlight and then independently derive the most important findings.

The prior assumptions and definitions allow us to derive the rates at which vehicles depart, $n(t)$, and the length of the queue, $q(t)$, for any time t between t_0 and t_f.

$$n(t) = \begin{cases} \frac{\alpha}{\alpha-\beta} D, \ t \le t^* \\ \frac{\alpha}{\alpha+\gamma} D, \ t > t^* \end{cases}$$

$$q(t) = \begin{cases} \frac{\beta}{\alpha-\beta} D(t-t_0), & t \le t^* \\ \frac{\beta}{\alpha-\beta} D(t^*-t_0) - \frac{\gamma}{\alpha+\gamma} D(t-t^*), & t \ge t^* \end{cases}$$

The first moment of departure is related to the ideal arrival time by the following expression:

$$\tau_0 = \tau^* - \frac{\gamma}{\beta+\gamma}\left(\frac{N}{D}\right).$$

Note that $n(t) > D$ at the beginning of the peak hour and so a queue accumulates. Because of this, the vehicle which arrives at τ^* must leave at some time prior to τ^* and this value is given by the expression:

$$t^* = \frac{(\alpha-\beta)\gamma}{\alpha(\beta+\gamma)}\frac{N}{D} + t_0.$$

The departure rates of the Vickrey bottleneck create the departure curves depicted in Figure 1 and help us to understand the shape of departures during the peak hour. The red and green lines are the departures from the origin of travelers who arrive at the destination early and late, respectively. The orange line is the arrivals of vehicles as they are discharged through the bottleneck. The line AE is the time spent in queue by the traveler who arrives at τ^* and the line AG is the length of the queue that must be traversed.

The area of triangle $OABA$ represents the total travel time of all travelers in the system and consequently the travel time costs (TTC) of the system can be found by multiplying this area by α. The area of $OEFO$ and $ECBE$ are the total schedule delay (SDC) of all travelers and so schedule delay costs can be found by multiplying these areas by β and γ, respectively.

If we rotate $OABO$ so that the arrival curve forms the x-axis, we find Figure 2, which shows the queue length facing travelers. The solid red line represents the time at which travelers should depart in order to arrive at the desired arrival time. If the total number of travelers increases, then the green triangle will grow, but its apex will remain along the red line.

Figure 1: Cumulative departure decisions of homogeneous travelers with to toll.

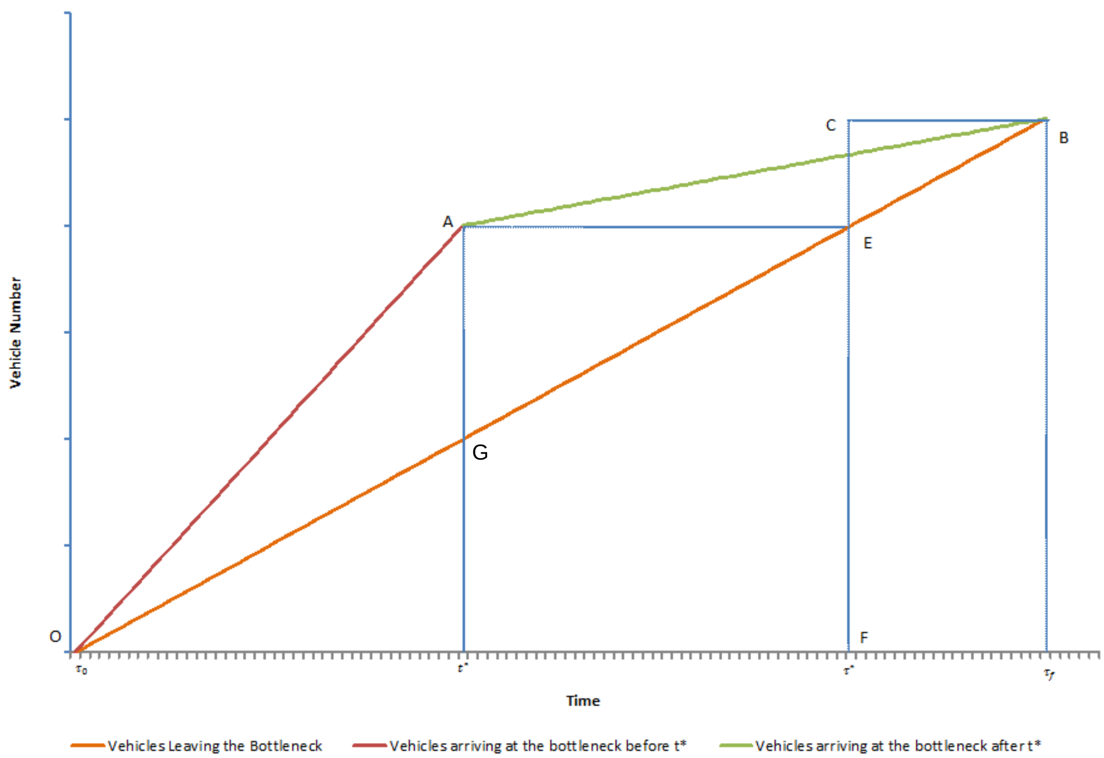
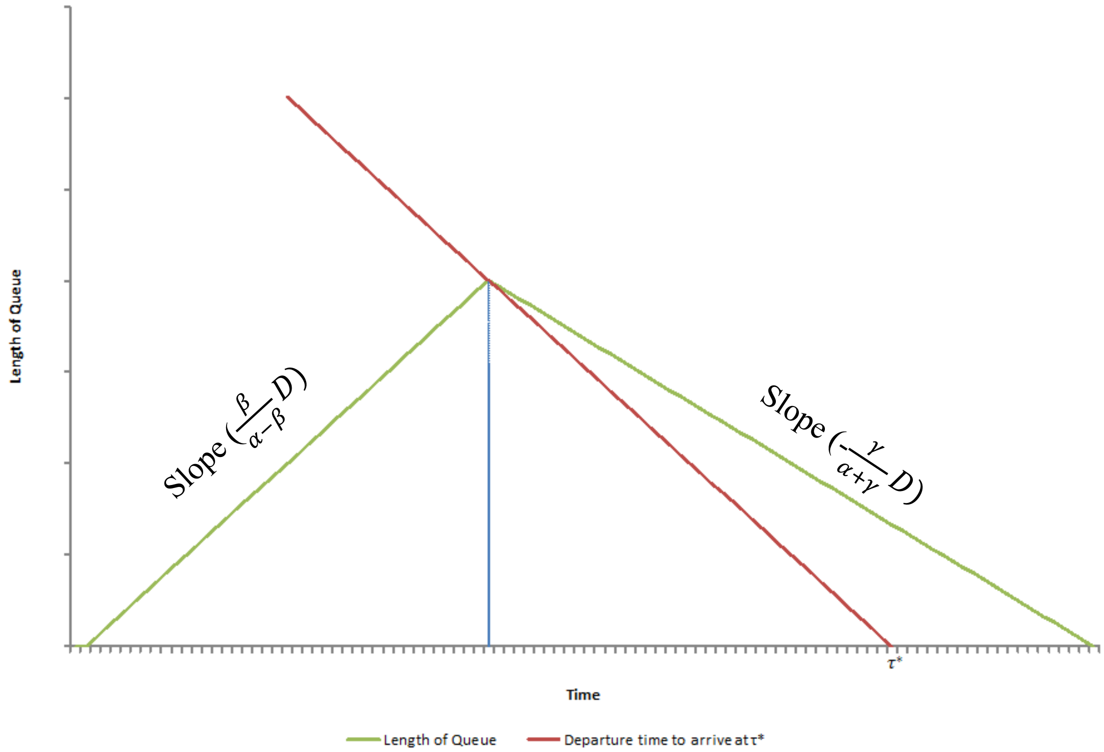


Figure 2: Cumulative queue facing travelers for all times, t.



We will now turn to a rigorous proof of the relations discussed at the beginning of the section. Because the expressions for departure rates and queue lengths are piecewise, let us prove them in a piecewise manner, beginning with finding the four "breakpoints" in the model: $\tau_0, \tau_f, \tau^*, t^*$.

$$\tau_f = \tau_0 + \frac{N}{D}$$

This is the minimum length of time that the peak hour could take because it is the shortest duration over which the entire traveling group could pass through the bottleneck and will occur when the bottleneck is used at capacity over the course of the peak period. τ_f will not occur later than $\tau_0 + \frac{N}{D}$ because, were the bottleneck not to be used at capacity, then some traveler would be able to reduce their schedule delay cost by changing their departure time to take advantage of the unused capacity.

We also know that at the margins of travel the only relevant costs are schedule delay costs. The first traveler does not face a queue, because *ex hypothesi*, she is first. The last traveler also will not face a queue.[14] We also know that the costs of the first and last travelers are equal (or else traveler with a higher cost would travel at a different time); this allows us to find a relationship between the time the first traveler departs, the last traveler departs, and the desired arrival time.

$$\beta(\tau^* - \tau_0) = \gamma(\tau_f - \tau^*)$$
$$\beta(\tau^* - \tau_0) = \gamma\left(\tau_0 + \frac{N}{D} - \tau^*\right)$$
$$(\beta + \gamma)\tau^* = (\beta + \gamma)\tau_0 + \gamma\left(\frac{N}{D}\right)$$
$$\boldsymbol{\tau_0 = \tau^* - \frac{\gamma}{\beta + \gamma}\left(\frac{N}{D}\right)\ Q.E.D.}$$

We now have a relationship between τ^*, τ_0, and τ_f, but because t^* is related to τ^* by $q(t^*)$, we must find a general expression for the length of the queue for all times $t \leq t^*$. Because the cost of travel at all times is equal, we can find the cost of travel at two arbitrary times.

$$C(t) = C(t + \delta t)$$
$$FF(t) + TT(t) + SD(t) + TR(t)$$
$$= FF(t + \delta t) + TT(t + \delta t) + SD(t + \delta t) + TR(t + \delta t)$$

No toll is levied in this scenario, so $TR(t) = 0$ and $SD(t) = \beta\left(\tau^* - t - \frac{q(t)}{D}\right)$

[14] Assume that the last traveler did depart at a time when she faced a queue of length $q(t)$, then she will arrive at the destination at $t + \frac{q(t)}{d}$ and face a total cost of $\alpha * \frac{n}{d} + \gamma * (\tau - \tau^*)$. This traveler could lower her individual cost by choosing to travel at τ and eliminating the cost for waiting in the queue. **Q.E.D.**

$$\beta\left(\tau^* - t - \frac{q(t)}{D}\right) + \alpha\left(\frac{q(t) + n(t) - D}{D}\right)$$
$$= \beta\left(\tau^* - (t + \delta t) - \frac{q(t + \delta t)}{D}\right) + \alpha\left(\frac{q(t + \delta t) + n(t + \delta t) - D}{D}\right)$$
$$\beta\left(\delta t + \frac{q(t + \delta t)}{D} - \frac{q(t)}{D}\right) = \alpha\left(\frac{q(t + \delta t) + n(t + \delta t) - q(t) - n(t)}{D}\right)$$

Because schedule delay costs are linear with respect to time, a constant rate of departure over this time period that increases $q(t)$ at a constant rate will preserve equal cost departures, *ergo* $n(t) = n(t + \delta t)$. Furthermore, by identity (iv) the above equations reduce quite nicely.

$$\beta\left(\frac{n(t)}{D}\right)\delta t = \alpha\left(\frac{n(t)}{D} - 1\right)\delta t$$
$$\boldsymbol{n(t) = \frac{\alpha}{\alpha - \beta} D \; Q.E.D.}$$

Given the rate of departures, the queue length faced by a traveler at any time t is directly calculable. The queue is the difference in the total number of vehicles that have entered the system and those that have left by time t.

$$q(t) = \int_{\mathrm{t=t_0}}^{\mathrm{t}=t} (n(t) - D)\, dt$$
$$q(t) = \int_{\mathrm{t=t_0}}^{\mathrm{t=t}} \left(\frac{\alpha}{\alpha - \beta} D - D\right) dt$$
$$\boldsymbol{q(t) = \frac{\beta}{\alpha - \beta} D(t - t_0) \; Q.E.D.}$$

Now that we have $q(t)$ it is possible to express the relationship between τ^* and t^*.

$$\tau^* = t^* + \frac{q(t^*)}{D}$$
$$\tau^* = t^* + \frac{\beta}{\alpha - \beta}(t^* - t_0)$$
$$\tau^* = \frac{\alpha}{(\alpha - \beta)} t^* - \frac{\beta}{\alpha - \beta}\left(\tau^* - \frac{\gamma}{\beta + \gamma}\frac{N}{D}\right)$$

$$\boldsymbol{t^* = \tau^* - \frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N}{D}} \ \textbf{Q.E.D.}$$

The same procedure provides $n(t)$ and $q(t)$ for $t > t^*$ if we start from the assumption that $C(t) = C(t+\delta t)$. No toll is levied in this scenario, so $TR(t) = 0$ and $SD(t) = \gamma\left(t + \frac{q(t)}{D} - \tau^*\right)$

$$\gamma\left(t + \frac{q(t)}{D} - \tau^*\right) + \alpha\left(\frac{q(t)+n(t)-D}{D}\right)$$

$$= \gamma\left(t + \delta t + \frac{q(t+\delta t)}{D} - \tau^*\right) + \alpha\left(\frac{q(t+\delta t)+n(t+\delta t)-D}{D}\right)$$

$$\gamma\left(\delta t + \frac{q(t+\delta t)-q(t)}{D}\right) = \alpha\left(\frac{q(t)-q(t+\delta t)}{D}\right)$$

$$\gamma(D + n(t) - D)\delta t = -\alpha(n(t) - D)\delta t$$

$$\boldsymbol{n(t) = \frac{\alpha}{\alpha+\gamma}D \ \ Q.E.D.}$$

This departure rate in turns entails the queue length for any time t, $t \geq t^*$.

$$q(t) = \int_{\mathrm{t=t_0}}^{\mathrm{t}=t} (n(t) - D)\, dt$$

$$q(t) = \int_{\mathrm{t=t_0}}^{\mathrm{t=t^*}} \frac{\beta}{\alpha-\beta} D\, dt + \int_{\mathrm{t=t^*}}^{\mathrm{t=t}} \left(\frac{\alpha}{\alpha+\gamma}D - D\right)\, dt$$

$$\boldsymbol{q(t) = \frac{\beta}{\alpha-\beta}D(t^* - t_0) - \frac{\gamma}{\alpha+\gamma}D(t - t^*) \ \ Q.E.D.}$$

We will later consider heterogeneous groups of travelers who differ in the value of time (α), early (β), and late (γ) arrival penalties. When that happens, it will be useful to know how many travelers in a group are arriving early and how many will choose a travel strategy to arrive late. We find this ratio by multiplying the departure rate for a group by the amount of early or late periods that they travel. Thus, the early arrival group is found by the following formula.

$$Total\ Early\ Arrivals = \int_{t_0}^{t^*} n(t)dt = \frac{\alpha}{\alpha-\beta}D * (t^* - t_0) = \frac{\gamma}{\beta+\gamma}N$$

The number of late arrivals follows.

$$Late\ Arrivals = Total\ Arrivals - Total\ Early\ Arrivals = \frac{\beta}{\beta+\gamma}N$$

The sum of the cost for all vehicles over the entire peak hour in a no-toll model to travel can be broken down into schedule delay and the travel cost.

$$TC = SDC + TTC$$

Let us begin with the schedule delay costs. Figure 2 is helpful in visualizing the integral.

$$SDC = \int_{t_0}^{t^*} \beta Dt\,dt + \int_{t^*}^{t_f} \gamma(N - Dt)\,dt = \frac{\beta D}{2}{t^*}^2 + \gamma N\left(t_f - t^*\right) - \frac{\gamma D}{2}(t_f^2 - {t^*}^2)$$
$$= \beta\frac{D}{2} * \left(\frac{\gamma N}{(\beta+\gamma)D}\right)^2 + \gamma N\left(\frac{N}{D} - \frac{\gamma N}{(\beta+\gamma)D}\right) - \gamma D\left(\frac{N^2}{2D^2} - \frac{1}{2}\left(\frac{\gamma N}{(\beta+\gamma)D}\right)^2\right)$$
$$= \frac{N^2}{D} * \left(\frac{\beta\gamma^2}{2(\beta+\gamma)^2} + \frac{\gamma}{2} - \frac{\gamma^2}{\beta+\gamma} + \frac{\gamma^3}{2(\beta+\gamma)^2}\right)$$
$$= \frac{N^2}{D} * \frac{\beta\gamma^2 + \gamma(\beta+\gamma)^2 - 2(\beta+\gamma)\gamma^2 + \gamma^3}{2(\beta+\gamma)^2}$$
$$= \frac{N^2}{2(\beta+\gamma)^2 D} * (\beta\gamma^2 + \beta^2\gamma + 2\beta\gamma^2 + \gamma^3 - 2\beta\gamma^2 - 2\gamma^3 + \gamma^3)$$
$$= \frac{N^2}{2(\beta+\gamma)^2 D} * (\beta\gamma^2 + \beta^2\gamma) = \frac{1}{2}\frac{\beta\gamma N^2}{(\beta+\gamma)D}$$

Queue time costs are found by multiplying traveler's value of time, α, by the length of time they spend in queue, i.e. length of queue divided by the discharge rate. It makes sense, then, that we should conceptualize this integral using the queue length graph (Figure 2).

$$TTC = \alpha * \frac{1}{2}(t_f - t_0)\left(\frac{\beta}{\alpha-\beta}D(t^* - t_0)\right) = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha-\beta}N\right)\left(\frac{(\alpha-\beta)\gamma}{\alpha(\beta+\gamma)}\frac{N}{D}\right)$$
$$= \frac{1}{2}\frac{\beta\gamma}{(\beta+\gamma)}\frac{N^2}{D}$$

Thus, $TC = C(SDC) + C(TTC) = \frac{\beta\gamma}{(\beta+\gamma)}\frac{N^2}{D}$.

We can verify the relationships between departure rates, queuing behaviors, and costs through a script that simulates the departure behavior described by the $n(t)$

equations and then evaluates TT(t) and SD(t). This numerical evaluation ensures that the departure profile results in equal costs trips for all travelers and fulfillment of the Wardrop conditions.[15,16]

Table 4: 1 group homogeneous travelers- Base case parameters.

Parameter	Value
D	6 (veh/min)
N	60 veh
α	6 (₪/min)[17]
α/β	3
γ/β	4

Using a base case with parameters described in Table 4 creates an appropriate departure curve (Figure 3) and queue profile (Figure 4). We can verify that every traveler chooses to depart at an equal and minimal cost time through inspection of Figure 5, which shows that travelers in the user equilibrium departure profile exchange schedule delay costs for travel time costs at an equal rate as they choose to arrive closer to τ^*.

Table 5 examines the sensitivity of the 1-group user equilibrium no-toll scenario to changes in the parameters. Case 1 is the base case against which all changes are individually measured (and corresponds to the relationships found in Table 4). The size of the variation tested in the table is not intended to represent "realistic" variations of the

[15] The code was developed in R-studio using R version 3.3.1 ("Bug in Your Hair"). Complete copies of the source code are available on request from the author. All scenarios were evaluated with 1,000,000 time steps.

[16] The current case is so simple it hardly warrants a numerical validation of the results. However, cases with heterogeneous travelers are less amenable to an analytic solution, so the development of a numerical solution in this case serves as a sanity check for later results.

[17] The Israeli shekel (₪) symbol is used to further underscore the break from empirical research and to guard against temptations to see these values in terms of US dollars or other familiar currency.

values, but solely to give an idea of the spread created by various values that are constrained by requirements of the model (e.g. $1 > \frac{\alpha}{\beta}$ and $1 < \frac{\alpha}{\gamma}$).

Figure 3: Departure and arrival profile for 1 group of travelers.

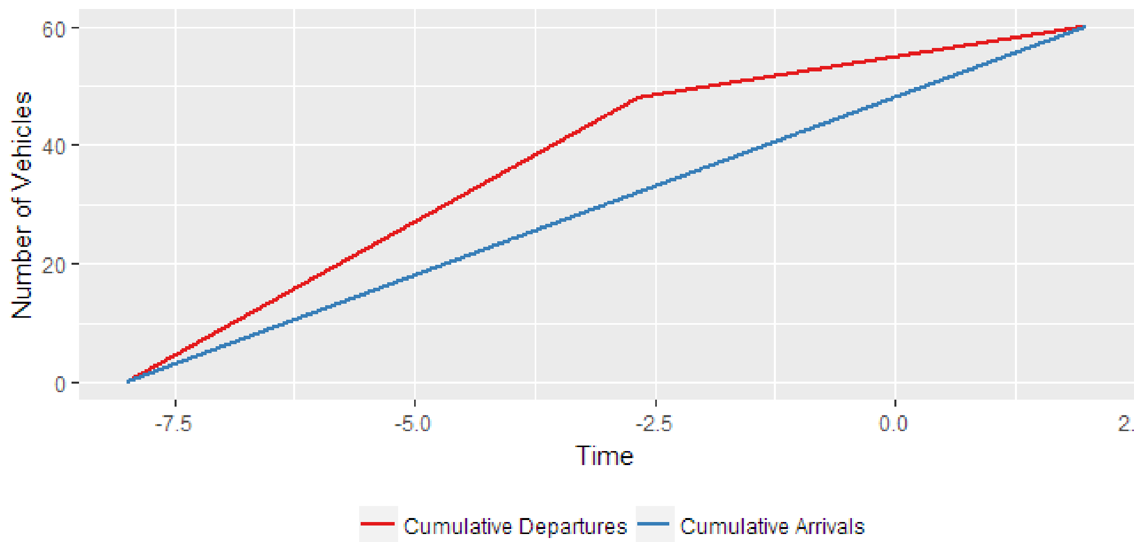


Figure 4: Queue length profile for 1 group of travelers.

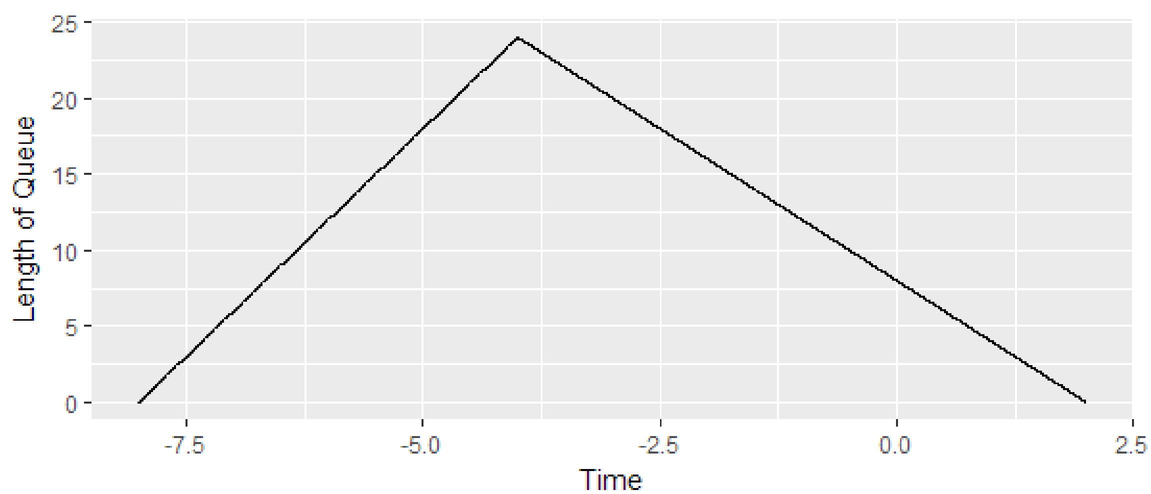
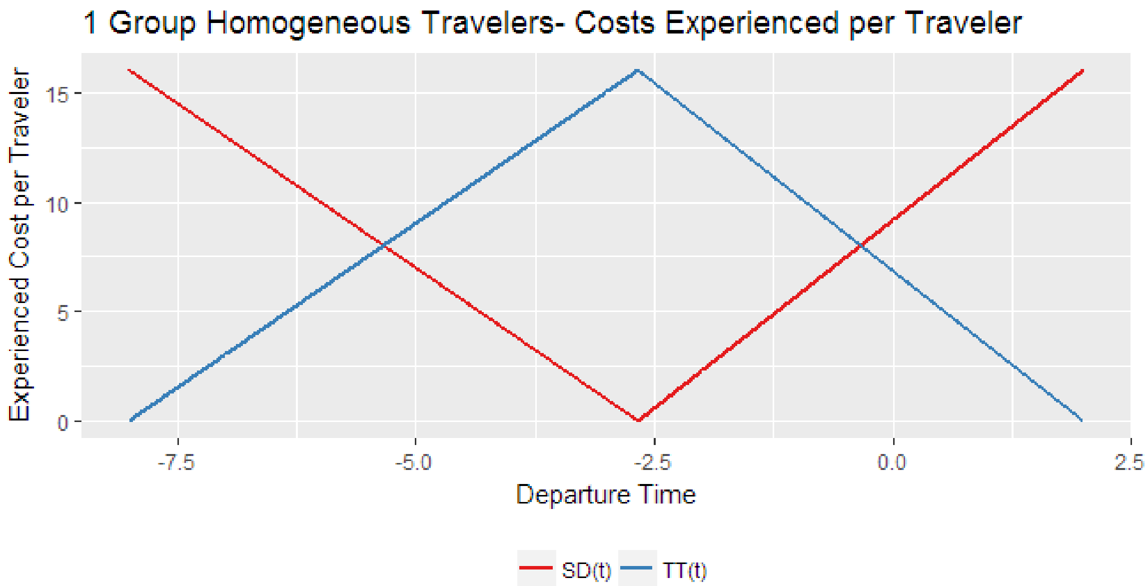


Figure 5: Schedule delay and travel time costs for 1 group of travelers.



The most interesting relationship occurs when the relationship between $\alpha, \beta, \& \gamma$ is varied. Because β controls the relative cost of spending money on a toll or arriving early, the social cost always decreases as β decreases relative to α; people are more willing to arrive early and so queuing costs and schedule delay costs decrease even as the total amount of time spent in schedule delay remains the same. As $\alpha/\beta \rightarrow \infty$, travelers will choose to depart at the discharge rate, D. The ratio of β/γ determines the number of travelers who choose to arrive before τ^* and those who choose to arrive later. As β/γ increases, more travelers choose to arrive at their destination late, and this decreases the social costs of all travelers as the arrival distribution becomes more normal.

Table 5: Sensitivity analysis for a 1 group, no-toll system user equilibrium.

No.	Discharge Rate	N	α	β	γ	SDC	TTC	TC
1	6	60	6.00	2.00	8.00	479.99	479.99	959.99
2	4	60	6.00	2.00	8.00	719.99	720.00	1439.99
3	8	60	6.00	2.00	8.00	359.99	360.00	719.99
4	6	40	6.00	2.00	8.00	213.32	213.33	426.65
5	6	80	6.00	2.00	8.00	853.32	853.33	1706.65
6	6	60	4.00	1.33	5.33	319.99	320.00	639.99
7	6	60	8.00	2.67	10.67	639.99	639.99	1279.98
8	6	60	6.00	2.40	9.60	575.99	576.00	1151.99
9	6	60	6.00	1.71	6.86	411.42	411.42	822.83
10	6	60	6.00	2.00	7.00	466.65	466.65	933.30
11	6	60	6.00	2.00	9.00	490.90	490.90	981.80

3.5 Analytical Solution and Discussion for Departure Rates, User Costs at t for a Single Traveler Class in User Equilibrium with a System Optimal Toll

The fixed arrival curve means that SDC is ineliminable and so provides a lower bound on TC. In order to move closer to this optimum, the next scenario to consider is to find a tolling scheme that eliminates queuing delay by users. This is done by creating a time-dependent toll. For travelers departing at $t \leq t^*$ the toll rate is $\beta(t - t_0)$ and that for travelers departing at $t > t^*$, the toll rate is $\beta(t^* - t_0) - \gamma(t - t^*)$. Because there is no queuing, in a system with a system optimal toll, $t^* = \tau^*$. For $t \leq \tau^*$, the objective of eliminating queuing occurs if $n(t) = D, \tau_0 \leq t \leq \tau_f$. We can find the rate of increase of the toll by examining the cost of traveling at any two arbitrary times and enforcing a queue length of 0 ($q(t) = 0$).

$$C(t) = C(t + \delta t)$$

$$FF(t) + SD(t) + TT(t) + TR(t)$$
$$= FF(t + \delta t) + SD(t + \delta t) + TT(t + \delta t) + TR(t + \delta t)$$

$$\beta(\tau^* - t) + TR(t) = \beta(\tau^* - t - \delta t) + TR(t + \delta t)$$

$$TR(t + \delta t) - TR(t) = \beta\ \delta t$$

The toll increases at a rate of β and is 0 when $t = 0$, so $TR(t) = \beta(t - t_0)$ up to a maximum of $TR(\tau^*) = \beta(\tau^* - \tau_0)$.

After t^*, the toll begins to decrease. For $t \geq t^*$:

$$C(t) = C(t + \delta t)$$

$$\gamma(t - \tau^*) + TR(t) = \gamma(t + \delta t - \tau^*) + TR(t + \delta t)$$

$$TR(t + \delta t) - TR(t) = -\gamma\ \delta t$$

Thus, $TR(t) = \beta(\tau^* - t_0) - \gamma(t - \tau^*)$ down to a minimum of $TR(\tau_f) = 0$.

The cost to travel when a system optimal toll is in place has two components.

$$TC = SDC + TRC$$

Because the arrival rate with a system optimal toll remains the same as under a no-toll user equilibrium, the schedule delay costs are the same.

$$SDC = \frac{1}{2} \frac{\beta\gamma}{(\beta + \gamma)} \frac{N^2}{D}$$

And, because the toll substitutes for the cost of congestion (the rate of increase for the toll is determined entirely by the cost required to prevent any traveler from having an incentive to queue), TRC is determined by the difference between TC and SDC.

$$TRC = TC - SDC = \frac{1}{2}\frac{\beta\gamma N^2}{(\beta+\gamma)D}$$

The difference between the toll and no-toll scenarios, of course, is that TTC in the no-toll scenario is a social dead loss. Any time spent in queue is time wasted and so α essentially represents the value of non-productive time. In contrast, TRC represents money collected by a tolling agency. And if the traveling public is reimbursed by the toll agency, then as a whole society will have suffered no loss. Obviously, travelers cannot be reimbursed by the exact amount that they paid into the system. A dollar for dollar reimbursement would mean that, although a traveler may need to wait a few weeks to receive their reimbursement check, they aren't actually paying the amount of the toll. Thus, travelers would begin to queue again.

This bug will appear in any reimbursement scheme that reimburses travelers according to the fees they pay. However, a flat reimbursement rate to all travelers will not create an incentive to spend more in tolls and so would preserve the social gain of preventing congestion while returning as much as $\frac{1}{2}\frac{\beta\gamma}{(\beta+\gamma)}\frac{N}{D}$ to each traveler. This number would be reduced by any overhead expenses and inescapable inefficiencies required to process the payments, but would nonetheless represent a significant rebate for all travelers. Litman (1996; 2015; 2016) argues that, because road networks are already subsidized by taxes on non-drivers and through various externalities, the toll-payers are not even justified in demanding that the rebate be given to them as a class. Instead it ought to be spent on social priorities that benefit all of the public, such as water infrastructure, mass transit or even parks. Skipping a detailed debate on this reimbursement scheme, the system optimal tolling scheme provides a strategy for capturing the social cost of queuing congestion and distributing that as a benefit.

Table 6 examines the sensitivity of the 1-group tolling scheme to changes in the parameters. As with Table 5, case 1 is the base case against which all changes are individually measured. The parameter variations appear in the same order as Table 5 to facilitate comparison between the no-toll and tolled equilibria.

Table 6: Sensitivity analysis for a 1 group, tolled system optimal scheme.

No.	Discharge Rate	N	α	β	γ	SDC	TRC	TC
1	6	60	6.00	2.00	8.00	480.00	480.00	960.00
2	4	60	6.00	2.00	8.00	720.00	720.00	1440.00
3	8	60	6.00	2.00	8.00	360.00	360.00	720.00
4	6	40	6.00	2.00	8.00	213.33	213.33	426.66
5	6	80	6.00	2.00	8.00	853.33	853.33	1706.66
6	6	60	4.00	1.33	5.33	320.00	320.00	640.00
7	6	60	8.00	2.67	10.67	640.00	640.00	1280.00
8	6	60	6.00	2.40	9.60	576.00	576.00	1152.00
9	6	60	6.00	1.71	6.86	411.43	411.43	822.86
10	6	60	6.00	2.00	7.00	466.67	466.67	933.33
11	6	60	6.00	2.00	9.00	490.91	490.91	981.82

TRC perfectly substitutes (within rounding error) toll costs for travel time costs, TTC. The most interesting relationship occurs by changing the relationships between $\alpha, \beta, \& \gamma$. Because β controls the relative cost of spending money on a toll or arriving early, the social cost always decreases as β decreases relative to α; people are more willing to arrive early and so less of a toll is needed to deter travelers from queuing. The ratio of β/γ determines the number of travelers who choose to arrive before τ^* and those who choose to arrive later. As β/γ increases, more travelers choose to arrive at their destination late, and this decreases the social costs of all travelers as the cost distribution becomes more normal.

3.6 Analytical and Numerical Solution and Discussion for Departure Rates, User costs at t for Two Traveler Classes in User Equilibrium with No Toll

The prior two sections presented the results of no-toll and system optimal toll equilibria for a group of traveler class with identical $\alpha, \beta, \& \gamma$. But the world is not filled by identical clones who all want to travel to the same destination at the same time. Next consider a scenario where there are several distinct classes of travelers, each with their own $\alpha, \beta, \& \gamma$. Examining this scenario will shine a light on potential equity issues with dynamic toll rate proposals because it will be possible to understand the disaggregated effects of such a proposal on each group.

Let there be K distinct classes of travelers of size $n_1, n_2, \dots, n_k$which are characterized by differing $\alpha, \beta,$ and γ, but constrained so that $\eta = \frac{\beta_k}{\gamma_k}\ \forall k \in K$. Let the value of time and schedule penalties be assigned to groups such that the groups can be ordered $\alpha_1 > \alpha_2 > \cdots > \alpha_k$. The intuitive representation of such a case is that the absolute value of time decreases as the number of the group increases and the ratio of the early to late schedule delay penalties are constant across all traveler groups. Having made these assumptions, let us then consider two cases: 1) $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2} > \cdots > \frac{\alpha_k}{\beta_k}$ and 2) $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2} < \cdots < \frac{\alpha_k}{\beta_k}$. The ratio of $\frac{\alpha}{\beta}$ represents the relationship between the absolute value of time while traveling and the early arrival schedule delay penalty; this ratio will always be greater than 1. Were it to be less than 1, then we would be modeling travelers who derive more benefit from waiting in traffic than arriving early to do paperwork or even sitting in the company parking lot. The shape of the peak rush hour and the fact that most workers will do what they can to arrive at work a few minutes early shows that the assumption is reasonable.

In the first case, the traveler group with the highest absolute value of time has the lowest relative early (and by extension late) schedule delay penalty of all the traveler groups. Think here of a white-collar worker whose work is highly flexible. They would prefer to come to the office from 9am-5pm, say, but would be willing to shift their day's schedule to 8-4 if that meant they would have a shorter commute. At the other end of the spectrum are workers who have a low value of time while delayed in traffic in absolute terms, but whose early delay penalty is almost as large as their overall value of time. This situation might most closely describe a blue collar shift worker who is not paid for arriving early but will be heavily penalized for arriving late. In the second case, although group 1 has the highest absolute value of α and so possesses the greatest distaste for waiting in traffic overall, they are, relatively speaking, more amenable than group 2 to arriving early in order to avoid that traffic.

Arnott, de Palma, and Lindsey (1987a) prove that the departure profile of heterogeneous travelers is highly analogous to the homogeneous cases examined in Section 3.4 & Section 3.5. Consider an early traveler in group 1 as though all travelers had her characteristics. This traveler is able to depart at any time prior to t^* where $n(t) = \frac{\alpha_1}{\alpha_1 - \beta_1} D$. If $n(t)$ is greater than this, then the length of the queue will grow so quickly that $TT(t + \delta t) + SD(t + \delta t) > TT(t) + SD(t)$ and group 1 will stop traveling. Conversely, if $n(t)$ is less, then more travelers from group 1 will have an incentive to travel at t until $n(t)$ rises. This is exactly the same behavior as was modeled in Section 3.4.

These incentives don't change for the individual just because other individuals have different priorities. Each individual minimizes her travel cost by choosing to travel with other individuals who have the same characteristics. Thus, the size of K is unimportant, because the structure of the departure choice model means that any conclusions we draw from a case with even two distinct traveler groups can be extended to an arbitrarily large number of groups.

It is possible to derive the departure profiles through the same methods employed in Section 3.4 and full details of the proof can be found in Appendix B of Arnott, de Palma, and Lindsey (1987a). As α/β decreases, $n(t)$ consistent with an equal cost departure increases. Thus, if $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$, then group 1 will depart first *regardless of whether* $\alpha_1 \geq \alpha_2$. If $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2}$, then group 2 will depart first. Under both of these scenarios the two groups will depart at separate times and there will be no overlap. But, if $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$, the two groups will behave as though they are identical, because although the absolute cost of traveling at any arbitrary t will differ across groups, the cost of traveling at t to each group will be equal and minimal over the entire peak hour. Thus, each group's departure rates are described by the following equation.

$$n_i(t) = \begin{cases} \dfrac{\alpha_i}{\alpha_i - \beta_i} D, & \text{when early} \\ \dfrac{\alpha_i}{\alpha_i + \gamma_i} D, & \text{when late} \end{cases}$$

Note that the conditions under which the early and late departure rates for group i are no longer explicitly indexed to t^*. This is because when even two groups depart, there will be four departure rates over the peak hour: group 1 early, group 2 early, group 2 late, and group 1 late. Call the transition time when the early travelers from group 1 cease traveling and the early travelers from group 2 begin t_{12} and let t_{21} be the time when the reverse occurs. Peak hours with more groups will have correspondingly more departure rates.

Because there are more pieces to the piecewise function describing departure rates in the system, it is now more convenient to express the queue length as an integral or summation rather than in its most expanded form.

$$q(t) = \int_{t_0}^{t} n(t)\, dt$$

The ratio of early to late travelers for each group remains the same and so because all traveling groups in this scenario possess the same ratio of β/γ, the first and last moments of departure is related to the ideal arrival time in the same way as though there were a single group traveling whose size is equal to the sum of all individual groups. ($N = \sum_i N_i$.)

$$\tau_0 = \tau^* - \frac{\gamma}{\beta+\gamma}\left(\frac{N}{D}\right)$$
$$\tau_f = \tau^* + \frac{\beta}{\beta+\gamma}\left(\frac{N}{D}\right)$$

The other time-related "breakpoints" in the departure rates are given in a two group scenario by the following equations.

$$t^* = \tau^* - \frac{\beta_1}{\alpha_1}\frac{\gamma}{\beta+\gamma}\frac{N_1}{D} - \frac{\beta_2}{\alpha_2}\frac{\gamma}{\beta+\gamma}\frac{N_2}{D}$$
$$t_{12} = \tau^* - \frac{\beta_1}{\alpha_1}\frac{\gamma}{\beta+\gamma}\frac{N_1}{D} - \frac{\gamma}{\beta+\gamma}\frac{N_2}{D}$$
$$t_{21} = \tau^* - \frac{\beta_1}{\alpha_1}\frac{\gamma}{\beta+\gamma}\frac{N_1}{D} + \frac{\beta}{\beta+\gamma}\frac{N_2}{D}$$

Let us closely examine a numerical case. As in Section 3.4, the chosen values of time and schedule delay were not selected based on real world conditions, but rather to better understand the tradeoffs involved among groups with differing preferences.

Table 7: 2 group heterogeneous travelers- Base case parameters.

Parameter	Value	Parameter	Value
D	6 (veh/min)		
N_1	30 veh	N_2	30 veh
α_1	24 (₪/min)	α_2	12 (₪/min)
α_1/β_1	3	α_2/β_2	2
γ_1/β_1	4	γ_2/β_2	4

Figure 6: Departure and arrival rate profile for 2 groups of travelers in a no-toll equilibrium.

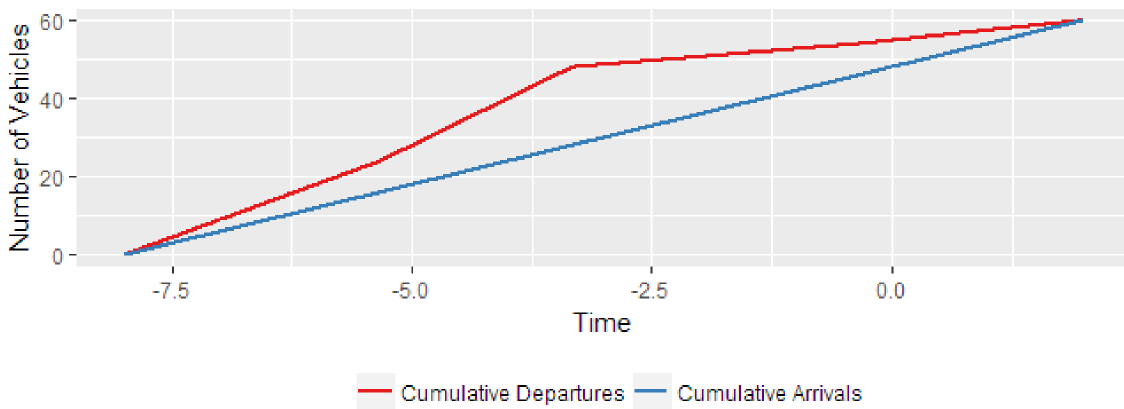


Figure 7: Queue length and composition for 2 groups of travelers in a no-toll equilibrium.

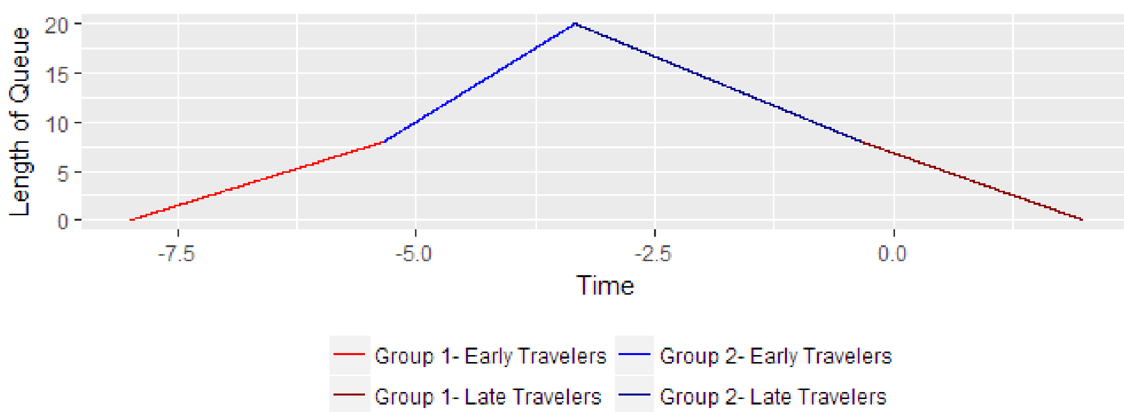


Figure 8, Figure 9, and Figure 10, display the schedule delay cost, travel time cost, and total costs profiles for base case. In each figure, the costs are displayed as the cost that an individual traveler would face should she choose to depart at that moment in time. SDC, TTC, and TC can be found by summing the area under the solid line and multiplying it by the appropriate value of time or schedule delay factor. In Figure 8, note that schedule delay costs for each group change linearly, but that there is a piecewise jump between groups, because $\beta_1 > \beta_2$ & $\gamma_1 > \gamma_2$ and that per traveler schedule delay costs find a minimum at t^* (-3.33 minutes) rather than τ^*. In Figure 9, note that $TT(t)$ is piecewise, which reflects the increase in the rate of queue accumulation aft t_{12}. Finally, Figure 10 shows the sum of values in Figure 8 and Figure 9. We see that each group travels at its minimal period and that every chosen travel time has minimal cost.

Figure 8: Schedule delay cost profile for 2 groups of travelers in a no-toll equilibrium.

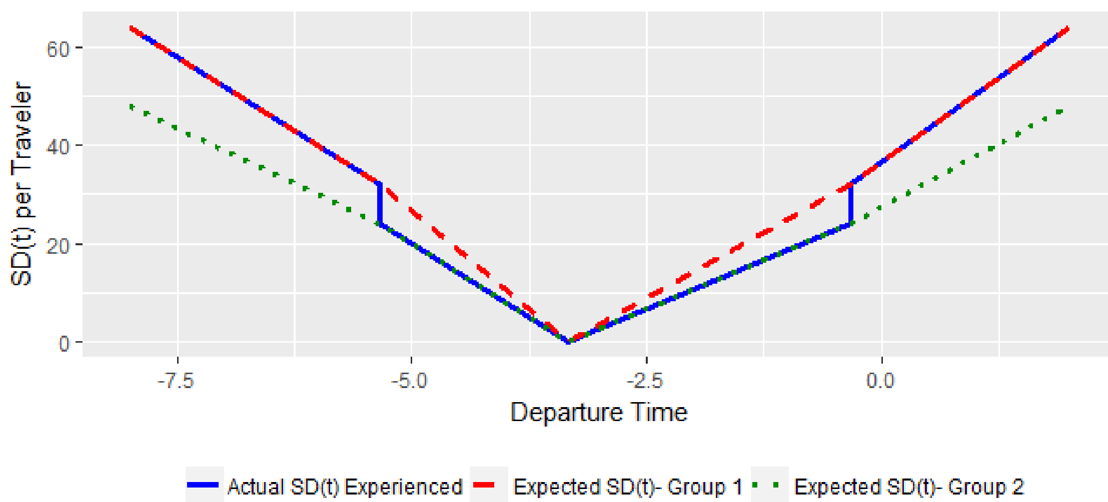


Figure 9: Travel time cost profile for 2 groups of travelers in a no-toll equilibrium.

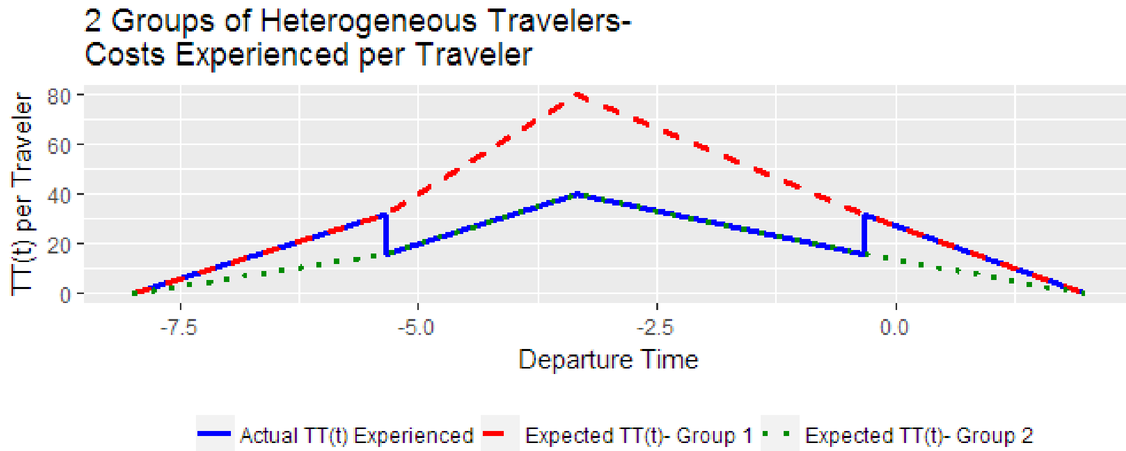


Table 8 provides a picture of the sensitivity of the model to changes in all parameters. In scenarios 1-19, $\alpha_1 > \alpha_2$ and is usually about twice as large. As a naïve rule, then, we might expect the total cost accrued by group 1, TC_1, to be about twice as large as the cost accrued by group 2, TC_2. In the base case, group 1 pays just under twice the cost of group 2 – 61.54% of TC is borne by group 1. Scenarios 2 and 3 show that a change in the discharge rate, D, proportionally increases or decreases the total cost of the system, but maintains the proportion of costs.

Figure 10: Total cost profile for 2 groups of travelers in a no-toll equilibrium.

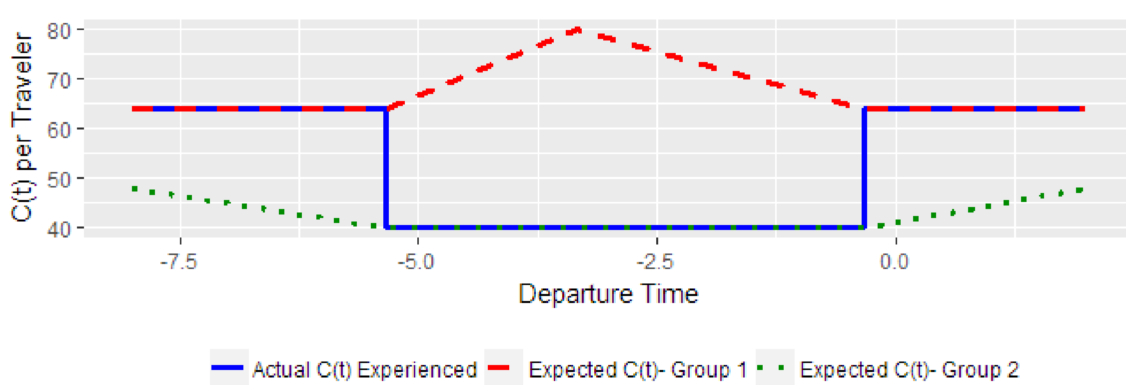


Table 8: Sensitivity of schedule delay, travel time, and total costs to parameters for the no-toll equilibrium. (Subscripts indicate group number.)

No.	D	N_1	α_1	β_1	γ_1	N_2	α_2	β_2	γ_2	SDC	SDC_1	SDC_2	TTC	TTC_1	TTC_2	TC	TC_1	TC_2
1	6	30	24	8	32	30	12	6	24	1800.0	1440.0	360.0	1320.0	480.0	840.0	3120.0	1920.0	1200.0
2	4	30	24	8	32	30	12	6	24	2700.0	2160.0	540.0	1980.0	720.0	1260.0	4680.0	2880.0	1800.0
3	8	30	24	8	32	30	12	6	24	1350.0	1080.0	270.0	990.0	360.0	630.0	2340.0	1440.0	900.0
4	6	30	16	5.3	21.3	30	8	4	16	1200.0	960.0	240.0	880.0	320.0	560.0	2080.0	1280.0	800.0
5	6	30	32	10.7	42.7	30	16	8	32	2400.0	1920.0	480.0	1760.0	640.0	1120.0	4160.0	2560.0	1600.0
6	6	30	24	8.0	28.0	30	12	6	21	1750.0	1400.0	350.0	1283.3	466.7	816.7	3033.3	1866.7	1166.7
7	6	30	24	8	36	30	12	6	27	1840.9	1472.7	368.2	1350.0	490.9	859.1	3190.9	1963.6	1227.3
8	6	30	18	6	24	30	12	6	24	1440.0	1080.0	360.0	1200.0	360.0	840.0	2640.0	1440.0	1200.0
9	6	30	30	10	40	30	12	6	24	2160.0	1800.0	360.0	1440.0	600.0	840.0	3600.0	2400.0	1200.0
10	6	30	24	9.6	38.4	30	12	6	24	2088.0	1728.0	360.0	1512.0	576.0	936.0	3600.0	2304.0	1296.0
11	6	30	24	6.9	27.4	30	12	6	24	1594.3	1234.3	360.0	1182.9	411.4	771.4	2777.1	1645.7	1131.4
12	6	30	24	8	32	30	6	3	12	1620.0	1440.0	180.0	900.0	480.0	420.0	2520.0	1920.0	600.0
13	6	30	24	8	32	30	18	9	36	1980.0	1440.0	540.0	1740.0	480.0	1260.0	3720.0	1920.0	1800.0
14	6	30	24	8	32	30	12	8	32	1920.0	1440.0	480.0	1440.0	480.0	960.0	3360.0	1920.0	1440.0
15	6	30	24	8	32	30	12	4.8	19.2	1728.0	1440.0	288.0	1248.0	480.0	768.0	2976.0	1920.0	1056.0
16	6	20	24	8	32	30	12	6	24	1213.3	853.3	360.0	893.3	213.3	680.0	2106.7	1066.7	1040.0
17	6	40	24	8	32	30	12	6	24	2493.3	2133.3	360.0	1853.3	853.3	1000.0	4346.7	2986.7	1360.0
18	6	30	24	8	32	20	12	6	24	1280.0	1120.0	160.0	960.0	480.0	480.0	2240.0	1600.0	640.0
19	6	30	24	8	32	40	12	6	24	2400.0	1760.0	640.0	1760.0	480.0	1280.0	4160.0	2240.0	1920.0
20	6	30	12	4	16	30	24	12	48	1440.0	720.0	720.0	1920.0	240.0	1680.0	3360.0	960.0	2400.0
21	6	30	24	12	48	30	12	4	16	2400.0	2160.0	240.0	1680.0	720.0	960.0	4080.0	2880.0	1200.0

Cases 16-19 show that changing the number of travelers in one group but not the other creates some changes in cost for both groups, but that the effects are largely confined to the group whose numbers change. For instance, a 33% drop in the number of group 1 travelers (Case 16) decreases SDC for group 1 but not group 2 as would be expected since the portion of the peak period where group 2 arrives does not change. But the gains in welfare from decreased TTC are shared. Of the 32.3% decrease in TTC overall, group 1 experiences a 55.6% decrease in its own TTC, while group 2 experiences a 19% decrease in TTC. Scenarios 8, 9, 12, and 13 show that when α varies (and this change is carried over into the schedule delay terms), the corresponding decrease in social cost is confined to the group whose α changes. This result makes perfect sense since the variation does not affect a group's internal relative penalties to queue or arrive early.

Finally, I want to draw the reader's attention to cases 20 and 21, which examine traveler behavior when the lower wage group also has a lower relative schedule delay penalty. In cases 20 and 21, we see the results of a test where the departure order is not enforced. In each of these scenarios group 1 is manually compelled to travel first and group 2 travels at τ^*, regardless of whether $\frac{\alpha_1}{\beta_1} \geq \frac{\alpha_2}{\beta_2}$. Figure 11 displays the total cost profile when group 1 is constrained to travel first in case 21. Case 20 reports the cost results that result from following proper departure order. Case 21 reports the cost if they travel in the wrong order. Figure 11 shows the total cost to each traveler when they travel out of order. Each group travels during a time when its cost profile is equal but not minimal. Each group could lower their cost by choosing to travel at another time. In contrast, Figure 12 shows the total cost profile of Scenario 20 where the groups are allowed to travel in the order specified at the beginning of the section, i.e. in order of decreasing α/β. These two figures visually prove the correctness of the departure order and we can also verify from Table 8 that the correct order results in lower total costs than an insistence that the groups travel in some other order.

Figure 11: Total individual cost profile for 2 groups when $\alpha_1/\beta_1 < \alpha_2/\beta_2$ and group 1 is constrained to travel first. (Case 21.)

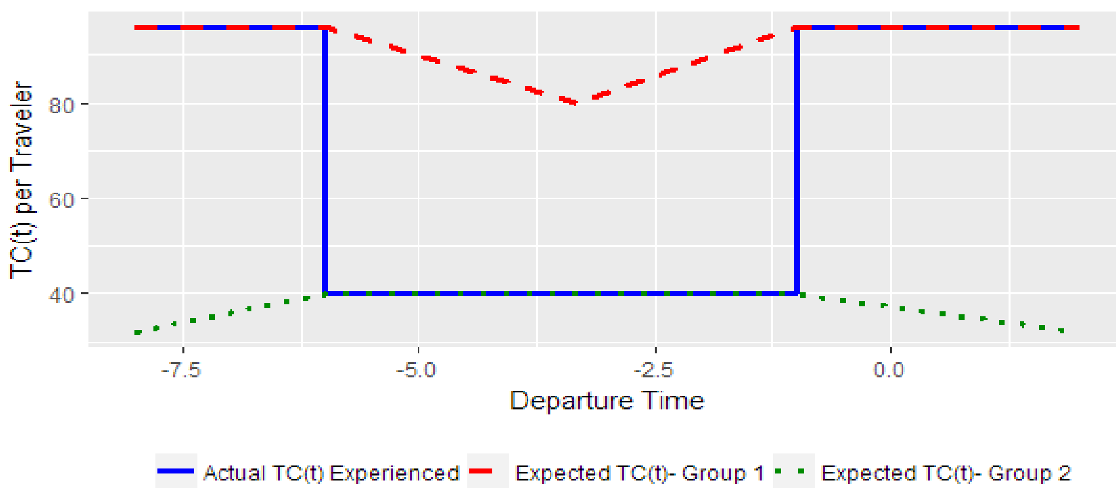
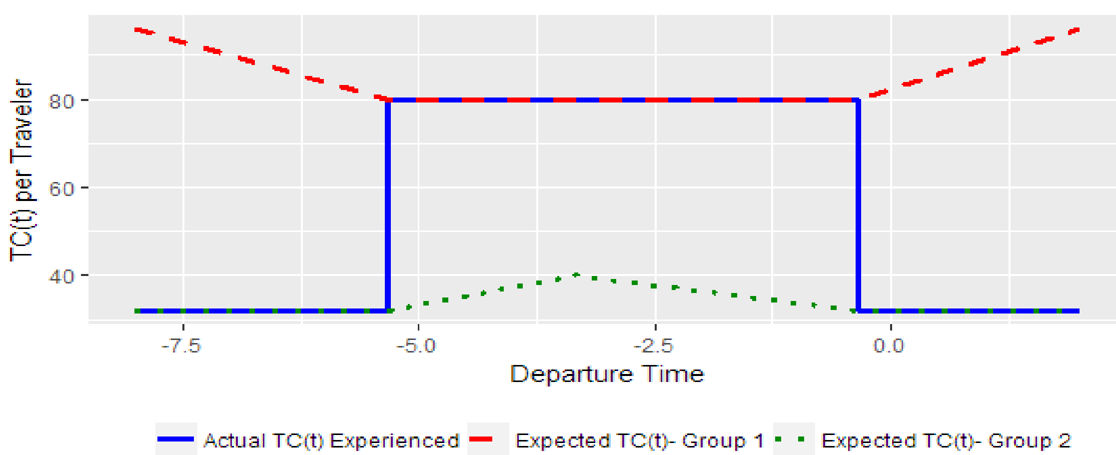


Figure 12: Total individual cost profile for 2 groups when $\alpha_1/\beta_1 < \alpha_2/\beta_2$, but groups travel with decreasing α/β. (Case 20.)



3.7 Analytical and Numerical Solutions and Discussion for Departure Rates, User Costs at t for Two Traveler Classes in User Equilibrium with a System Optimal Toll

Section 3.6 extended the single group no-toll equilibrium to a case with two distinct traveling groups. This section does the same for the Section 3.5 system optimal equilibrium. As in Section 3.5, the system optimal toll, which minimizes TC, considered by Arnott, de Palma, and Lindsey has two properties: 1) a single toll is charged to all travelers who decide to depart at t, call this $TR(t)$; 2) the optimal toll is one that increases quickly enough travelers will choose not to queue but not so quickly that the bottleneck will be used at less than full capacity.[18]

With these restrictions in mind and the lesson from Section 3.5 that the toll rate profile is determined by the schedule delay costs of the traveler group (and not the general value of time), the toll rate for two groups of travelers are given by the following set of equations. Let there be K distinct classes of travelers of size $n_1, n_2, \dots, n_k$which are characterized by differing $\alpha, \beta,$ and γ, but constrained so that $\eta = \frac{\beta_k}{\gamma_k}$ for all $k \in K$. The value of time and schedule penalties should be assigned to groups such that the groups can be ordered $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_k$.[19] Then it will be the case that the following toll rate schedule will combine with $SD(t)$ to maintain an equal and minimal $C(t)$ (Arnott, de Palma, & Lindsey, 1987a).

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[18] Note that in the model under consideration the absolute value of the toll is unimportant. Travelers do not have a choice to stay home even if the toll begins at \$1000. Since they *must* travel, their objective is to minimize that cost and their only choice the tradeoff they can make between the toll they pay and the time they arrive through the bottleneck. In this tradeoff, only the *rate of increase of the toll* is important to their decision.

[19] Think of this as considering the same groups who traveled in Section 3.6, but in a new order.

$TR(t)$

$$
= \begin{cases} \beta_1(t-\tau_0), & \tau_0 \le t \le \tau_{12} \\ \beta_1(\tau_{12}-\tau_0)+\beta_2(t-\tau_{12}), & \tau_{12} \le t \le \tau^* \\ \beta_1(\tau_{12}-\tau_0)+\beta_2(\tau^*-\tau_{12})-\gamma_2(t-\tau^*), & \tau^* \le t \le \tau_{21} \\ \beta_1(\tau_{12}-\tau_0)+\beta_2(\tau^*-\tau_{12})-\gamma_2(\tau_{21}-\tau^*)-\gamma_1(t-\tau_{21}), & \tau_{21} \le t \le \tau_f \end{cases}
$$

Should there more be more groups of travelers, the pattern can easily be extended. The toll schedule is set at a rate such that it will encourage $n(t) = D$ for all times $t_0 \le t \le t_f$. The same ratio of travelers will arrive early for each group as in Section 3.5, which simplifies finding the transition times between groups, because we need merely find the length of time required to discharge $\frac{\gamma}{\beta+\gamma} N_i$ at $n_i(t)$ rather than taking into account the effect created by an accumulated queue. The following equations describe the key points in time during the peak hour when there are two traveling classes.

$$\tau_0 = \tau^* - \frac{\gamma}{\beta+\gamma}\frac{N}{D}$$

$$\tau_{12} = \tau^* - \frac{\gamma}{\beta+\gamma}\frac{N_2}{D}$$

$$\tau_{21} = \tau^* + \frac{\beta}{\beta+\gamma}\frac{N_2}{D}$$

$$\tau_0 = \tau^* + \frac{\beta}{\beta+\gamma}\frac{N}{D}$$

In order to facilitate comparison between the results of Sections 3.6 and 3.7, we will examine the same base case laid out in Table 7. (The conditions for the order of travel will necessitate that the groups change their order of travel for most of the scenarios examined.) Figure 13 shows the schedule delay costs for travelers when the toll is enacted. The dashed lines express the schedule delay costs that would be incurred by a traveler of group k were they to travel at t. Obviously members of group 1 do not travel for the entire peak period, but we can see that the group with a lower slope travels further away from τ^*. Figure 14 displays the toll rate at every time, t, in the system optimal toll scheme. Because the toll is charged to whomever travels at t, there is only a single line, although that line changes colors to indicate which group chooses to pay the toll rate at a given t.

Figure 13: Schedule delay cost profile for 2 groups in a system optimal tolling scheme.

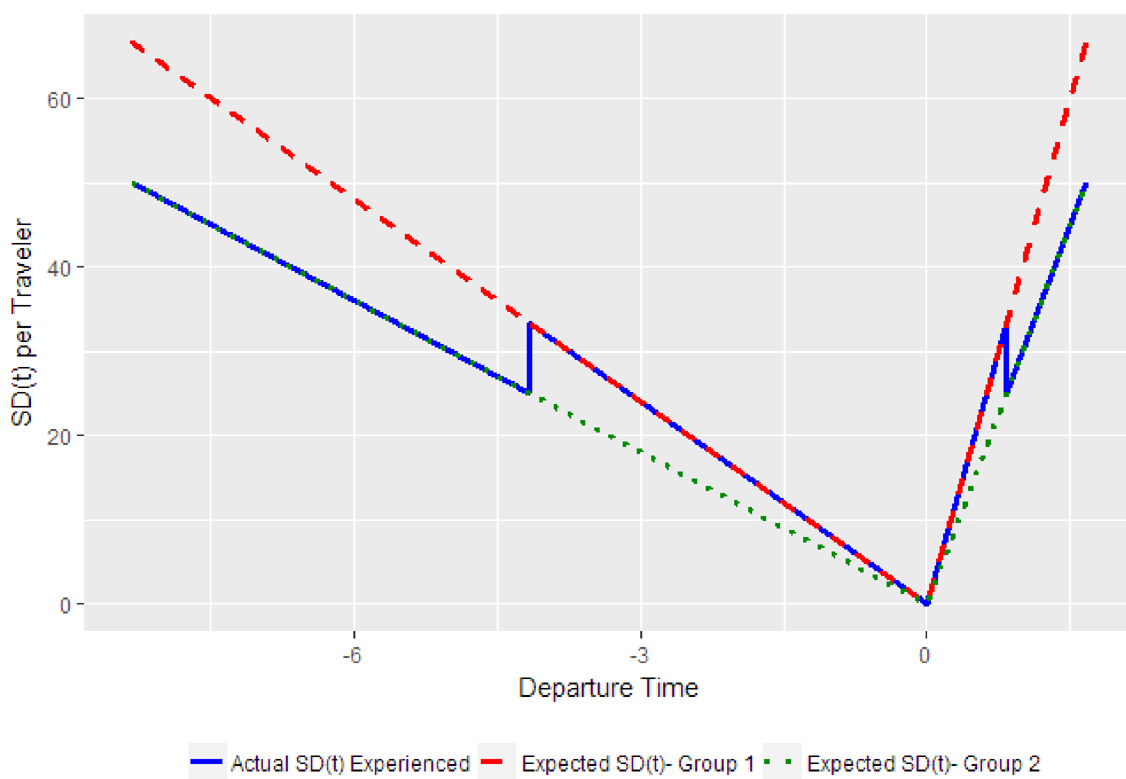

Figure 14: Toll rate for 2 groups in a system optimal toll scheme.

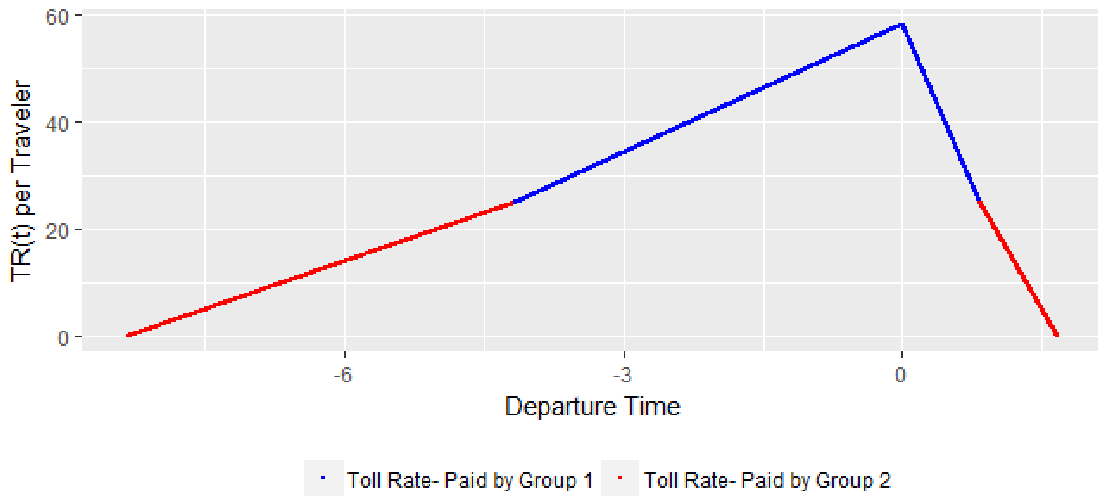

Figure 15 displays the total cost profiles of each group under a system optimal toll. The solid line indicates the time that travelers in each group choose to depart and so it is clear that each group chooses to travel at points in time with equal and minimal costs.

Figure 15: Total cost profile for 2 groups in a system optimal tolling scheme.

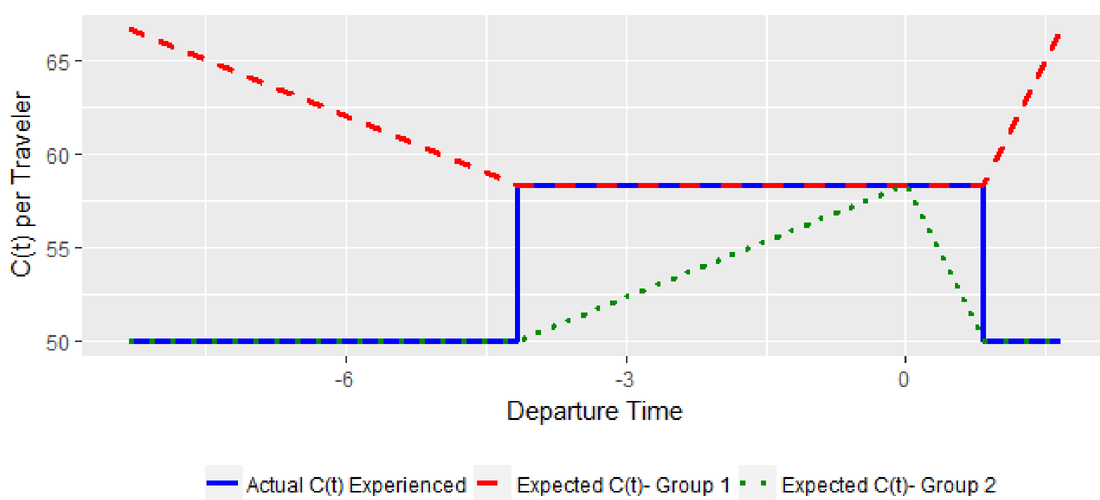

Table 9 displays the same range of sensitivity tests as Table 8.[20] There are a few important points that should be noted. First, while TC in the toll and no-toll scenarios may be the same, this is not always the case. Scenarios 10-21 are cases where the total cost of the system changes. This can occur because the change in the order of travelers means that schedule delay is minimized. Thus the eventual effect on TC may not yield equal costs between the scenarios. The imposition of a marginal toll to eliminate queuing behavior can even increase total system costs. In cases 8, 11, 13, 14, 16, and 19, the welfare of the whole system suffers from imposing a toll. When the value of time for group 1 decreases relative to group 2 in case 8 the toll creates a large loss of welfare when they are shifted to the margin of the peak hour. In case 13, a decrease in β_2 (without a decrease in α_2) relative to the base case means that group 2 would be willing to arrive early rather than queue, but imposition of a toll causes them to pay 225% than they would "pay" in time spent waiting in traffic in the no-toll scenario.

Second, the departure order is determined by the absolute value of the schedule delay parameters rather than by relative worth of schedule delay. In the system optimal tolling scheme proposed by Arnott, de Palma, and Lindsey, a traveler with $\alpha_1 = 100$ ₪$/min$ and $\beta_1 = 1$ ₪$/min$ would travel in the middle of the peak period over another traveler with $\alpha_2 = 1$ ₪$/min$ and $\beta_2 = 0.99$ ₪$/min$. This means that the departure order between the toll and no-toll scenarios is often flipped (as can be seen from the figures above).

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[20] I include cases 20 and 21 for completeness (along with the enforced traveling order), but draw no conclusions from them since they are intended to show what happens when travelers do *not* behave as according to Wardrop's principles in the no-toll case.

Table 9: Sensitivity of schedule delay, travel time, and total costs to parameters for tolled equilibrium. (Subscripts indicate group number.)

No.	D	N1	α_1	β_1	γ_1	N2	α_2	β_2	γ_2	SDC	SDC_1	SDC_2	TRC	TRC_1	TRC_2	TC	TC_1	TC_2
1	6	30	24	8	32	30	12	6	24	1560.0	480.0	1080.0	1560.0	1200.0	360.0	3120.0	1680.0	1440.0
2	4	30	24	8	32	30	12	6	24	2340.0	720.0	1620.0	2340.0	1800.0	540.0	4680.0	2520.0	2160.0
3	8	30	24	8	32	30	12	6	24	1170.0	360.0	810.0	1170.0	900.0	270.0	2340.0	1260.0	1080.0
4	6	30	16	5.3	21.3	30	8	4	16	1040.0	320.0	720.0	1040.0	800.0	240.0	2080.0	1120.0	960.0
5	6	30	32	10.7	42.7	30	16	8	32	2080.0	640.0	1440.0	2080.0	1600.0	480.0	4160.0	2240.0	1920.0
6	6	30	24	8	28	30	12	6	21	1516.7	466.7	1050.0	1516.7	1166.7	350.0	3033.3	1633.3	1400.0
7	6	30	24	8	36	30	12	6	27	1595.5	490.9	1104.5	1595.5	1227.3	368.2	3190.9	1718.2	1472.7
8	6	30	18	6	24	30	12	6	24	1440.0	360.0	1080.0	1440.0	1080.0	360.0	2880.0	1440.0	1440.0
9	6	30	30	10	40	30	12	6	24	1680.0	600.0	1080.0	1680.0	1320.0	360.0	3360.0	1920.0	1440.0
10	6	30	24	9.6	38.4	30	12	6	24	1656.0	576.0	1080.0	1656.0	1296.0	360.0	3312.0	1872.0	1440.0
11	6	30	24	6.9	27.4	30	12	6	24	1491.4	411.4	1080.0	1491.4	1131.4	360.0	2982.9	1542.9	1440.0
12	6	30	24	8	32	30	6	3	12	1020.0	480.0	540.0	1020.0	840.0	180.0	2040.0	1320.0	720.0
13	6	30	24	8	32	30	18	9	36	2100.0	480.0	1620.0	2100.0	1560.0	540.0	4200.0	2040.0	2160.0
14	6	30	24	8	32	30	12	8	32	1920.0	480.0	1440.0	1920.0	1440.0	480.0	3840.0	1920.0	1920.0
15	6	30	24	8	32	30	12	4.8	19.2	1344.0	480.0	864.0	1344.0	1056.0	288.0	2688.0	1536.0	1152.0
16	6	20	24	8	32	30	12	6	24	1120.0	480.0	640.0	1120.0	960.0	160.0	2240.0	1440.0	800.0
17	6	40	24	8	32	30	12	6	24	2080.0	480.0	1600.0	2080.0	1440.0	640.0	4160.0	1920.0	2240.0
18	6	30	24	8	32	20	12	6	24	1053.3	213.3	840.0	1053.3	693.3	360.0	2106.7	906.7	1200.0
19	6	30	24	8	32	40	12	6	24	2173.3	853.3	1320.0	2173.3	1813.3	360.0	4346.7	2666.7	1680.0
20	6	30	12	4	16	30	24	12	48	2400.0	240.0	2160.0	2400.0	1680.0	720.0	4800.0	1920.0	2880.0
21	6	30	24	12	48	30	12	4	16	1440.0	720.0	720.0	1440.0	1200.0	240.0	2880.0	1920.0	960.0

Third, although costs as a whole decline, the costs to group 2 are almost uniformly higher. In cases 1-7, i.e. in the base case, as D varies, and as the overall value of all traveler's time changes, group 1 experiences a 67% decrease in SDC, a 150% increase tolls paid versus TTC, and a 12% decrease in TC. On the other hand group 2 travelers face a 200% increase in SDC, a 57% decrease in tolls paid versus TTC and a 20% increase in overall TC. In fact, the only cases where group 2 has a lower cost than in the no-toll scenario is when the number of group 2 travelers decreases and when the number of group travelers increases, which indicates that as group 1 becomes a smaller proportion of the traveling public they may benefit from tolls. (However please note that as group 2 grows as a share of the traveling public, their incentive to accept a toll decreases since their costs rise dramatically.)

Because travelers in different classes experience different results from the imposition of a tolling scheme, we ought to examine whether the scheme equitably allocates the welfare savings from the toll. On the one hand, the toll payment is horizontally equitable. Everyone faces an identical option to pay the face value of the toll and can choose to travel whenever they believe they will experience a minimal cost. On the other hand, it is clear that the welfare savings from the toll are not equally distributed amongst *groups*. When fully half the travelers face roughly 20% increases in their own commuting costs in order to create a 12% surplus in the welfare of the other half of travelers, the toll scheme effects a transfer of welfare to the richer travelers. This is an inequitable result.

Fourth, even if we set aside the inequitable welfare results, there is still the further inequity in the way the toll changes the patterns of travel. When a toll is present, groups with a higher value of β and γ will travel at the center of the peak hour regardless of the relative weight of their schedule delay penalty and groups with a lower value of time will be shifted to the margins of the peak period. Thus, on top of having their actual costs increased by the toll, low-income travelers are further harmed by being made more likely to experience additional time poverty. Leisure time, as discussed in Section 2.4, is a primary social good. The opportunity to eat breakfast with one's kids or head home to pick them up from soccer practice is as valuable to low-income families as high income families in relative terms, even though low-income families are not in a position to spend money in a way that reveals this preference on the toll road.

We can see some of the inequitable time costs imposed on group 2 relative to group 1 by examining the change in the composition of SDC when the number of travelers changes, i.e. who bears the burdens of additional travelers and who receives benefits from their reduction. What we see in cases 16-19 is that group 2 loses out on these terms every time. Because group 2 is shifted from the middle of the peak hour, where they were willing to wait because of their relatively high schedule delay penalty, they experience large cost increases while the cost to group 1 decreases. When the number of group 1 travelers increases, SDC for group 1 decreases by 77% relative to the no-toll equilibrium but SDC for group 2 increases by 344%. But when the size of group 1 decreased, the benefits again flow in an outsized way to group 1, where SDC for group 1 decreases by 44% and SDC for group 2 increases by 78%. These same patterns hold when the size of group 2 is changed.

One response to this line of reasoning might be the claim that an examination of SDC has limited value and is not the proper frame of comparison. After all, SDC only measures the cost a group faces for the disutility its member face for failing to arrive at τ^*. Perhaps a better comparison would the total cost of travel in the no-toll scenario

($SDC + TTC$, which encompasses all of the *time* travelers spend) and the time cost in the toll scenario (SDC). The critic may reason that the toll paid is not relevant to the time poverty of travelers. Two responses are in order. First, if we make this comparison we can see that although both mostly groups benefit from the imposition of the toll, group 1 receives far more of the benefit than group 2. In the majority of scenarios, the overall time cost of group 1 decreases by 75% while group 2 decrease by 10%. Again, cases where the size of the two travel groups vary create overwhelming benefits for group 1 at the expense of group 2. Second, we should not be so quick to disregard the price of the toll. I have not offered a reimbursement scheme to suggest how the money collected in the toll scenario ought to be redistributed, but redistributing more of it to poorer travelers is not naturally connected goal of the system optimal toll scheme, since the only requirement is to decrease TC. Any further distribution of welfare would be undertaken for independent reasons. Paying for the toll will require group 2 travelers to work hours whose goal is to enable the traveler to pay for their work trip in order to earn other money. If no toll is levied, then group 1 travelers would presumably have the option to not work those hours and remain just as well off as before.

3.8: Analytical and Numerical Solutions and Discussion for Departure Rates, User Costs at t for Two Traveler Classes in User Equilibrium with a Time Equitable Toll

The time inequity of the system optimal tolling scheme is a feature, not a bug of that system. Low-income travelers, who are already more likely to experience time poverty, are shunted to the margins of the peak travel period in an effort to decrease the sum of the costs of travel for all citizens. Wealthier travelers with a higher α experience a higher cost for a given level of congestion (by definition) and so efforts to decrease total system costs are most effective when they focus on the large marginal effects that come from changing the costs facing those with the highest value of time.

The original contribution of this thesis is the equity analysis of the system optimal dynamic toll and a proposal for what I will call a time-equitable tolling scheme that addresses these inequities. A time-equitable toll addresses the problems by meeting three key desiderata.

First, the tolling scheme should preserve the same order of departures as in the no-toll scenario. In the no-toll case, the departure order is a reflection of the relative weight a group places on their schedule delay. An equitable program would show respect for all travelers by preserving this ordering of preferences rather than attempting to coerce travelers to choose times based on their absolute value of time.

Second, a time-equitable toll rate should be a function of the difference between the values of time (α) of the traveling groups. If $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$ and $\alpha_1 = \alpha_2$, then no subsidy should be provided. Our equity concern grows as the gulf between groups increases, because larger wealth disparities will lead to more unjust switches between groups (e.g. the case described in Section 3.7.)

Third, a time-equitable toll should generate the same revenues as the system optimal toll. As was discussed in Chapter 1, cities and states use tolls both to ease congestion and to shift the burden of paying for infrastructure onto those who use it. Alternatives to the system optimal toll should raise at least as much money so that it does not shift the cost of the infrastructure back onto non-travelers. The idea of the system optimal toll is to make travel at the middle of the peak period affordable and fair, not to make it “free”.

Meeting these three requirements is best achieved by charging different rates to different travelers. Let there be K distinct classes of travelers of size $n_1, n_2, \dots, n_k$which are characterized by differing $\alpha, \beta,$ and γ, but constrained so that $\eta = \frac{\beta_k}{\gamma_k}\ \forall k \in K$. The value of time and schedule penalties be assigned to groups such that the groups can be ordered $\frac{\alpha_1}{\beta_1} \geq \frac{\alpha_2}{\beta_2} \geq \cdots \geq \frac{\alpha_k}{\beta_k}$. Let $\epsilon_n = \frac{\alpha_n}{\beta_n}$ (note that the size of ϵ is less than or equal to k). Let there also be a toll rate escalator, ESC such that $ESC > 1$. During a length of time sufficient for travelers with ϵ_1 to travel ($\frac{\gamma}{\beta+\gamma}$ before t^* and $\frac{\beta}{\beta+\gamma}$ afterwards and determined by the sum of all travelers in all groups with ϵ_1) the toll rate will increase by β during the early period and decrease by γ for late arrivals, but during periods closer to t^* that rate of increase or decrease will increase by a further factor of ESC.

The tolling authority divides up the peak hour into early and late arrival periods corresponding to each ϵ. For travelers in $\epsilon_i$the toll rate will increase by β_i during early period travel up through t_{ij} and decrease by γ_i during late period travel after t_{ji}. During $t_{ij} \leq t \leq t_{ji}$ the toll will increase by $ESC * \beta_i$ for early travel and decrease by $ESC * \gamma_i$ for late travel. If there are two groups of travelers in ϵ_i (i.e. they have different α & β) then each group will be charged a rate determined by its own schedule delay penalty. In the highly deterministic model described in this thesis where travelers have perfect knowledge of the networks costs for all travel times and insist on traveling at absolute minimum cost, ESC can be literally any value greater than 1. Although it would be possible to maintain a system with no queues while not charging any toll prior to the first moment travelers in a group would choose travel if there were no prior travelers, the requirement to maintain revenues leads us to make the toll charged sensitive to the total number of travelers in the system. It is also more equitable to begin charging from τ_0

because this leads the toll charged to group j to be sensitive to the size of N. If group j's toll begins at t_{ij}, then they will evade the costs that their travel imposes on other groups.

The easiest way to flesh out this toll rate scheme is to return the 2 group Vickrey bottleneck. Let there be two groups where $\beta_1 > \beta_2$ and $\frac{\alpha_1}{\beta_2} > \frac{\alpha_2}{\beta_2}$.[21] Then the toll rate schedule of the two groups is given by the following schedule.

$$TR_1(t) = \begin{cases} \beta_1(t-\tau_0), & t_0 \le t \le t_{12} \\ \beta_1(t_{12}-t_0) + ESC * \beta_1(t-t_{12}), & t_{12} \le t \le t^* \\ \beta_1(t_{12}-t_0) + ESC * \left(\beta_1(t^*-t_{12}) - \gamma_1(t-t^*)\right), & t^* \le t \le t_{21} \\ \beta_1(t_{12}-t_0) + ESC * \left(\beta_1(t^*-t_{12}) - \gamma_1(t_{21}-t^*)\right) - \gamma_1(t-t_{21}), & t_{21} \le t \le t_f \end{cases}$$

$$TR_2(t) = \begin{cases} \beta_2(t-\tau_0), & t_0 \le t \le t_{12} \\ \beta_2(t^*-t_0) - \gamma_2(t-t_{21}), & t_{21} \le t \le t_f \end{cases}$$

The cost to group 2 is the same as if all travelers in the peak period had the same preferences as group 2, which means that they don't receive a "free lunch" as they would if their toll were set to 0 until the start of their travel period. The goal of the time equitable toll is not to absolutely minimize traveler costs, but instead to ensure that they are reasonable while preserving a departure order determined by travelers' relative cost of schedule delay.

Figure 16, Figure 17, and Figure 18 display the schedule delay, toll rate, and total cost profiles, respectively, for the time-equitable tolling pattern. Travelers in group 2 pay a cost equal to the cost they would bear if all N travelers possessed the same traits and preferences as themselves. This ensures that they pay their fair share for the congestion

[21] We choose this relationship in order to demonstrate the effect of travelers changing their order. If traveler preferences are differently ordered then the time equity issues central to this thesis are not encountered.

that they might cause while not being priced out of the ability to travel in the middle of the peak period.

Figure 16: Schedule delay cost profile for 2 groups in a time-equitable tolling scheme.

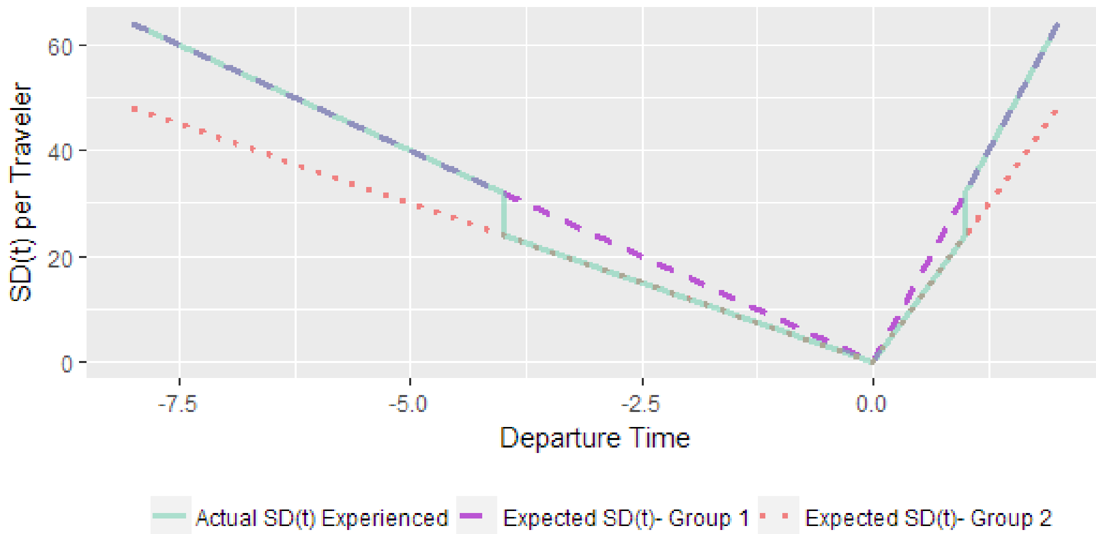
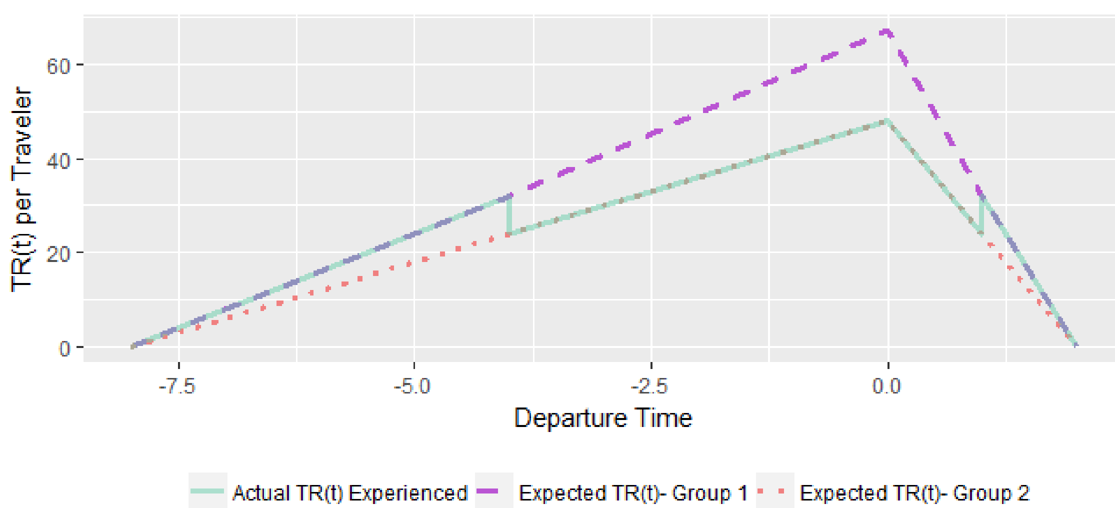
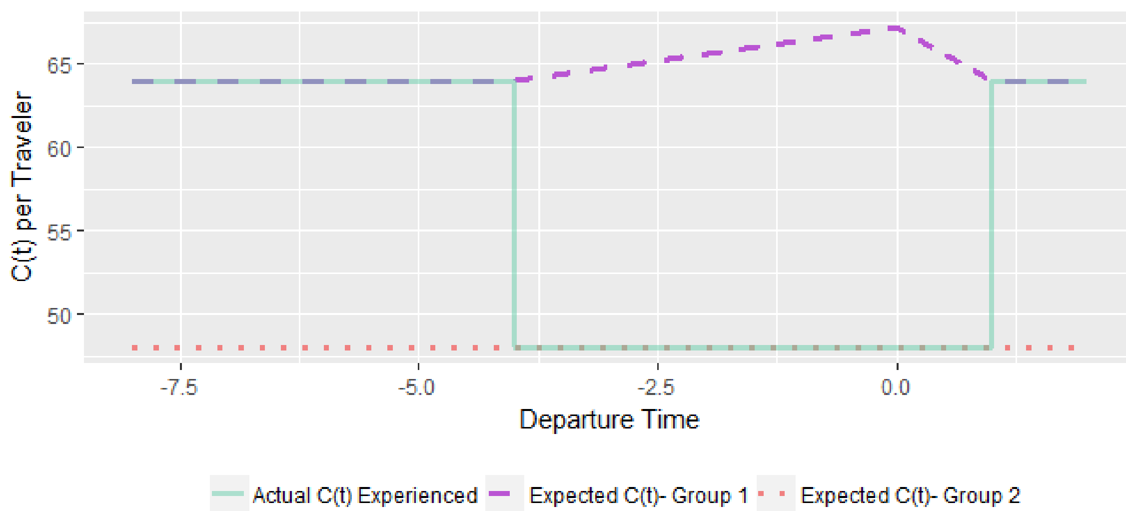


Figure 17: Toll rate profile for 2 groups in a time-equitable tolling scheme.



Setting a continuously increasing toll for group 2 from τ_0 raises their costs as can be seen from Figure 17. If we decided to substantially increase the subsidy to group 2 travelers we could do so by dropping their toll to zero outside of $\tau_{12} \leq t \leq \tau_{21}$though the third condition on an acceptable alternative to the system optimal toll does not permit such a subsidy. Figure 18 would then show a decreasing cost function for group 2 during the period they do not travel rather than the constant function in the time-equitable toll policy I have proposed.

Figure 18: Total cost profile for 2 groups in a time-equitable tolling scheme.



We will now discuss the time-equitable tolling scheme in relation to both the no-toll and the queue eliminating toll proposed by Arnott, de Palma, and Lindsey. The results of the time-equitable tolling model are found in Table 10. The first result that must be acknowledged is that the time-equitable toll always increases the total welfare cost relative to the no-toll scheme, although the effect is slight. The maximum increase in cost is 14% (case 14) comes when β_2 rises relative to α_2 because tolls depend on β while TTC depends upon α. Nonetheless, the effect on net social welfare is not large.

The second point to note is that SDC stays constant between the no-toll and time-equitable toll, which occurs because the travelers maintain their order of departure. This means that any change in the aggregate welfare of the scenario comes from the difference in the travel time cost and monetary cost of the toll. Third, note that the cost to group 1 stays constant between the no-toll and time-equitable tolls. SDC_1 is the same for all cases between the two schemes and $TTC = TRC$. Thus, increases in TC come from increases in TC_2, which are driven by the higher cost of tolls relative to travel time, i.e. $TRC_2 - TTC_2$. The result is that, relative to the no-toll scenario all additional costs are borne by group 2, but on the other hand group 1 does not benefit in its total welfare by the imposition of the toll as occurs with the system optimal toll.

This uneven bearing of the costs of the toll does raise an equity concern. After all, if the point of the time-equitable toll is to be *equitable*, then a toll that eliminates congestion by charging low-income travelers more than they believe their time waiting in traffic is worth while substituting the cost of queuing for high income travelers with a toll is certainly suspicious. However, there are two ways in which the proposed tolling scheme is more equitable as becomes apparent when we compare the outcomes of the queue eliminating toll and the time-equitable toll.[22]

[22] Note that the toll is being proposed on the supposition that eliminating congestion is a worthwhile goal. This is eminently reasonable. TTC is a complete social loss, while TRC represents money that can be spent on other goods, even subsidizing group 2 as a class. The time-equitable toll is also being proposed on the further supposition that the no-toll scenario is the proper reference point for our analysis. Any proposal where we define equity as the preservation of the distribution of welfare through a change in the system (e.g. using a toll to eliminate congestion) will necessarily hinge a great deal on whatever we define as the starting point. In this case I believe beginning with the welfare distribution found in the no-toll scheme is appropriate. In particular, it is an appropriate starting point because the no-toll scheme sorts travelers by their relative desire to arrive on time. All travelers are shown equal respect in that, if they wish to arrive at τ^* they are all equally able to use their time for that purpose.

Table 10: Sensitivity of schedule delay, travel time, and total costs to parameters for time-equity equilibrium. (Subscripts indicate group number.)

No.	D	N1	α_1	β_1	γ_1	N2	α_2	β_2	γ_2	SDC	SDC_1	SDC_2	TRC	TRC_1	TRC_2	TC	TC_1	TC_2
1	6	30	24	8	32	30	12	6	24	1800.0	1440.0	360.0	1560.0	480.0	1080.0	3360.0	1920.0	1440.0
2	4	30	24	8	32	30	12	6	24	2700.0	2160.0	540.0	2340.0	720.0	1620.0	5040.0	2880.0	2160.0
3	8	30	24	8	32	30	12	6	24	1350.0	1080.0	270.0	1170.0	360.0	810.0	2520.0	1440.0	1080.0
4	6	30	16	5.3	21.3	30	8	4	16	1200.0	960.0	240.0	1040.0	320.0	720.0	2240.0	1280.0	960.0
5	6	30	32	10.7	42.7	30	16	8	32	2400.0	1920.0	480.0	2080.0	640.0	1440.0	4480.0	2560.0	1920.0
6	6	30	24	8	28	30	12	6	21	1750.0	1400.0	350.0	1516.7	466.7	1050.0	3266.7	1866.7	1400.0
7	6	30	24	8	36	30	12	6	27	1840.9	1472.7	368.2	1595.5	490.9	1104.5	3436.4	1963.6	1472.7
8	6	30	18	6	24	30	12	6	24	1440	1080	360	1440	360	1080	2880	1440	1440
9	6	30	30	10	40	30	12	6	24	2160	1800	360	1680	600	1080	3840	2400	1440
10	6	30	24	9.6	38.4	30	12	6	24	2088.0	1728.0	360.0	1656.0	576.0	1080.0	3744.0	2304.0	1440.0
11	6	30	24	6.9	27.4	30	12	6	24	1594.3	1234.3	360.0	1491.4	411.4	1080.0	3085.7	1645.7	1440.0
12	6	30	24	8	32	30	6	3	12	1620	1440	180	1020	480	540	2640	1920	720
13	6	30	24	8	32	30	18	9	36	1980	1440	540	2100	480	1620	4080	1920	2160
14	6	30	24	8	32	30	12	8	32	1920.0	1440.0	480.0	1920.0	480.0	1440.0	3840.0	1920.0	1920.0
15	6	30	24	8	32	30	12	4.8	19.2	1728.0	1440.0	288.0	1344.0	480.0	864.0	3072.0	1920.0	1152.0
16	6	20	24	8	32	30	12	6	24	1213.3	853.3	360.0	1053.3	213.3	840.0	2266.7	1066.7	1200.0
17	6	40	24	8	32	30	12	6	24	2493.3	2133.3	360.0	2173.3	853.3	1320.0	4666.7	2986.7	1680.0
18	6	30	24	8	32	20	12	6	24	1280.0	1120.0	160.0	1120.0	480.0	640.0	2400.0	1600.0	800.0
19	6	30	24	8	32	40	12	6	24	2400.0	1760.0	640.0	2080.0	480.0	1600.0	4480.0	2240.0	2240.0

First, the time-equitable toll is a *more* equitable distribution of TC than the system optimal toll. Take the base case. In both the system optimal and time-equitable tolls group 2 experiences a 20% increase in C_2 , but in the system optimal toll all of this welfare is transferred to group 1 (which results in a net wash of TC). Although group 2 experiences higher costs, they are not compelled to transfer their costs into a benefit for other travelers. Should group 2 travelers decide that they do not have the funds to pay a toll in the center of the peak hour, they always have the option to forego the time benefits by choosing to travel outside of their group's time slot since their costs are flat over the entire peak hour. If this happened in significant numbers a real system may not be able to maintain the no-queue requirement, but such as a case would also mean that the estimates of $\alpha_2, \beta_2,$ and γ_2 were improperly calibrated.

Second, the time-equitable toll provides group 2 with a benefit by decreasing SDC_2 while eliminating congestion. The time-equitable toll preserves the correct order of departures by tying departure patterns to a relative desire to arrive on time. For all scenarios where the $N = 60$, SDC_2 decreases by 67% relative to the system optimal toll. The time-equitable toll thus decreases the incidence of time poverty amongst group 2 travelers. We have eliminated the wasted TTC for all travelers and passed that time savings on to all travelers but particularly to those who value it most relative to their ability to pay.

Chapter 4: Conclusion

The true costs of travel are always paid. Individual travelers may pay their cost by arriving early or late, by waiting in traffic, or by paying a toll. And they may end up paying queuing costs created by other travelers who have externalized their own cost of travel. The road itself may either be paid through user fees or through area wide taxes insensitive to travel behavior. Dynamic tolling schemes offer a way to pay for infrastructure while eliminating the deadweight loss of travel time costs. System optimal tolls have been widely proposed as a panacea to the ills of modern day travel because they minimize the cost of all travel over the network.

In this thesis I argue that system optimal tolls present two distinct equity concerns. First, they achieve their efficient outcome by shifting a greater percentage of system costs to low income travelers. Reducing travel costs facing high income travelers generates the largest marginal decrease in travel costs, so policy makers get the biggest bang for their buck by decreasing the costs of these travelers, even if that harms low income travelers. Second, system optimal tolls are likely to increase time poverty among low income travelers. System optimal tolls order traveler departures on the absolute value of traveler time rather than the relative value of schedule delay to travel time delay costs that orders no-toll equilibria. Combine this with the fact that low income families require extensive amounts of “free time” in order to perform household chores and meet basic needs that wealthier families are able to accomplish using money. The result is that structuring traveler departures so that low income travelers depart at the margins of the peak hour means that already overburdened families face significant burdens of time poverty.

In order to prove that system optimal tolls raise equity concerns, I present a theoretical foundation to argue that equitable policies transportation policies are ones that show sufficient respect to all travelers, argue that the dual disproportionate burdens of system optimal tolls fail to show sufficient respect to low income travelers, and conclude by proposing a time-equitable tolling scheme that ameliorates the time poverty problems and some of the welfare inequities of a system optimal toll. The definition that equitable policies show sufficient respect for travelers helps to explain the three different, sometimes exclusive, conceptions of equity deployed in the literature. Horizontal equity ought to be applied when travelers have equal status, bear roughly equal costs, and receive roughly equal benefits from a policy. Vertically equitable policies are required when the stakes are higher and the costs and benefits are distributed unequally even if all travelers are subject to the same rules and opportunities.

I contend that the burdens of time poverty are the precisely the kinds of costs that require special concern for low income travelers. The time equitable toll treats all travelers with sufficient respect by ordering departures in a way that respects the relative weight that groups place on their schedule delay costs, is sensitive to the gap in resources between different traveling groups, and generates the same amount of revenue as the system optimal toll.

Big data raises the likelihood that toll road companies will be able to target travelers according to their value of time and schedule delay costs. They are likely to adopt either profit maximizing algorithms or (if pushed by political pressure) system optimal tolls. Any tolling scheme will have manifold effects on travelers and it is important that we begin to have a discussion of these effects before they happen. This thesis is written in the hopes of being a catalyst for such a discussion on toll schemes, equity, and time poverty within transportation policy. Although the literature has discussed all three of these issues and even brought some of them together, there has to this point been no sustained discussion of how transportation policies can exacerbate or ameliorate inequitable distributions of time poverty. By adopting a straightforward model with a limited number of parameters, I attempt to raise these questions and to keep equity at the forefront of the model. I hope to continue this work by pursuing three distinct, but interdependent, strains of research.

First, I believe future work to more fully explore complex Vickrey bottleneck models is warranted. Introducing networks with alternate paths, networks with multiple destinations, travelers who desire to arrive throughout the peak hour, and travelers who can choose not travel to the basic Vickrey model considered in this thesis will provide fruitful material to future discussions of tolls and equity. It will be particularly important that these wrinkles to the model do not lead us to lose sight of the fact that we must be concerned with the inequitable distribution of the costs of time poverty and social welfare that system optimal tolls can create. To this end, the second piece of future research engendered by this thesis will be a thorough investigation into empirical estimations of time poverty parameters. Travelers with high relative schedule delay penalties experience real costs when their financial means do not permit them to travel in the middle of the peak hour. Low income parents who struggle to manage their schedule in order to pick their children up from school and then to cook meals from scratch are not abstract victims of toll schemes that price them out of the market. We should not pretend that they are. However, a great deal of work remains to be done to quantify these effects and place them in a context that policy makers will understand and appreciate.

The final step of this research program is to bring together real world implementations of dynamic tolls with the equity analysis proposed in this thesis. The real world is messy. Real cities have hundreds of thousands of travelers who all have hundreds of variables, most of which are unobservable, affecting their decision to travel and this greatly complicates the problem of determining how tolls should be priced in order to equitably minimize congestion. An important piece of this puzzle, however, is to dive into the details of actual traveler decisions. This was a project too sprawling for a thesis but remains an important next step in this research program.

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