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Improvements and Extensions of Dynamic Traffic Assignment in Transportation Planning

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Improvements and Extensions of Dynamic Traffic Assignment in Transportation Planning

by

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Improvements and Extensions of Dynamic Traffic Assignment in Transportation Planning

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The University of Texas at Austin, 2013

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A comprehensive approach is conducted to better utilize dynamic traffic assignment (DTA) in transportation planning by investigating its role in: (1) high-order functions, (2) project evaluation, and (3) traffic assignment. A method is proposed to integrate DTA and the four-step planning model such that traffic assignment is conducted at the subnetwork level while the feedback process occurs at the regional level. By allowing interaction between the subnetwork and regional area, the method is shown to be more beneficial than previous integration structures. Additionally, DTA is applied to a case study involving the proposed urban rail system in Austin, TX. The case study showcases the benefits and capabilities of DTA when analyzing traffic impacts caused by transit rail facilities. Multiple equilibria are shown to arise in simulation-based DTA models due to simplified fundamental diagrams. Piecewise linear diagrams are introduced to eliminate unlikely equilibria. Game theory is also applied to DTA; it is shown that an equilibrium solution is guaranteed to exist for general networks in mixed strategies, and unrealistic equilibria are reduced using the trembling hand refinement.

Table of Contents

List of Tables

List of Figures

Chapter 1: Introduction

1.1 Background

The ability to accurately model project alternatives and future scenarios is crucial to the transportation planning process. Several travel demand models have been proposed, modified, replaced, and improved over time; the field has been in a continuous flux of evolution since its inception. However, the traditional four-step planning model has been the cornerstone of demand modeling for the last 50 years. It is used by nearly all transportation planning agencies. The model is based on static travel behavior assumptions among all of the four processes, creating a theoretically consistent, well-designed iterative system.

Recently, there has been a dramatic change in the mentality of transportation planners across the United States due to lack of funding and the realization that congestion needs to be adeptly controlled. Planners are shifting their focus from the supply side of transportation (e.g., constructing an additional lane) to the demand side of transportation (i.e. influencing traveler behavior as to maximize the efficiency of the existing system). In order to evaluate the vast spectrum and growing trend of time-dependent demand policies in practice, time must be variable in the modeling process, since the majority of demand polices is aimed at preventing congestion formation – which only occurs in relatively small time periods. The standard four-step model with static traffic assignment is incapable of evaluating these policies. For this reason, dynamic traffic assignment (DTA) has been developed and is the next logical evolutionary step in the transportation planning process.

Dynamic traffic assignment accurately models traffic in time-varying demand interactions and has addressed shortcomings of the traditional static traffic assignment model, such as the representation of individual lanes, bottlenecks, queues, and congestion formation. DTA is mesoscopic in nature, analyzing flows at a fine time scale across large spatial areas. It is fully capable of being implemented at the regional level due to recent improvements and the availability of efficient software programs. DTA produces space-time vehicular trajectories, which completely describe the state of the transportation system. From these trajectories, route choice behavior, queue formation, and dynamic travel times can be observed. In short, dynamic traffic assignment is an invaluable tool to practitioners, especially when applied to unsteady demand conditions and in conjunction with modern technologies such as ITS devices.

1.2 Motivation

Despite the benefits of dynamic traffic assignment and its necessity, agencies have been slow to adopt DTA methods in their decision analyses for several reasons. It is a relatively new field and has not been researched or established as sufficiently as more traditional methods. There are several well-known models with distinct solution methodologies (i.e., there is no consensus on a standard model), making the decision to implement a particular model difficult. Its theoretical basis is different from static travel behavior assumptions upon which traditional planning processes are based (e.g., the assumption of steady-state conditions). Thus, few planning entities have active, calibrated DTA models. Furthermore, agencies that do, tend to use dynamic traffic assignment as a standalone tool, only implementing DTA on a case-by-case basis (e.g., only for project analysis in urban areas or for evaluating bottleneck mitigation strategies). Outputs from the model are rarely applied outside of the project scope or integrated with higher-order planning processes; a large amount of useful and detailed data is wasted. Essentially, DTA is heavily underutilized in practice. The aim of this thesis is to make DTA more usable and beneficial to transportation practitioners by: (1) proposing methods to integrate dynamic traffic assignment with other planning processes so as to effectively exploit and employ its abilities, (2) showcasing the capabilities of DTA through a detailed case study, and (3) increasing the theoretical integrity and fundamental knowledge of dynamic traffic assignment outputs and solution process.

1.3 Contributions

This thesis takes a comprehensive approach to improve the practicality of dynamic traffic assignment by conducting research at three distinct functional levels: (1) high-order, long-term regional transportation planning, (2) project impact analysis and evaluation, and (3) examination of the fundamental assumptions in modern DTA models. It is an investigation of how DTA can be improved in high-level to low-level transportation planning functions. The main contributions of this thesis are described in the following sections.

1.3.1 High-level Transportation Planning

A method is proposed to integrate dynamic traffic assignment and the traditional four-step planning model. The more detailed and accurate information from DTA can therefore be utilized in high-level analyses (e.g., in the trip distribution and mode choice processes). A comprehensive, system wide analysis of time-varying demand policies can be conducted, which would otherwise be impossible using the more traditional planning model. Despite the benefits from this joined system, there has been limited research in the area. The proposed method is believed to be the first work where traffic assignment is conducted at the subnetwork level while integration and the overall iterative planning model is conducted at the regional level. This structure is beneficial for several reasons and has improved upon previous studies in the following manner: (1) it avoids the long convergence time of regional DTA models, (2) detailed information is retained only where detail is needed – in the area of interest, the subnetwork, and (3) it maintains a connection between the subnetwork and regional network, thus regional impacts caused by subnetwork modifications can be modeled. Furthermore, this subnetwork-regional connection can be used to reduce the size of the subnetwork saving computational time and effort.

1.3.2 Individual Project Analysis

A detailed case study is presented to demonstrate the capabilities and benefits of dynamic traffic assignment when applied to project impact analyses. The study focuses on identifying and quantifying route changing behavior, which is vital to estimating the benefits/costs of a project. The case study expands how DTA is currently used in practice by investigating traffic impacts caused by transit rail facilities. In the past, estimating traffic impacts due to rail services was conducted via microsimulation or regional planning. Microsimulation is limited in spatial area and does not consider route choice changes. Impacts outside of the corridor being analyzed will not be captured. Regional planning typically lacks detailed inputs and does not directly model transit impedances in the traffic assignment process – giving a rough aggregate estimation of the true impact. This thesis presents a new methodology by applying DTA, and thus improving upon the two previous methods by modeling route choice behavior using realistic inputs at a fine time scale across a large spatial area. Therefore, traffic impacts can be accurately modeled far from the physical location where modifications have occurred (i.e., network-wide impacts can be captured at a high level of detail).

1.3.3 The Dynamic Traffic Assignment Process

A major reason for the slow adoption of DTA in practice stems from the complicated nature of its process and the complicated behavior of its results. Traffic assignment is based on the assumption that drivers choose routes in order to minimize their travel time. The state of the network is in user equilibrium when all travelers are on their respective shortest path and cannot decrease travel time by switching routes. In modern DTA models, multiple user equilibrium solutions are possible and equilibrium may not exist. This is a significant issue, since the output traffic flows from the model may not be representative of the actual traffic situation. Two approaches are presented in this thesis to limit the number of unrealistic results from DTA: (1) computational game theory – traffic assignment is formulated as an economic game and (2) relaxation of simplifying assumptions of current modeling techniques.

This work is the first to apply game theory refinements to dynamic traffic assignment equilibrium. By utilizing game theory techniques, this thesis illustrates that the number of unrealistic pure strategy results can be reduced and that an equilibrium solution is guaranteed to exist for general networks in mixed strategies. This report is also the first of its kind to identify assumptions in current DTA modeling methods that cause multiple user equilibrium results. By relaxing these assumptions, all unlikely equilibrium solutions were eliminated on the studied network. These results are extremely beneficial to transportation planners, since projects are evaluated and compared at a single equilibrium state. If the equilibrium state does not reflect the true traffic situation, planning decisions may not only lead to inefficient spending of funds but may negatively impact the network. Identifying solutions to reduce the number of user equilibria is a starting point in the development of more accurate and trusted DTA models.

1.4 Organization

The thesis is organized as follows. Chapter 2 presents the proposed method of integrating dynamic traffic assignment and the traditional four-step planning model. The method is illustrated using a simple example network. A detailed comparison of static and dynamic traffic assignment is also included. Chapter 3 comprises of a comprehensive case study, where traffic impacts caused by transit facilities are modeled using dynamic traffic assignment. The study showcases the capabilities of DTA by analyzing route changing behavior. Chapter 4 examines existence and uniqueness issues associated with dynamic traffic equilibrium. Several small networks are presented showcasing the complicated nature of dynamic user equilibria. Two methods are proposed that reduce the number of unrealistic equilibrium solutions: computational game theory and relaxation of certain modeling assumptions. Chapter 5 summarizes the contributions of this report and discusses possible directions for future research.

Chapter 2: Integration of Dynamic Traffic Assignment and the Traditional Planning Model

2.1 Introduction

Beginning in the 1990s, several studies have been conducted demonstrating dynamic traffic assignment (DTA) as a successful traffic operations tool. For example, DTA has been applied to tolling, congestion mitigation strategies, and ITS technologies. These applications use DTA as a standalone device; DTA outputs are not fed back into the transportation planning model and are not used outside of the project scope. There has been limited research on the integration of dynamic traffic assignment and high-order planning processes – one reason being the high cost of altering an agency's travel demand model. For example, conversion to an activity-based model with DTA integration is impractical and nearly impossible for many agencies due to extensive data requirements and long running times. Combining the traditional four-step planning model with DTA is the most cost-effective approach (and may be the only available approach) to add detailed temporal dynamics to existing planning processes.

The goal of this chapter is to develop a working, efficient, uncostly, and intuitive approach to use the detailed information from dynamic traffic assignment in the traditional transportation planning model. The proposed method allows the agency to conduct traffic analysis at the subnetwork level while integrating DTA and the four-step model at the regional level. This chapter conducts a detailed comparison between static and dynamic traffic assignment, discusses previous literature, and presents the proposed method through an illustrative example.

2.2 Background

2.2.1 Overview of the Four-Step Planning Model

The four-step planning model is shown in Figure 2.1. It includes four iterative processes: trip generation, trip distribution, mode choice, and traffic assignment.

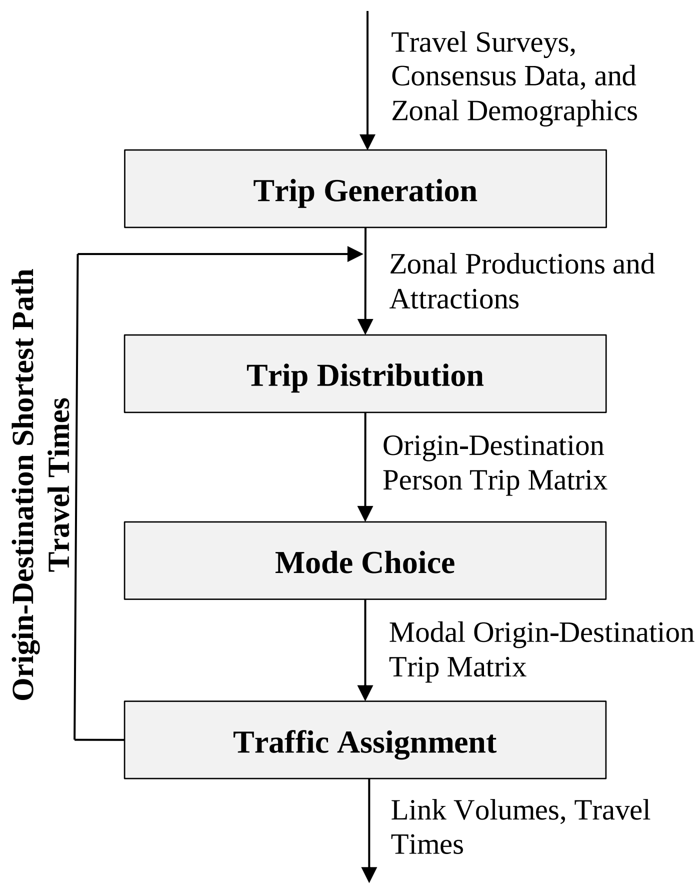

Figure 2.1: The traditional four-step model

Trip generation uses demographic and survey data to determine how many trips are being attracted and produced in each zone. Common practice is to divide trips into trip purpose categories: home-based work, home-based other, non-home-based, etc. The number of trips being produced in each zone is modeled with local survey data at the household level, relating trip production with income, vehicle ownership, household size, and other demographics. Linear regression is commonly used in practice to correlate these independent variables with the number of produced trips. The linear model is used mainly for simplicity; only the average value of the independent variables for each zone is needed as input. Attracted trips can be modeled in the same fashion or can be estimated from the *Trip Generation* handbook published by the Institute of Transportation Engineers (2008).

Trip distribution uses the attractions and productions from the trip generation step and distributes them among the traffic analysis zones in the planning area. This results in the origin-destination matrix, which contains the total number of trips starting in each zone and ending in every other zone. Some version of the gravity model is typically used as shown below.

$$T_{ij} = P_i \frac{(A_j * FF_{ij})}{\sum_{j \in J}(A_j * FF_{ij})}$$

T_{ij} represents the number of trips from origin i to destination j. P_i is the amount of trips zone i produces. A_j is the total number of attracted trips to zone j. FF_{ij} is the friction factor between zone i and zone j. It is typically assumed to be a decreasing function of the shortest path travel time from i to j.

Mode choice converts the person-trips from the trip distribution step into vehicle or other modal trips. This is typically done through a utility function, which measures how satisfied a person is with each mode choice. Utility functions may include the comfort, in-vehicle and out-of-vehicle travel time, cost, and reliability of the mode. A multinomial logit model can be estimated from these utility functions to determine the mode split for each origin-destination pair.

Traffic assignment distributes the vehicular origin-destination matrix from the previous step onto the transportation network using the principle of user equilibrium (PUE). PUE states that every used path between the same origin and destination must have minimal and equal travel time. In static traffic assignment, link performance functions are used. A link performance function relates link volumes to link travel times. Common practice is to assume the link performance function as the Bureau of Public Roads (BPR) delay function given below.

$$t(v) = t_o\left(1 + \alpha\left(\frac{v}{c}\right)^{\beta}\right)$$

$t(v)$ is the link's travel time with a volume of v, a free flow travel time of t_o, and a capacity of c. α and β are parameters which are used to fit observed data, but are usualy taken as the default values of $\alpha = 0.15$ and $\beta = 4$ (Bureau of Public Roads, 1964).

2.2.2 Foundations of Static and Dynamic Traffic Assignment

Comparing DTA and static traffic assignment is not a trivial exercise due to their fundamental difference in theoretical base and solution methodology. In static traffic assignment (STA), there is no temporal dimension; every user enters and is distributed onto the network at the same time. Therefore, STA models do not limit the actual flow on each link. Demand can exceed capacity. Consequently, true roadway capacities should not be used in static assignment – a caveat which has been overlooked by several transportation agencies. Practical capacities are used and the "excess" demand is accommodated by the dramatic increase in travel time when v/c exceeds one in the BPR function. Practical capacity is defined as the capacity at LOS C.

Dynamic traffic assignment models can be grouped into two broad categories: analytical and simulation-based. Currently, simulation-based models are the only methods suitable for practical use. Many models are based on traffic flow dynamics and use the Lighthill-Whitman-Richards (LWR) theory developed by Lighthill and Whitman (1955) and Richards (1956). The LWR model develops relationships between roadway speed (u), density (k), and flow (q). Specifically, the model is characterized by the fundamental equation (2.1), the fundamental relationship (2.2), and the conservation equation (2.3) as shown below.

$$q = uk \tag{2.1}$$

$$q = f(k) \tag{2.2}$$

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad (2.3)$$

The fundamental relationship relates flow and density. It is a continuous nonnegative function that is zero when $k = 0$ and $k = k_j$. k_j is the jam density and is defined as the maximum density of the roadway (i.e., the condition where traffic is completely stopped due to congestion). Nearly all modern DTA software uses either a triangular or trapezoidal fundamental diagram. The diagrams are completely defined by: k_j, q_{max} (capacity), u_f (free flow speed), and w (backward wave speed). Figure 2.2 depicts a typical triangular fundamental diagram.

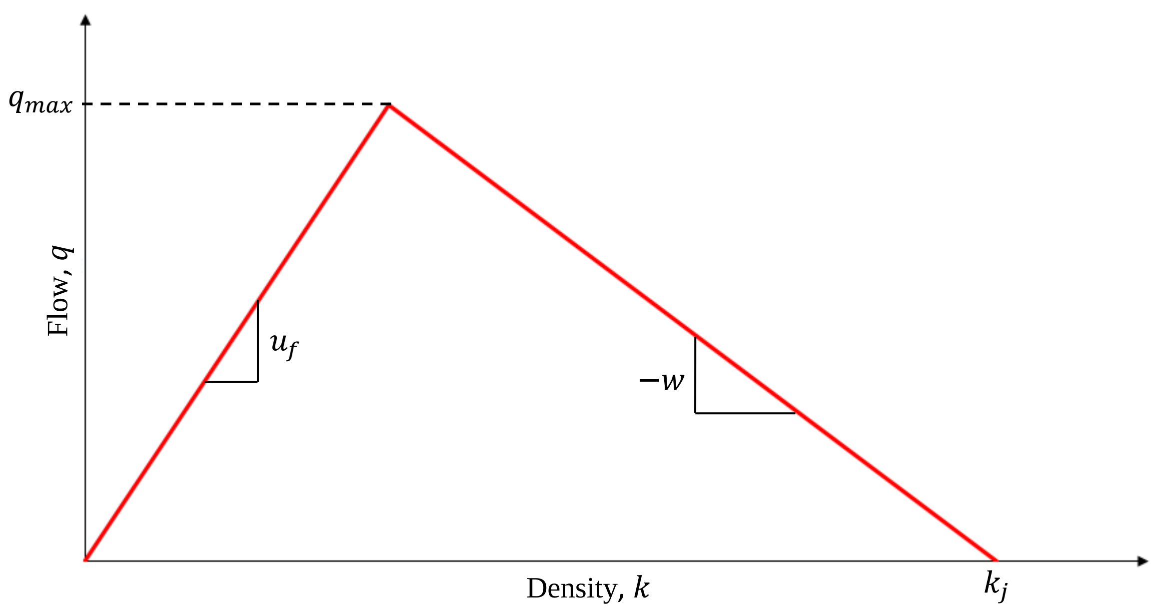

Figure 2.2: Triangular fundamental diagram

The conservation equation enforces that all vehicles are kept in the system (i.e., vehicles do not disappear or appear spontaneously). t represents time, and x represents distance. Traffic is simulated through the network such that equations (2.1), (2.2), and (2.3) are satisfied. Users still try to minimize travel time but now flows are constrained to fundamental laws; capacity constraints are strictly enforced. Figure 2.3 shows the relationship between travel time and link volume when employing the LWR model with a triangular fundamental diagram. Figure 2.4 shows the relationship between travel time and volume in static traffic assignment (i.e., the BPR function). The differences are apparent. For example, the BPR function is convex and differentiable; the function presented in Figure 2.3 is neither. This affects modeling techniques since convexity is a requirement when using optimizing methods to solve global minimization problems. Therefore (due to other properties as well), static traffic assignment can be formulated as a mathematical program and solved with standard optimization techniques. Simulation-based DTA models cannot.

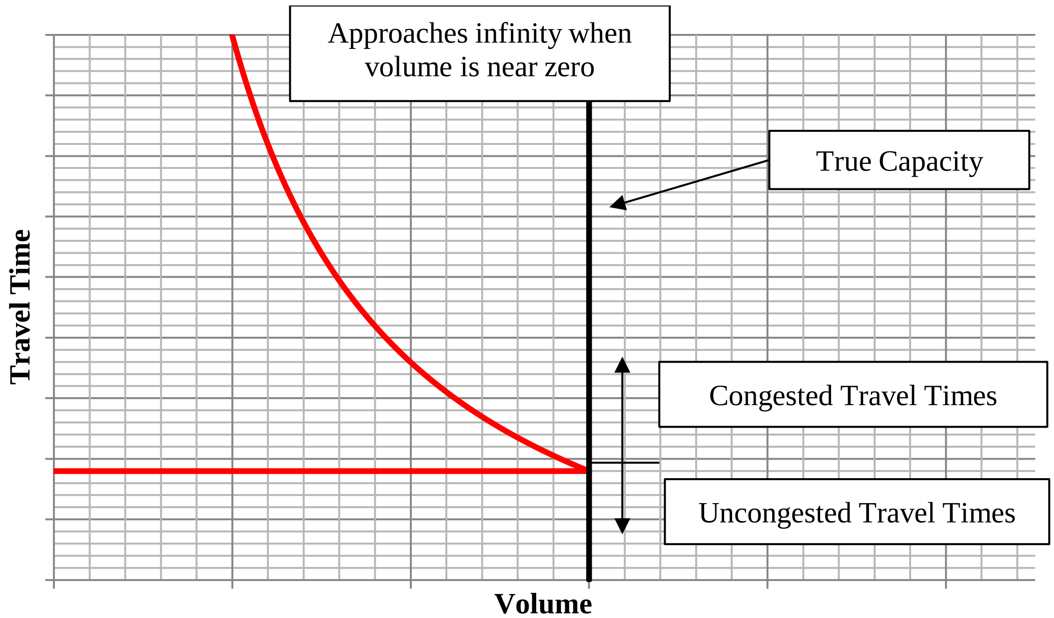

Figure 2.3: Travel time and link volume relationship in DTA

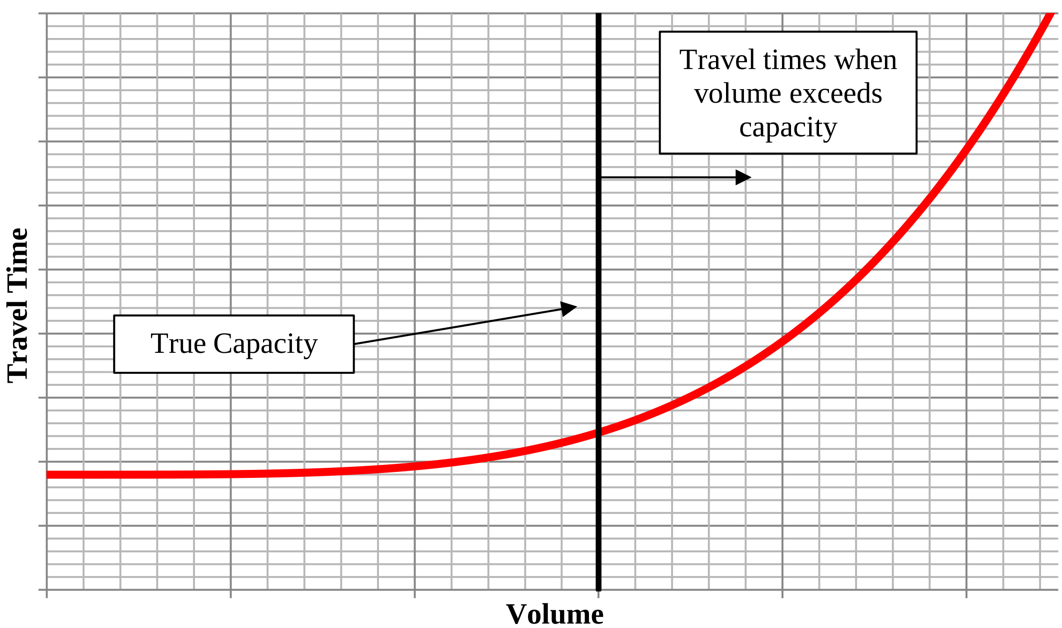

Figure 2.4: Travel time and link volume relationship in STA – BPR function

2.2.3 Comparison of Static and Dynamic Traffic Assignment

This section examines outcome differences between dynamic and static traffic assignment using the downtown Austin, TX network shown in Figure 2.5. The network consists of 1,253 links, 456 nodes, 86 centroids, and 2,542 origin-destination pairs. Standard BPR functions with default α and β values are used. For dynamic traffic assignment, the Visual Interactive System for Transportation Algorithms (VISTA) software is used. VISTA is based on the cell transmission model (CTM). CTM divides the network into sections (cells). The length of the cells are equal to the distance traveled by a typical vehicle in the assignment period. The cell's capacity, free flow speed, and vehicle occupancy are known for each time period. Therefore, the model can track the inflow and outflow of each cell. Several refinements of the original CTM have been made to VISTA, including the incorporation of traffic signals, advanced intersection movements, and fixed route transit. Ziliaskopoulos and Waller (2000) describe VISTA and its web-based interface in detail. A thorough explanation of CTM is given by Daganzo (1994, 1995).

Figure 2.5: Downtown Austin, TX network

Every link in the downtown network can be divided into the following functional classifications: principal arterial directionally divided, principal arterial undivided, minor arterial undivided, and collector undivided. By grouping the network under these categories, differences in vehicular flows from DTA and STA become apparent. Roadways with large capacities have much higher traffic volume when using the static method compared to the dynamic method. Similarly, smaller roadways (minor arterials and collectors) have larger volumes in the dynamic case versus STA as shown in Table 2.1.

Table 2.1: Comparison of link volumes based on functional classification type

Functional Classification	Total Volume (STA)	Total Volume (DTA)	Total Difference in Volume (STA - DTA)	Average Difference in Volume per Link
Principal Arterial Directionally Divided	1,630,710	1,395,627	235,083	353
Principal Arterial Directionally Undivided	181,920	167,064	14,856	56
Minor Arterial Directionally Undivided	38,229	62,898	-24,669	-64
Collector Directionally Undivided	861	2,097	-1,236	-309

Users want to take high-mobility roadways due to higher speeds and added comfort. However, they cannot realistically all use high-mobility roadways due to capacity constraints and congestion; they distribute themselves throughout the entire network, including minor roadways to minimize travel time. This behavior is captured in dynamic traffic assignment and is further shown in Table 2.2. Table 2.2 shows the average percent change in volume of a link when static traffic assignment is used compared to DTA. Links are segmented by their v/c ratio. The v/c values were calculated using static traffic assignment flows. As shown in Table 2.2, STA may be over-predicting vehicular volume on congested streets and severely under-predicting flows on uncongested streets.

Table 2.2: Average percent change in link volume

V/C Ratio	Average Percent Change of Link Volume (Static as Base)
> 1.0	-28.62 %
< 1.0 - 0.75	-24.42 %
< 0.75 - 0.50	-10.06 %
< 0.50 - > 0.0	907.34 %

Directly comparing link travel times between the two differing traffic assignment methods is not trivial. VISTA simulates traffic signals. Therefore, the travel time of a link with a signalized intersection varies depending on which turning movement the user takes. Individual link travel time comparisons will not be conducted. Instead, travel times on three corridors (where turning movements are specified) will be analyzed. The corridors are shown in Figure 2.6. Each corridor illustrates a key behavior of static and dynamic traffic assignment.

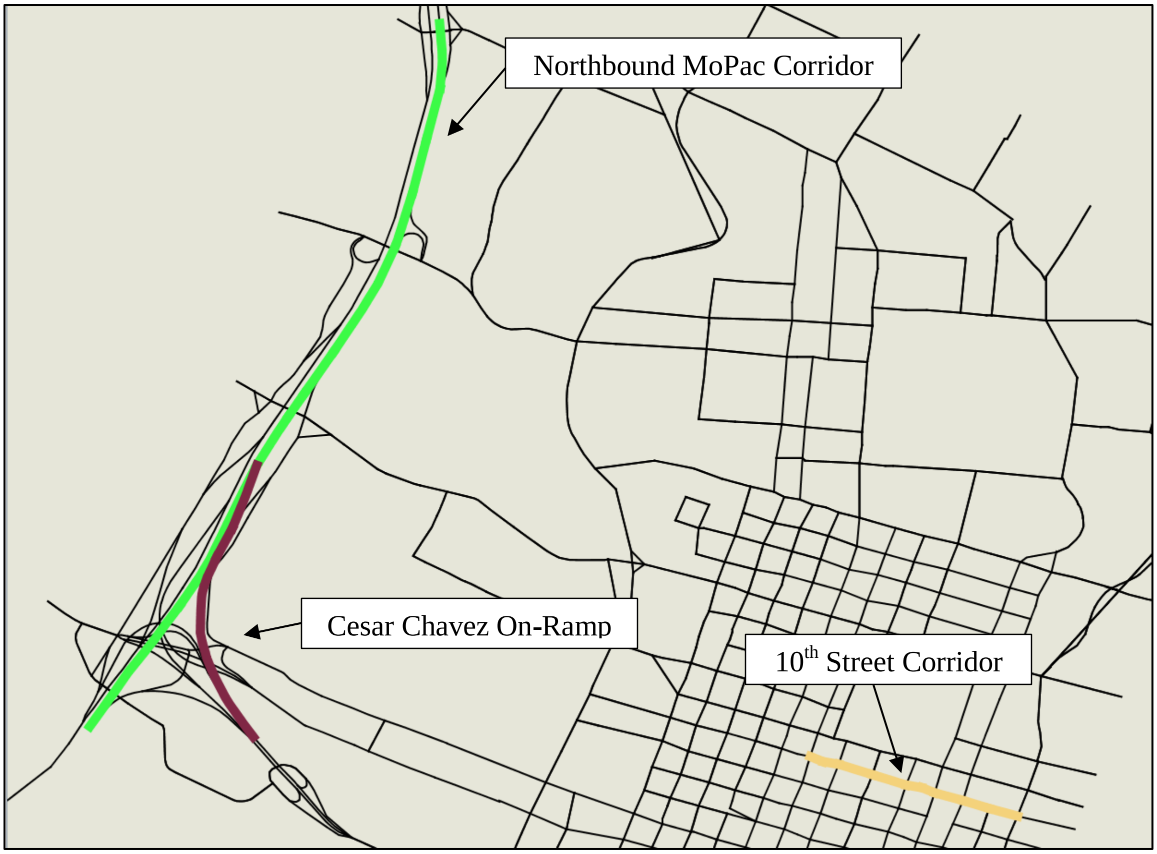

Figure 2.6: Location of corridors in travel time analyses

Figure 2.7 shows the dynamic and static travel times along the northbound MoPac corridor. The corridor exhibits the typical peak period curve and becomes heavily congested during the simulation period; travel times vary from 2 minutes to over 30 minutes in the dynamic case. This section of MoPac becomes congested mainly due to the large bottleneck that forms further north and steadily travels downstream. As shown in Figure 2.7, static traffic assignment does not capture this congestion buildup and queuing. Its travel time is roughly 3 minutes compared to the average dynamic travel time of 11 minutes. Based on this example, static traffic assignment may be under-predicting travel times in heavily congested areas – mainly because capacity constraints are not strictly enforced.

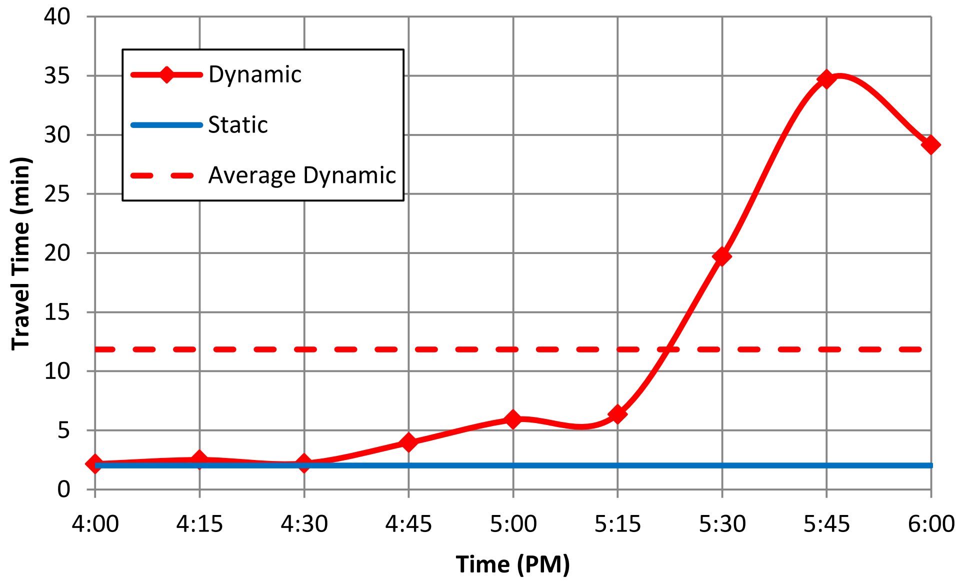

Figure 2.7: Northbound MoPac corridor dynamic and static travel times

The 10th Street corridor was included in the analysis to demonstrate travel time differences on uncongested links. As shown in Figure 2.8, dissimilarities between the two methods are still apparent. Static travel time is nearly 2 minutes shorter than the average dynamic travel time. There is limited variation in the dynamic travel times, indicating that these links are indeed uncongested. The stark difference can be explained through signalized intersection delays that are modeled in DTA but are nonexistent in most static traffic assignment models. This suggests that static assignment may be further under-predicting travel times due to intersection delay.

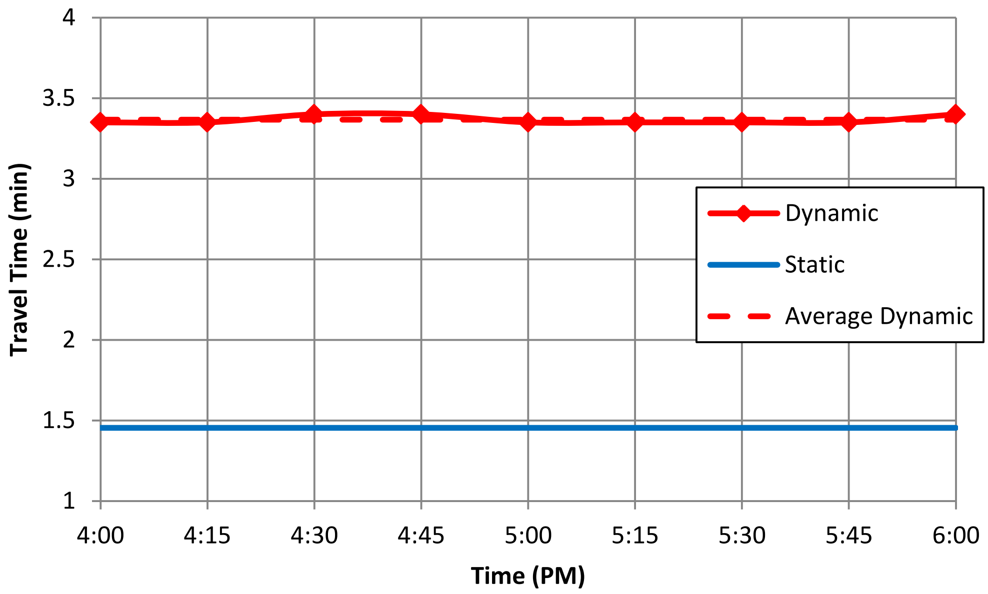

Figure 2.8: 10th Street corridor dynamic and static travel times

As shown in Figure 2.9, there is little variation in travel time on the Cesar Chavez entrance ramp. In fact, static and dynamic travel times are very similar; there is less than a 3 second difference between the static and average dynamic travel time. This suggests that static traffic assignment can more accurately predict travel times where there is limited congestion and no signalized intersection delay.

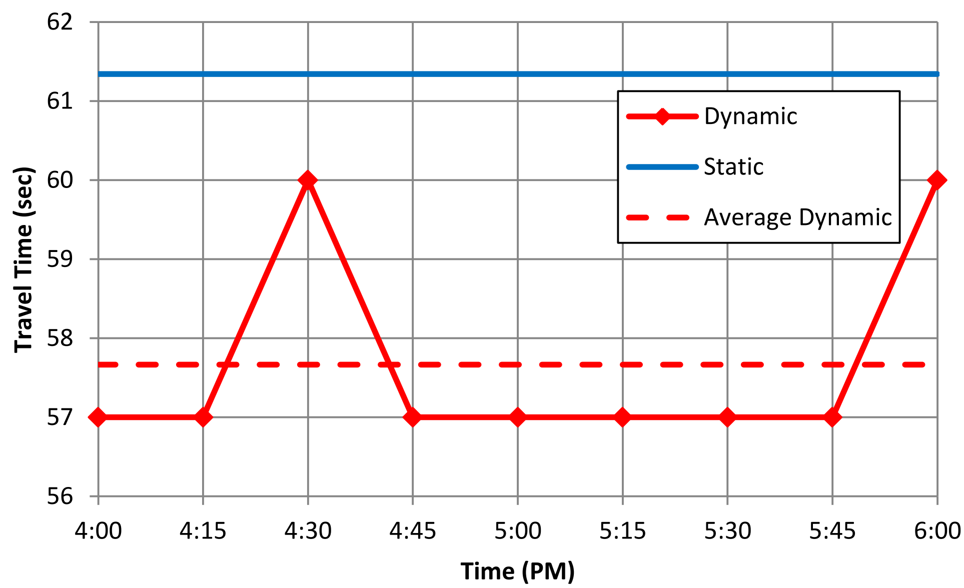

Figure 2.9: Cesar Chavez on-ramp dynamic and static travel times

2.3 Literature Review

Though there have been several studies involving dynamic traffic assignment and activity-based modeling integration (Lam and Yin, 2001; Lin et al., 2008; Lin et al., 2009; Hao et al., 2010; among others), the author knows of only two studies incorporating DTA in the traditional planning model. One study integrates DTA at the regional level, the other at the subnetwork level.

2.3.1 Regional Level Integration

Tung et al. (2012) integrated DTA with the Puget Sound Regional Council (PSRC) travel demand model. The model is based on the four-step process and encompasses the city of Seattle and surrounding areas. Tung et al. replaced the static traffic assignment step with dynamic traffic assignment while leaving the other processes and the feedback mechanism intact. The PSRC model includes a time-of-day discrete choice model which operates between the mode choice and traffic assignment step. The time-of-day process divides the 24-hour modal trip table into finer time intervals: one for every 30 minute period. The study is ongoing, and results have not been published. The report does not specifically state what is being fed back into the four-step model. However, the utility functions in the time-of-day model include a travel time delay term leading the reader to believe that the average travel time for each 30 minute interval for every origin-destination pair obtained from the dynamic traffic assignment process is used as input. The report also indicates feedback loops from DTA to the trip distribution and mode choice steps, but no details are given.

2.3.2 Subnetwork Level Integration

Pool (2012) implemented DTA in a standard four-step model: a basic gravity model is used in the trip distribution step and mode choice consists of a simple discrete choice logit model between vehicular and bus use. Like Tung et al. (2012), Pool replaced static traffic assignment with dynamic traffic assignment leaving the other steps and the main feedback system unchanged. Pool applied the integrated model at the subnetwork level using the downtown Austin, TX network shown in Figure 2.5. Pool simply replaced the shortest path travel time of each origin-destination pair that would originally be obtained from STA with the average shortest path travel time over the entire simulation period from DTA. These values were used as input to the trip distribution step – in the friction factors – and in the mode choice step – in the utility functions. To convert the static vehicle trip table (i.e., the total amount of origin-destination trips made during the simulated peak period) from the mode choice step to dynamic origin-destination matrices, a constant demand profile was assumed. Pool used this integrated planning model to converge on the peak period, subnetwork static trip table.

The two discussed methods have their own benefits and limitations. The main drawbacks of regional level implementation are: (1) the long convergence time of regional DTA models (the Austin regional model takes five days to converge) and (2) agencies typically apply DTA to smaller spatial area analyses creating wasted detail and data if applied regionally. Integration at the subnetwork level will not capture effects outside of the study area. These impacts can be significant especially if major connectors (e.g., highways or regional light rail lines) pass through the subnetwork. The proposed method in this thesis retains the advantages of both methods, while avoiding their disbenefits.

2.4 Methodology

Like Tung et al. (2012) and Poo1 (2012), the proposed method involves replacing static traffic assignment with dynamic traffic assignment. The goal is to utilize the more detailed and accurate traffic information from DTA while minimally changing the existing four-step process. Significantly altering an agency's travel demand model is costly – in terms of development, time, calibration, retraining, data collection, and monetary expenditures. This is one of the reasons why regional integration is important; many traditional planning models in practice converge and operate with regional metrics. For example, the Capital Area Metropolitan Planning Organization (CAMPO) in Austin, TX uses the regional trip length frequency distribution in their travel demand model convergence process.

Assume a regional DTA network with sets O and D of origin and destination nodes, respectively. $O_s \subseteq O$ and $D_s \subseteq D$ are the set of origins and destinations in the subnetwork being analyzed. Set E represents external connectors in the subnetwork. External links are located on the study boundary and connect the subnetwork to the regional network as shown in Figure 2.11. Assume P_{od} is the set of used paths from origin o to destination d. The proposed integration scheme first requires complete regional implementation for the base condition (such as the base year network). From these regional DTA outputs, four items are recorded:

1. The origin-destination travel times for every $o \in O, d \in D: \{o \notin O_s, d \notin D_s, E \notin P_{od}\}$. These travel times are assumed constant regardless of changes to the subnetwork.
2. The travel time of the portion of used paths connecting *to* an external connector for every $p_{od} \in P_{od}, o \in O, d \in D: \{o \notin O_s, d \in D_s\ or\ e \in p_{od}\}$. These paths are entering the subnetwork. The travel time of the portion of the path located outside of the subnetwork is assumed constant.
3. The travel time of the portion of used paths *from* an external connector for every $p_{od} \in P_{od}, o \in O, d \in D: \{o \in O_s\ or\ e \in p_{od}, d \notin D_s\}$. These paths leave the subnetwork. The travel time of the portion of the path outside the subnetwork is assumed constant.

4. The proportion of total demand for each origin-destination pair that use an external connector. This proportion is assumed constant.

Essentially, paths or portions of paths outside the subnetwork are assumed to have constant travel time. After the initial base condition, dynamic traffic assignment is implemented at the subnetwork level. Travel times within the subnetwork will vary at each iterative loop of the four-step process. The shortest path travel time from each external connector to each subnetwork centroid (and vice versa) is determined. This information is then used to update regional path travel times as shown in Figure 2.10.

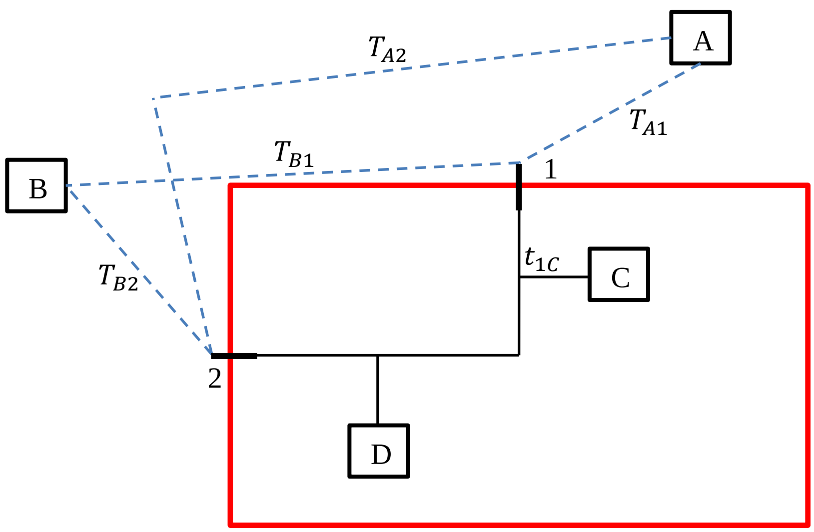

Figure 2.10: Demonstration of the regional path travel time update procedure

The network in Figure 2.10 has two external connectors (1 and 2) and two external centroids (A and B). The subnetwork boundary is in red. T_{B1} represents the constant shortest path travel time from centroid B to external link 1 as recorded from the regional DTA model. Assume an agency is analyzing changes to the subnetwork (perhaps a link is added, a lane is dropped, or capacity decreased). Dynamic traffic assignment is applied at the subnetwork level, and travel times within the subnetwork are updated. Based on these travel times the regional shortest path travel times are calculated. For example, t_{1C} represents the shortest path travel time from external link 1 to centroid C. The shortest path travel time from origin B to destination C, t_{BC}, is: $t_{BC} = T_{B1} + t_{1C}$. These regional origin-destination pair travel times can then be used in the friction factor values of the gravity model or in the mode choice utility functions.

Items (1) – (3) are produced by current DTA software automatically or can easily be determined from given outputs. Item (4) is calculated directly from DTA results and requires minimal computational effort. Since origin-destination demand can change at each iteration, assumption (4) is required to convert the regional origin-destination matrix into the subnetwork origin-destination matrix. By allowing some interaction between the subnetwork and regional outputs, more information can be used in the four-step planning convergence loop (e.g., transit lines extending outside of the subnetwork can be somewhat modeled via the mode choice step occurring at the regional level). This interaction is nonexistent in the method proposed by Pool (2012). Also by allowing this interaction, subnetwork regions may be modeled at a finer spatial area saving time, effort, and storage from unwanted data. The proposed method is summarized in Figure 2.11 using the regional Austin, TX network.

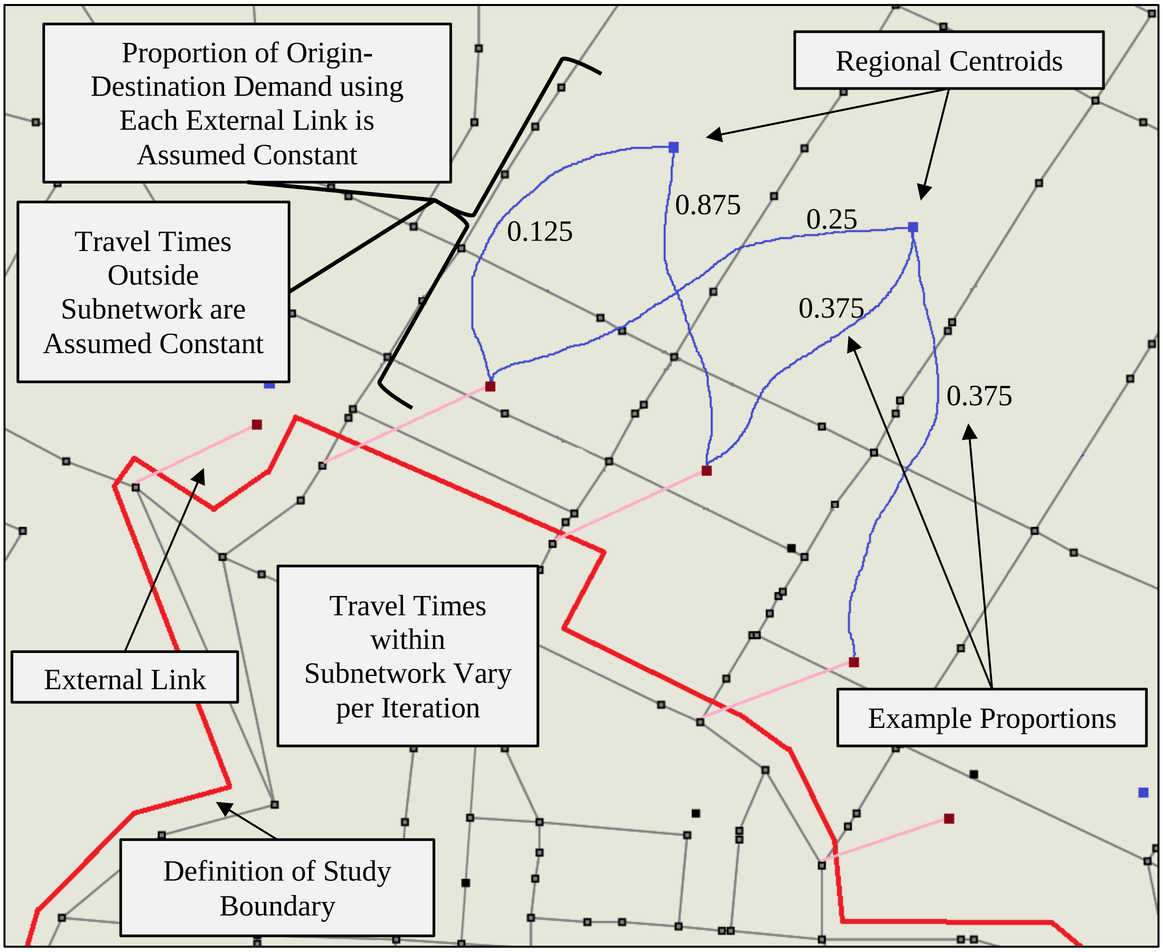

Figure 2.11: Summary of proposed method using the regional Austin, TX network

The integration method discussed above is adaptable and can be applied to an agency's unique travel demand model. For example, as in Pool (2012), average dynamic travel times over entire simulation periods can be used in the feedback process. Like Tung et al. (2012), average travel times over finer intervals can be used. Travel time information can feedback to the trip distribution, mode choice, or time-of-day model. The four-step model convergence metric can be the static trip table, regional trip length frequency distribution, and/or shortest path travel times. The procedure to update regional path travel times is also flexible. For example: (1) regional paths using an external link can be assumed to use the same link after modifications to the subnetwork, (2) a subset of paths (i.e., paths using different external connectors) can be analyzed and the shortest path among the subset used in the four-step process, or (3) the travel time of each possible path (i.e., the shortest path from the regional centroid to each external link, then from each external link to the subnetwork centroid) can be determined and the minimum used as the shortest path. If agencies are concerned the altered subnetwork may significantly divert users – drivers bypass the subnetwork due to increased travel time, the regional path update procedure can include a comparison of the travel time of the shortest path through the subnetwork and the shortest path outside the study boundary. To determine the path travel time outside the study area, DTA can be applied to a reduced base case regional network, which excludes subnetwork arcs and nodes. The next section further demonstrates the proposed method using a small example network.

2.5 Illustrative Example

The network shown in Figure 2.12 is used to compare three implementation methods: the proposed method, subnetwork integration only (Pool, 2012), and complete regional integration (Tung et al., 2012). There are two regional centroids (A and B), four subnetwork centroids (C, D, E, and F), and four external links (4 ↔ C, 5 ↔ D, 7 ↔ C, 9 ↔ E). The study boundary is indicated in red and represents a downtown area with regional connections via a rail service and surrounding highway network.

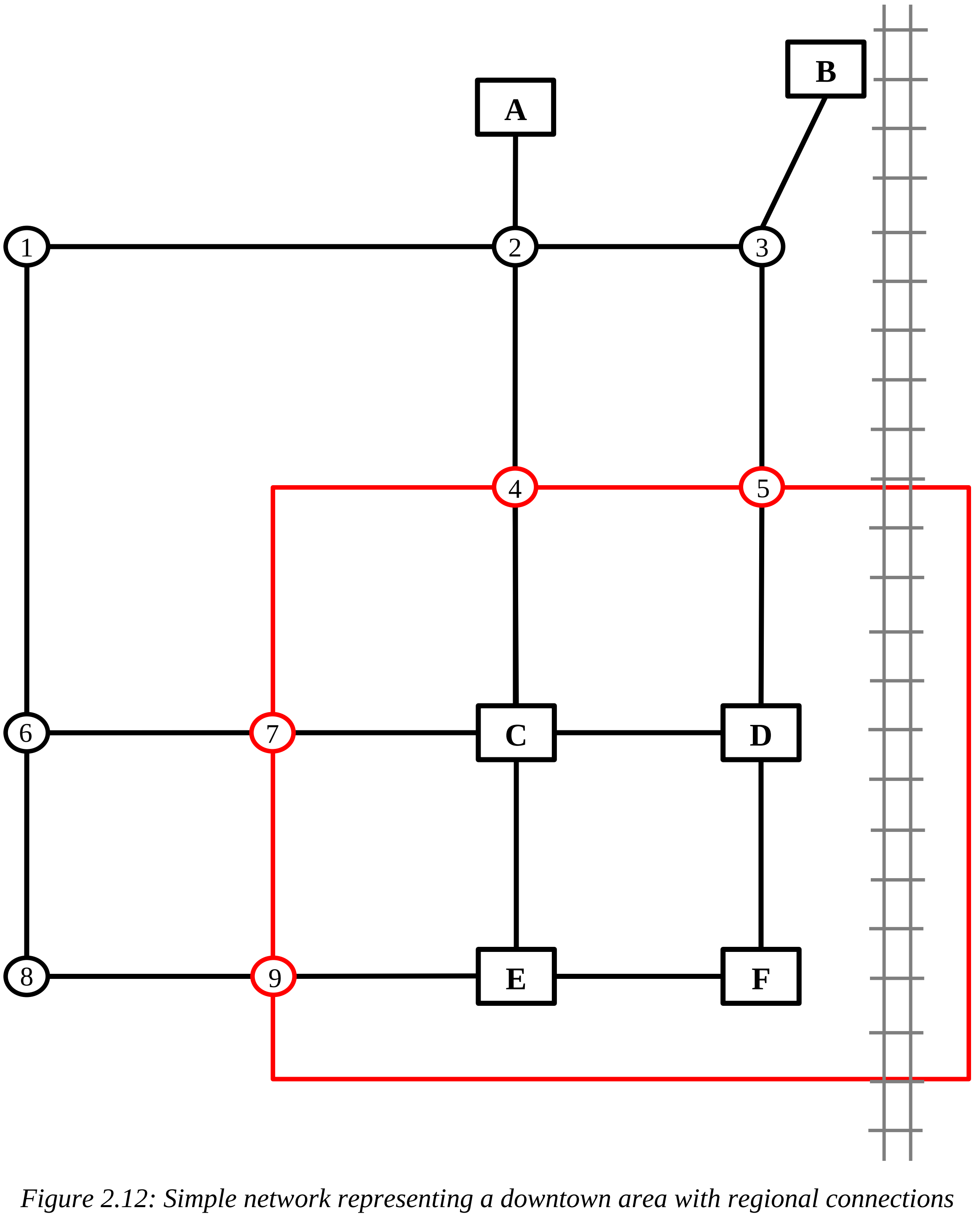

Figure 2.12: Simple network representing a downtown area with regional connections

The rail service is available to centroids B, D, and F. The subnetwork links, link 2 ↔ 4, and link 3 ↔ 5 have free flow speeds of 30 mph and capacities of 2,000 vph per traffic direction. Regional links have free flow speeds of 75 mph and capacities of 3,000 vph per direction. For simplicity, the iterative process of the integrated travel demand model occurs over a single time period (e.g., the AM peak period) and convergence is measured by the root-mean-square error of the peak period static trip table. Productions and attractions are shown in Table 2.3. As shown in the table, centroids A and B have high productions but few attractions. Centroids D and F have high attractions but few productions.

Table 2.3: Peak period productions and attractions

Centroid	Productions	Attractions
A	4,200	700
B	2,380	840
C	1,400	1,680
D	1,260	3,780
E	1,680	1,680
F	1,400	3,640

Dynamic traffic assignment is conducted using the VISTA software (see Section 2.2.3 for details). The average travel time among all dynamic paths in the peak period for each origin-destination pair is fed back into the trip distribution and mode choice steps. A standard gravity model is used in the trip distribution process. Mode choice consists of a discrete choice logit model between vehicular and rail use, where available. Parameters of the utility functions were chosen such that 8% of travelers use the rail service if both modes had equal travel time. The regional rail line has constant travel time approximately 3x longer than free flow conditions traveling by car. Dynamic origin-destination matrices were determined by applying the demand profile shown in Figure 2.13 to the static peak period trip table.

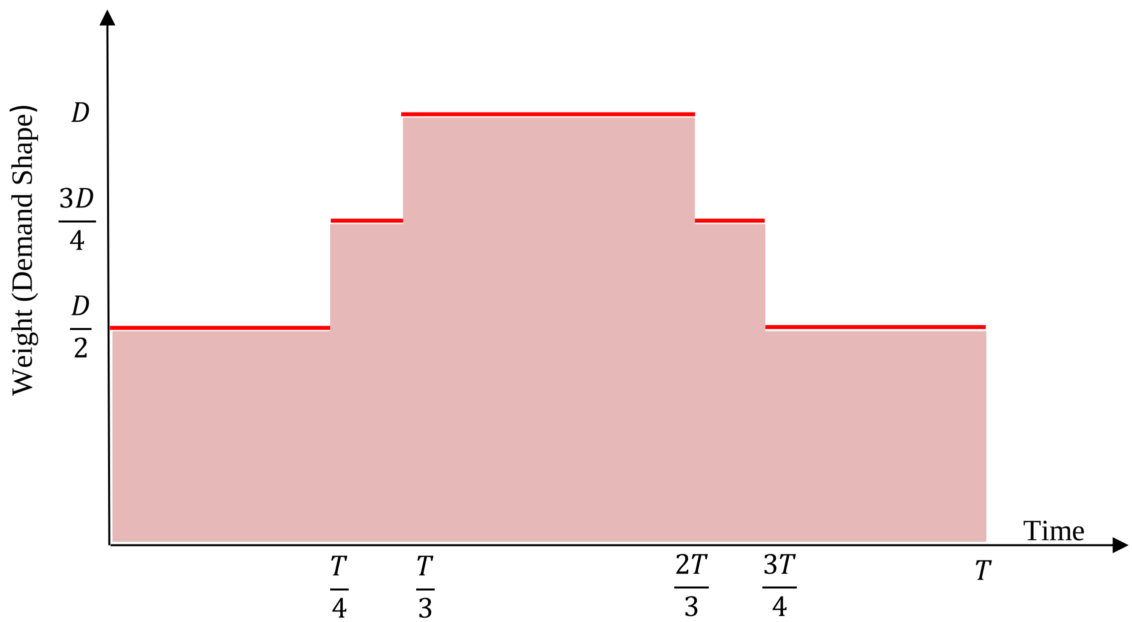

Figure 2.13: Demand profile

The integrated demand model was applied to the network shown in Figure 2.12. The resulting link volumes at convergence (totaled over the peak period) are shown in Table 2.4. Free flow speeds of links C ↔ E and E ↔ F were then decreased to 10 mph. As in the method proposed by Tung et al., the demand model was applied regionally again. The total link volumes of the modified network are also shown in Table 2.4.

Table 2.4: Total link volumes of the initial and modified regional network

	Initial Network	Modified Network	(Modified – Initial)
Link	**Total Volume**	**Total Volume**	**Volume Difference**
1 → 2	28	0	-28
1 ← 2	63	111	48
2 → 3	719	938	219
2 ← 3	1192	1287	95
2 → 4	1964	1831	-133
2 ← 4	246	307	61
3 → 5	1900	2108	208
3 ← 5	217	233	16
1 → 6	63	111	48
1 ← 6	28	0	-28
4 → C	1953	1831	-122
4 ← C	232	307	75
5 → D	1886	2107	221
5 ← D	203	233	30
6 → 7	83	25	-58
6 ← 7	51	7	-44
7 → C	69	25	-44
7 ← C	37	7	-30
C → D	1345	1379	34
C ← D	626	664	38
6 → 8	59	93	34
6 ← 8	57	0	-57
C → E	1314	1052	-262
C ← E	498	455	-43
D → F	1258	1493	235
D ← F	866	900	34
8 → 9	59	93	34
8 ← 9	57	0	-57
9 → E	45	93	48
9 ← E	43	0	-43
E → F	1509	1071	-438
E ← F	701	378	-323
Total	**19,371**	**19,139**	**-232**

As shown in Table 2.4, the decrease in free flow speed caused several changes to travel behavior. There are significantly fewer users traveling on the modified links C ↔ E and E ↔ F. Instead, these users, caused by route choice behavior and changing demand, shift to links 2 ↔ 3, 3 ↔ 5, 5 ↔ D, and D ↔ F (especially to links 2 → 3, 3 → 5, 5 → D, and D → F). Likewise, this causes a major decrease in flow on 4 → D and the regional link 2 → 4. More travelers are using the regional rail due to the increased vehicular travel times. However, this modal shift is not significant due to the predisposition toward vehicular use assumed in the population.

To conduct integration at the subnetwork level only as in Pool (2012), subnetwork productions and attractions were determined from the regional unmodified network solution and are assumed constant (i.e., they do not vary from iteration to iteration). The network is then modified. Link volumes resulting from this and the proposed method are shown in Table 2.5. Subnetwork link volumes are compared to the true solution (the modified network integrated at the regional level – the method proposed in Tung et al. (2012)).

As shown in Table 2.5, integration at the subnetwork level leads to dramatically different results. The method predicts a larger impact; there are significantly fewer vehicles on links 4 ↔ C, C ↔ E, and E ↔ F. In fact, links connecting centroids C, D, E, and F have markedly different flows. These differences are caused by the assumption that productions and attractions are constant and directly calculated from the unmodified network. Using the proposed method, subnetwork productions and attractions vary at each iteration; only regional productions and attractions are assumed constant. This variation, this subnetwork-regional network interaction, allows for a more accurate prediction of flow as shown in Table 2.5. The error associated with the proposed method – as measured by the root-mean-square error (RMSE) of total link demand using the modified regional network solution as the true solution – is much smaller than the error associated with integration at the subnetwork level only.

Table 2.5: Results from the proposed method and integration at the subnetwork only

	Regional Network	**Subnetwork Only**	**Proposed Method**	**(Subnetwork – Regional)**	**(Proposed – Regional)**
Link	**Total Volume**	**Total Volume**	**Total Volume**	**Volume Difference**	**Volume Difference**
4 → C	1831	1027	1819	-804	-12
4 ← C	307	205	216	-102	-91
5 → D	2107	2485	1757	378	-350
5 ← D	233	2322	189	2089	-44
7 → C	25	22	64	-3	39
7 ← C	7	0	34	-7	27
C → D	1379	1427	1253	48	-126
C ← D	664	153	584	-511	-80
C → E	1052	795	1225	-257	173
C ← E	455	53	464	-402	9
D → F	1493	1259	1171	-234	-322
D ← F	900	631	807	-269	-93
9 → E	93	8	42	-85	-51
9 ← E	0	0	40	0	40
E → F	1071	766	1406	-305	335
E ← F	378	203	653	-175	275
Total	**11,995**	**11,356**	**11,724**	**-639**	**-271**
RMSE	**(Base)**	**151.98**	**43.83**		

2.6 Conclusion

This chapter proposed a method to integrate dynamic traffic assignment into the traditional four-step planning process, thus utilizing the detailed and accurate results of DTA to inform high-order planning decisions. The proposed structure allowed traffic analysis to be conducted at the subnetwork level, while integration occurred at the regional level. This avoided the long convergence time of regional DTA models while keeping a vital connection between the subnetwork and regional area. As demonstrated via a small example network, the connection is imperative for accurate results. This chapter ends the investigation of dynamic traffic assignment in high-order planning processes. The next chapter studies DTA as an operational tool, showcasing its capabilities when analyzing traffic impacts through a detailed case study.

Chapter 3: Modeling the Traffic Impacts of Transit Facilities using Dynamic Traffic Assignment

3.1 Introduction

The purpose of this chapter is to demonstrate the usefulness, benefits, and capabilities of dynamic traffic assignment (DTA) when applied to traffic impact analysis (i.e., how the versatile outputs of DTA can quantify travel behavior changes and aid practitioners in project evaluation). A proposed urban rail system in downtown Austin, TX is used as a case study. In the past, transportation planning agencies have adopted two basic types of analyses to estimate vehicular impacts caused by rail facilities: corridor-specific analysis and regional planning. This chapter presents a new methodology by implementing dynamic traffic assignment. DTA provides a connection between the two methods: it can model route choice behavior with realistic, detailed inputs at the large-scale level. Thus, the new methodology retains the benefits of the past two procedures while avoiding their shortcomings.

The subsequent sections detail the corridor-specific analysis and regional planning methods, define the geometric and operational characteristics of the proposed urban rail system, describe how the rail facility was modeled and inputted into the dynamic traffic assignment program, and conduct a traffic impact analysis focusing on route choice behavior.

3.2 Background

Implementation of new or expansion of existing transit systems can have significant and wide-ranging impacts on traveler behavior. With the serious investment in light rail over the past two decades (Pucher, 2002), it is imperative for planners to quantify and fully understand these impacts – especially the effects on vehicular traffic. Rail transit systems can affect automobile users in several ways: signal priority or at-grade rail crossings can cause extra driver delay, users may switch from automobile use to the transit mode helping to mitigate congestion, drivers may switch their route (perhaps avoiding corridors with reduced vehicular capacity), or the transit system can fundamentally change user travel patterns. Planning agencies as well as research entities have used several methods to estimate these traffic impacts. As mentioned before, they can be broadly grouped into two categories: corridor-specific analysis and regional planning. Past studies in each category are discussed in the next section.

3.3 Literature Review

3.3.1 Corridor-Specific Analysis

One of the first documented corridor-specific studies to estimate vehicular impacts due to rail facilities was conducted by Cline et al. (1989). The study used microsimulation software to quantify the delay experienced by drivers at intersections with LRT crossings. Since then, microsimulation has become standard practice in transportation planning. Several studies have continued to investigate delay at at-grade rail intersections and where rail corridors share right-of-way (ROW) with automobile traffic (Tidewater Transportation District Commission et al., 2000; U.S. Department of Transportation et al., 2003; Chandler and Hoel, 2004; among others). A plethora of research has also been aimed at using microsimulation software to determine the transit signal priority scheme that minimizes automobile delay (Dale et al., 1999; Davol, 2001; Abdulhai et al., 2002; Ngan et al., 2004; among others). Furthermore, microscopic analysis has been applied to situations where existing right-of-way is converted from vehicular use to transit use (Dawson et al., 2011). The majority of corridor-specific studies only investigated automobile delay due to transit impedances. Delay is only a small portion of all traffic impacts; in microscopic level analysis, route choice is non-existent or greatly simplified. Due to the nature of users interacting with each other across the entire network, traffic impacts can be felt far from the actual point of impedance. For instance, delay may cause automobile users to change their route choice producing effects in areas outside the analyzed corridor.

3.3.2 Regional Planning

There is limited research on regional level traffic impact analysis. Most transportation planning agencies indirectly capture changes in vehicular use through the mode choice step in the traditional four-step model (Ang-Olson and Mahendra, 2011). Essentially, planners estimate the number of users who will switch from the automobile mode to the new transit rail mode. This may be done through surveys, utility functions, or through elasticity values from the literature. Once this is done one can estimate the reduction in vehicle miles traveled – which can be used to quantify changes in vehicle operating, emissions, and crash costs. Traffic assignment is then conducted capturing changes in route choice behavior. The inclusion of route choice modeling is important as shown in a recent case study in Melbourne, Australia (Aftabuzzaman et al., 2010). Changes to the rail system caused dramatic effects not just in the central business district but also, and more significantly, in the suburban areas. However, most regional level traffic impact analyses are not detailed enough to directly model transit impedances during the traffic assignment process. Sometimes, though rarely, corridors with transit activity may be modeled with reduced capacity.

Dynamic traffic assignment provides what both standard corridor-specific and regional planning analyses have been lacking. It models route choice behavior at a fine time scale across a large spatial area using realistic, comprehensive inputs.

3.4 Methodology

3.4.1 Proposed Urban Rail System

The case study consists of a proposed urban rail system through downtown Austin, TX. It connects the three major employers of Austin: the University of Texas at Austin (UT-Austin), the State of Texas, and the downtown central business district (CBD). A central hub is located in the downtown area, providing access to the Austin-Bergstrom International Airport as well as the Red Line commuter rail. Access is also provided to the north-eastern Mueller development area as shown in Figure 3.1 (Urban Rail Partners, 2008).

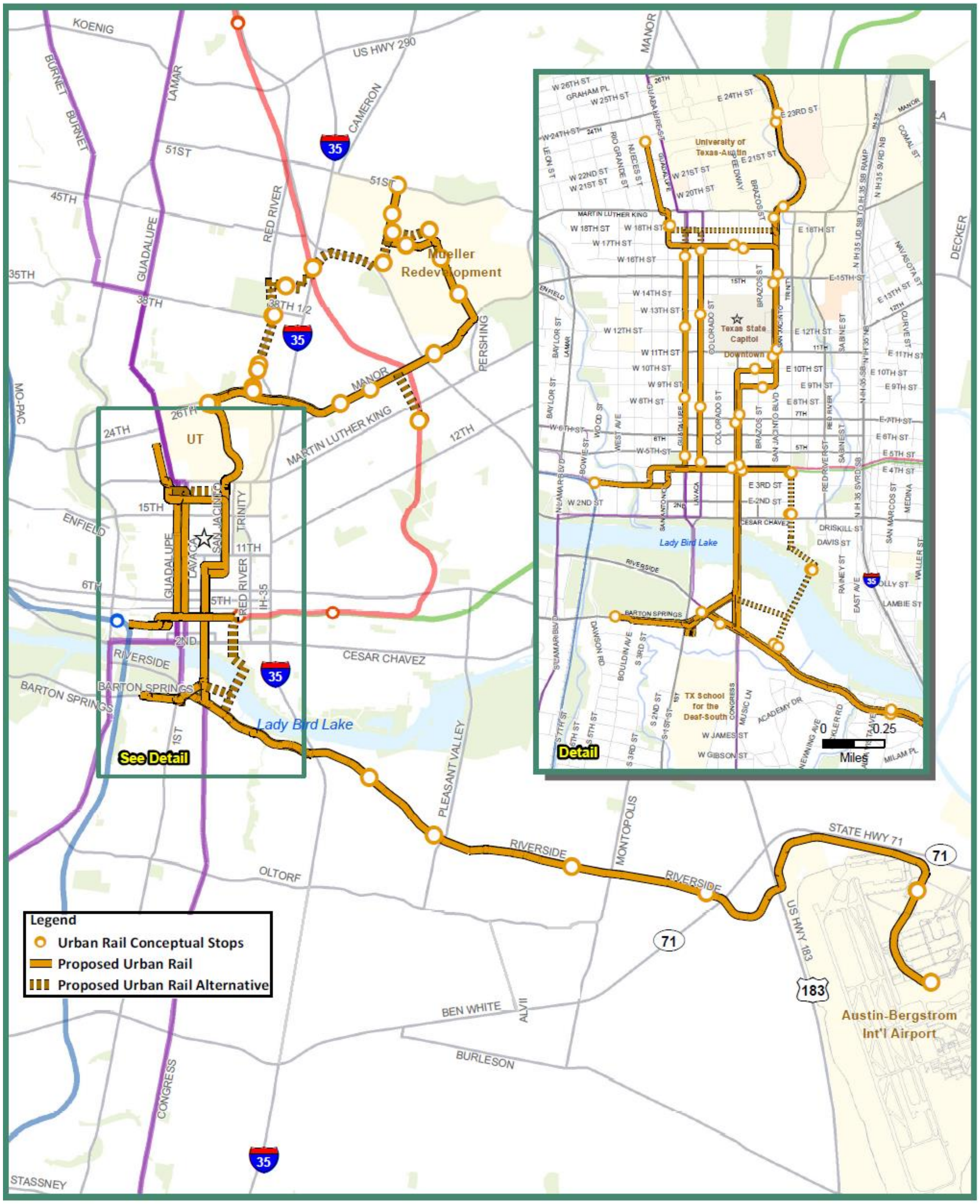

Figure 3.1: City of Austin proposed urban rail system

From the airport to the CBD hub as well as from UT-Austin to the Mueller development, the rail service acts as a light rail system with limited stops. Through campus and the downtown area, the urban rail acts as a streetcar. The system would provide two-way service for each corridor shown in Figure 3.1, except at the following one-way streets: Guadalupe, Lavaca, 9th, 10th, 17th, and 18th Streets. However, limited geometric changes occur to the existing transportation system; the rail shares ROW with traffic in the outside lane in each direction. The streetcar will be powered by an overhead electrical system. Each car has a capacity of 140 passengers, is 66 ft long, 11.4 ft high, and 8 ft wide (City of Austin and Capitol Metropolitan Transportation Authority, 2008). Four major changes occur to the existing network:

1. Guadalupe/Lavaca Corridor – one lane is converted into a shared urban rail and bus rapid transit lane; no automobiles are allowed. This occurs at 4th Street through 17th Street.
2. San Antonio Street – one lane is converted into a dedicated rail guideway.
3. 4th Street – one lane is added in the westbound direction.
4. San Jacinto Boulevard – one lane is added in the southbound direction between M.L.K Boulevard and Dean Keeton Street. It has dedicated ROW for the rail service.

3.4.2 Dynamic Traffic Assignment Process

Most available DTA software programs are fully capable of producing the traffic impact analysis conducted in this chapter (e.g., Dynameq, DynaMIT, DYNASMART, and DynusT). The inputs necessary for the DTA process include: the transportation network with known link capacities and free-flow speeds, traffic signal timings, transit schedules, and an origin-destination matrix for each assignment period. This dynamic origin-destination table may be estimated from an activity-based model or converted from a traditional four-step model through demand profiling as illustrated in Section 2.5. For each assignment/departure period (which ranges from several seconds to several minutes depending on the level of detail needed), traffic is loaded onto the network and user equilibrium is approximated. Therefore, traffic assignment for each period is dependent on the previous traffic distributions.

User equilibrium is attained through an iterative process. Once an initial traffic distribution has been assigned to the network, path travel times (including the shortest path) are calculated for each origin-destination pair. Then a percentage of traffic not located on the shortest path is switched onto the shortest path. This can be done through a variety of methods. The Method of Successive Averages (MSA) is the approach used in this chapter. MSA shifts users onto the shortest path each iteration in predetermined percentages. The typical ratio used is $1/p$, where p is the iteration number. As is common with all traffic equilibrium problems, convergence to the equilibrium is only attainable in the limit. Therefore, a convergence criterion is used to stop the process once traffic flows are reasonably close to equilibrium.

The simulation-based DTA software, Visual Interactive System for Transportation Algorithms (VISTA), is used in the traffic impact analysis. VISTA is based on the cell transmission model (CTM). The reader is referred to Section 2.2.3 for additional details regarding VISTA and CTM.

3.4.3 Description of Base Network

The base network is the downtown Austin network analyzed in Section 2.2.3. It contains 1,253 links, 456 nodes, 86 centroids, and 2,542 origin-destination pairs. The model covers the CBD, State Capitol, and UT-Austin campus as shown in Figure 3.2.

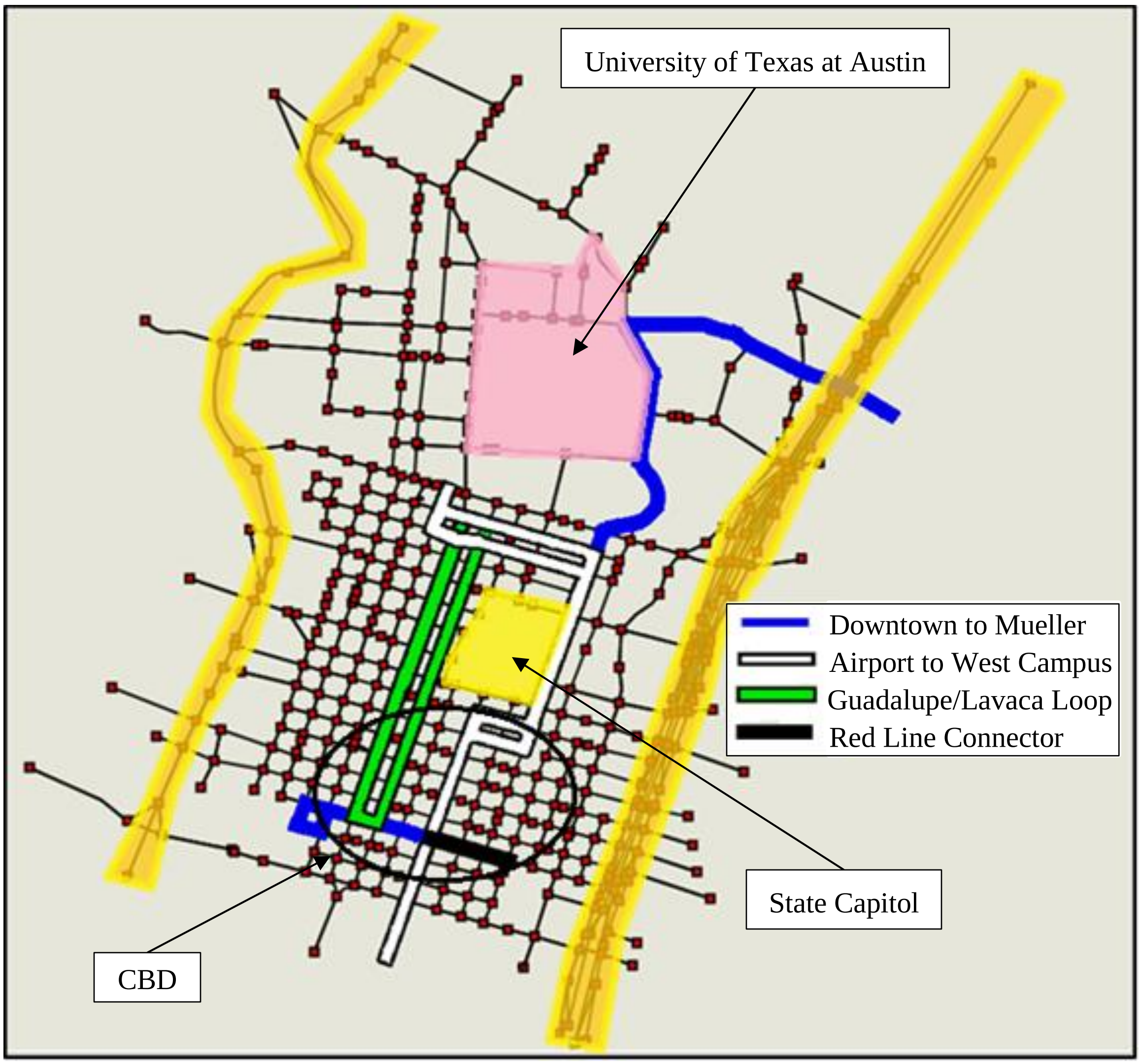

Figure 3.2: Base case network and urban rail routes

3.4.4 Creation of Routes

The urban rail preliminary design reports only identified proposed corridors; no specific route scheme was discussed (City of Austin and Capitol Metropolitan Transportation Authority, 2008). Therefore, routes were created from the identified corridors based on engineering judgment. As shown in Figure 3.2, there are four urban rail routes: Downtown to the Mueller development, Austin Airport to West Campus, the Guadalupe and Lavaca Loop, and the Downtown Hub to the Red Line connector. The design headway on all routes is 10 minutes. Where routes overlap, headways are 5 minutes (such as where the Austin Airport to West Campus route overlaps with the Downtown to Mueller route at San Jacinto).

3.4.5 Scenarios

Five scenarios were modeled each with a differing percentage of drivers switching from the automobile mode to the new urban rail:

1. Base Case Scenario – no rail implementation.
2. Worst Case Scenario – rail implementation, but no automobile users switch their mode.
3. 4% Scenario – 4% of vehicle users switch their mode, only where service is available.
4. 8% Scenario – 8% of drivers switch to the rail service, where available.
5. 16% Scenario – 16% of drivers switch to the rail service, where available.

Availability was determined by the location of each zone's centroid in relation to the proposed urban rail stops (see Figure 3.1). For the downtown and campus areas, urban rail transit is available to the user if his/her origin and destination centroids are within a 1/3 mile radius of a rail stop. One-quarter mile is the typical design walking distance for bus transit (Kittelson and Associates, Inc., 2003). Since the urban rail provides greater comfort and potential travel time savings compared to bus, users are assumed to be willing to walk a greater distance. In areas where the urban rail acts as a light rail transit service a 1/2 mile buffer was used, as recommended by Kittelson and Associates, Inc. (2003).

As noted in Section 2.4, the base network is a subnetwork of a larger regional model of Austin. Since the external nodes of the subnetwork represent many centroids in the regional network, estimating the mode split was more complicated for areas outside the downtown and university regions. For each external link, an origin-destination path analysis was conducted in the regional model – meaning that every path using a particular external link was determined. From this information, one can calculate the fraction of travelers using that link who have access to the urban rail. This percentage is multiplied by the mode split (4%, 8%, and 16%) to determine the overall mode split of that external connector. ArcGIS was used to determine which centroids were in the buffer area of the rail stops as shown in Figure 3.3.

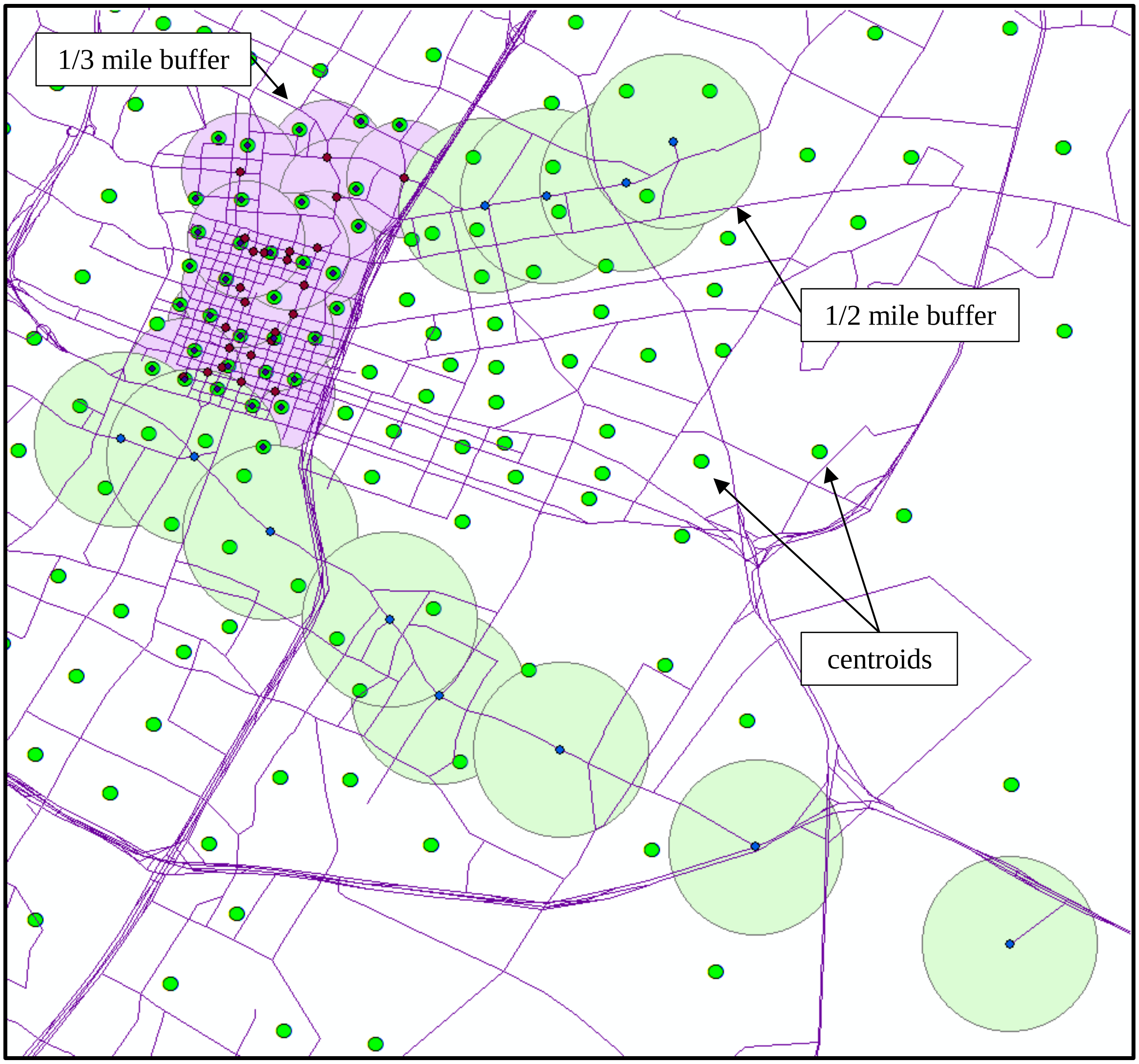

Figure 3.3: Determining availability of the urban rail service

3.5 Results

One hundred iterations were performed for each scenario. The simulation period is from 7:00 AM to 9:00 AM. Convergence was measured through the *cost gap percentage*, which can be interpreted as the percent increase of travel time an average user feels over the shortest path travel time. The cost gap and the user's average travel time at convergence for each scenario are shown in Table 3.1. Also shown in Table 3.1 are the cost gap percentages when fixed, equilibrium flows from the corresponding original network (the network before the urban rail is implemented) are applied to the equivalent modified network – as would be done in microsimulation. These values are consistently higher showcasing the limitation and potential error of microsimulation analysis.

Table 3.1: Convergence measures and average travel times

Scenario	Cost Gap Percentage	Cost Gap Percentage with Fixed Flows	Average Total Travel Time (min)
Base Case	1.1834	-	26.68
Worst Case	1.8185	1.9750	26.88
4% Scenario	1.4186	1.9286	27.34
8% Scenario	1.7122	2.4463	17.03
16% Scenario	0.6790	2.9367	20.61

The average total travel time is heavily dependent on convergence and slight differences should not be viewed as significant. Therefore, the Base Case, Worst Case, and the 4% Scenario have essentially the same total system travel time (TSTT), while the 8% Scenario and 16% Scenario have considerably lower TSTT. The cost gap percentage is similar in value across all scenarios. Table 3.2 compares the average travel times (across the entire simulation period) of the major east-west corridors.

Table 3.2: Average travel times on east/west streets

Corridor	Average Travel Time on Corridor (min)				
	Base Case	**Worst Case**	**4% Scenario**	**8% Scenario**	**16% Scenario**
Cesar Chavez EB	6.45	6.03	6.44	5.93	6.05
Cesar Chavez WB	7.81	7.77	7.77	7.98	7.28
5th Street	5.82	5.67	5.78	5.57	5.54
6th Street	6.44	6.44	6.44	6.64	6.65
8th Street WB	3.36	3.34	3.49	3.48	3.28
7th Street EB	3.78	3.80	3.79	3.79	3.76
9th Street EB	2.99	2.81	2.98	2.81	2.98
10th Street WB	3.45	3.64	3.49	3.46	3.46
11th Street EB	5.83	5.56	5.50	5.04	5.39
11th Street WB	4.88	5.05	5.19	5.00	4.84
12th Street EB	3.64	3.30	3.23	3.54	3.14
12th Street WB	2.88	2.84	2.87	2.86	2.84
15th Street WB	6.00	6.16	6.35	5.96	5.98
15th Street EB	11.40	8.86	17.21	6.41	6.34
MLK Boulevard WB	4.21	4.34	4.19	4.30	3.99
MLK Boulevard EB	3.60	3.50	3.56	3.60	3.61

As shown in Table 3.2, most of the east/west streets have roughly the same travel times across all scenarios. This makes intuitive sense since the majority of the urban rail runs in the north and south direction, and the significant changes to the existing transportation system occur along north/southbound streets. There are markedly lower travel times on eastbound 15th Street in the 8% Scenario and 16% Scenario. This occurs because 15th Street is a major arterial near one of the largest employers in Austin: the State Capitol. The travel times on major north-south corridors were also similar among the scenarios – even on the Guadalupe/Lavaca route where a lane drop occurred. This is shown in Figure 3.4.

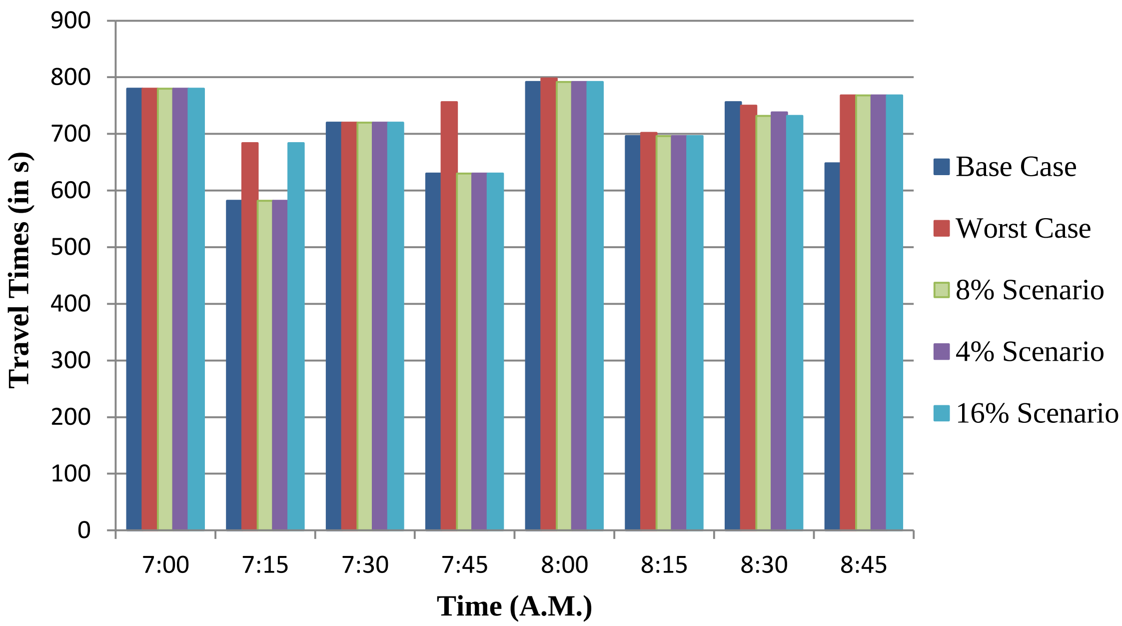

Figure 3.4: Travel times on the Guadalupe Lavaca Route during various time periods

As shown in Figure 3.4, travel times on the Guadalupe/Lavaca route are essentially the same across all scenarios. There are only slight increases in travel time in the Worst Case. The cyclic pattern of the figure suggests free-flow traffic. Cycles occur because of timed traffic signals. If the route was congested, Figure 3.4 would show a single rising and falling peak (the typical peak period curve). The geometric changes to the transportation system are not causing significant impacts because Guadalupe and Lavaca are operating at free-flow speed (are not congested) even with the dropped lane. However, when one analyzes the more congested segments of the Guadalupe/Lavaca route, it is clear that travel patterns are changing among the scenarios. The congested segments only occur along Lavaca and are shown in Figure 3.5. They form the congested Lavaca corridor.

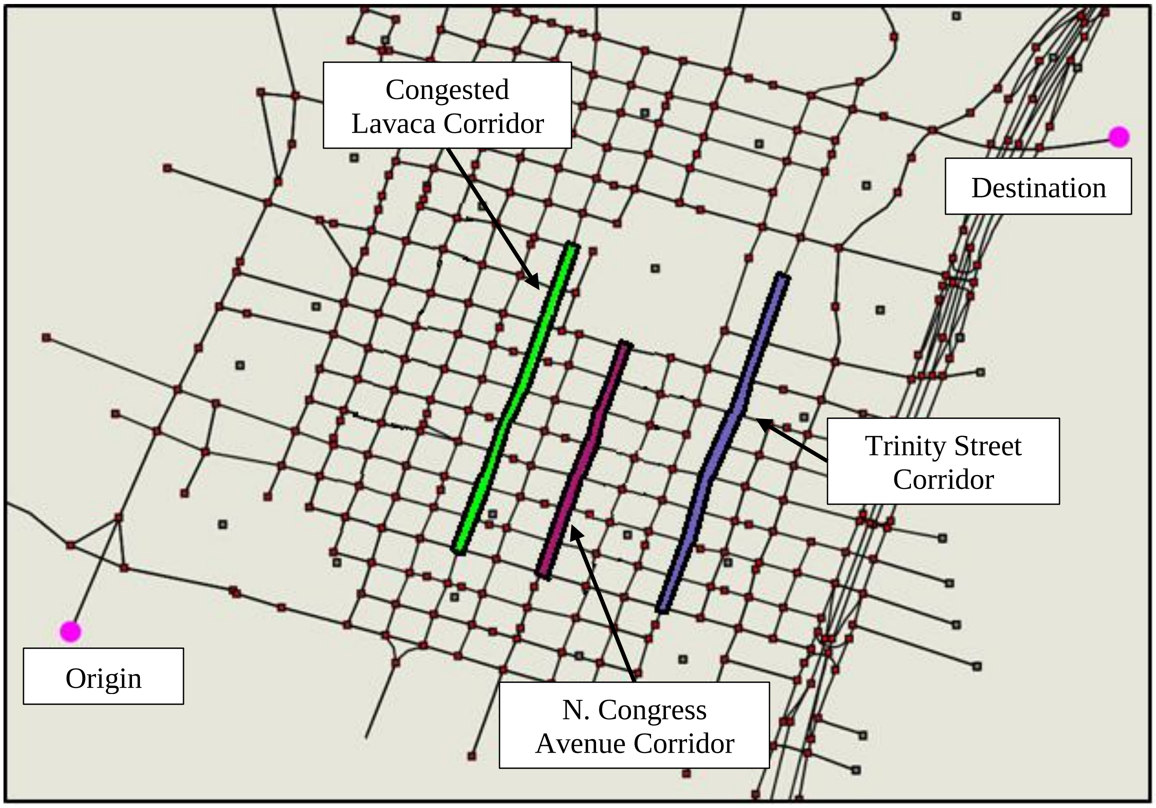

Figure 3.5: Parallel corridors near the congested segments of Lavaca

Figure 3.6 shows the average total volume on the congested Lavaca corridor. As shown in the figure, there is significantly less traffic on the corridor in the urban rail scenarios compared to the Base Case. This suggests that users are switching their route away from Lavaca due to congestion from the transit system changes. It is important to note that Figure 3.6 exhibits the typical peak period pattern. This pattern is also observed in Figure 3.7, further suggesting that these links are indeed congested – especially in the Worst Case.

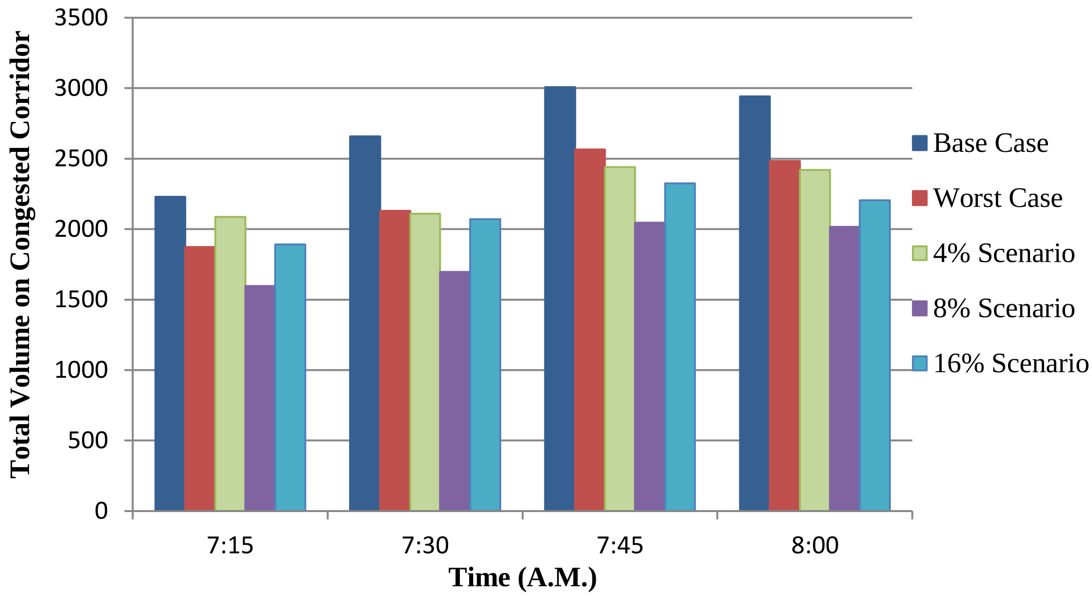

Figure 3.6: Volume on congested Lavaca corridor for each scenario

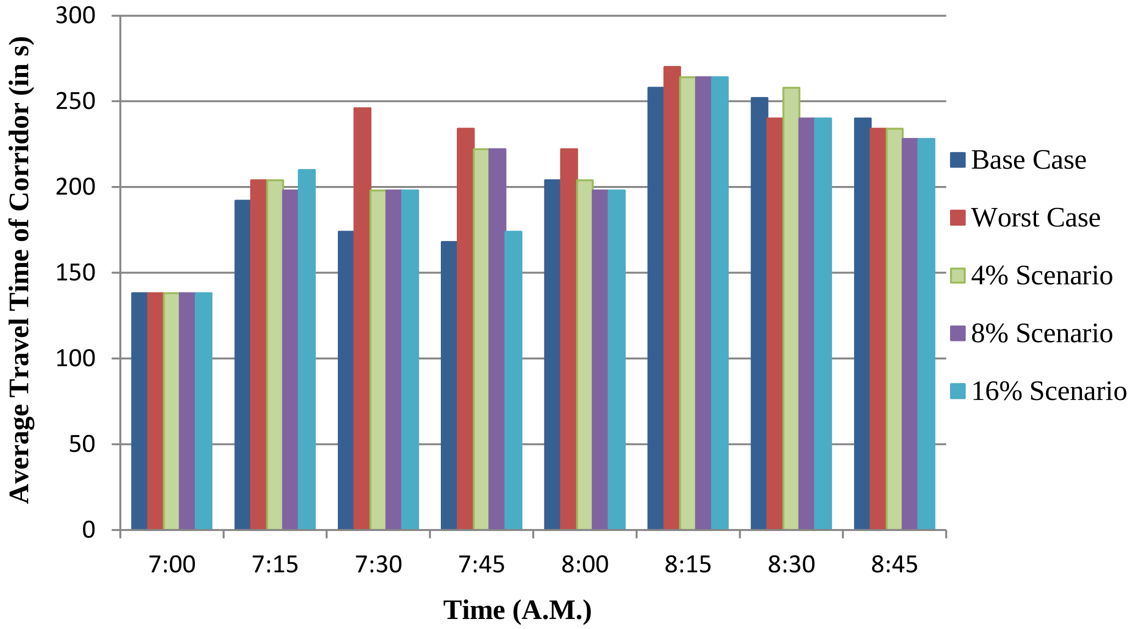

Figure 3.7: Congested Lavaca corridor travel times

Based on analyzing all the paths from the origin and destination shown in Figure 3.5, in the Worst Case and 4% Scenario traffic on Lavaca shifts to other parallel corridors – increasing the volume on North Congress Avenue, Nueces, Rio Grande, and Trinity Streets. This is shown in Figure 3.8, where link flows are the total number of vehicles over the analysis period that use a path connecting the origin-destination pair in Figure 3.5. The path analysis of the 4% Scenario was virtually the same as the Worst Case. This specific origin-destination was chosen because it had the highest demand of drivers using the congested Lavaca corridor. In the 8% and 16% Scenarios, traffic on Lavaca increases, lowering the volume on N. Congress, Trinity, and Nueces. See Figure 3.9. This occurred because the large number of drivers switching to the urban rail eases congestion on Lavaca and its parallel corridors. These findings are confirmed with Figures 3.10 and

3.11, which show the volume on North Congress and Trinity Street Corridors. These corridors are shown in Figure 3.5. As shown in Figures 3.10 and 3.11, volume on N. Congress and Trinity are much higher in the Worst Case and 4% Scenario.

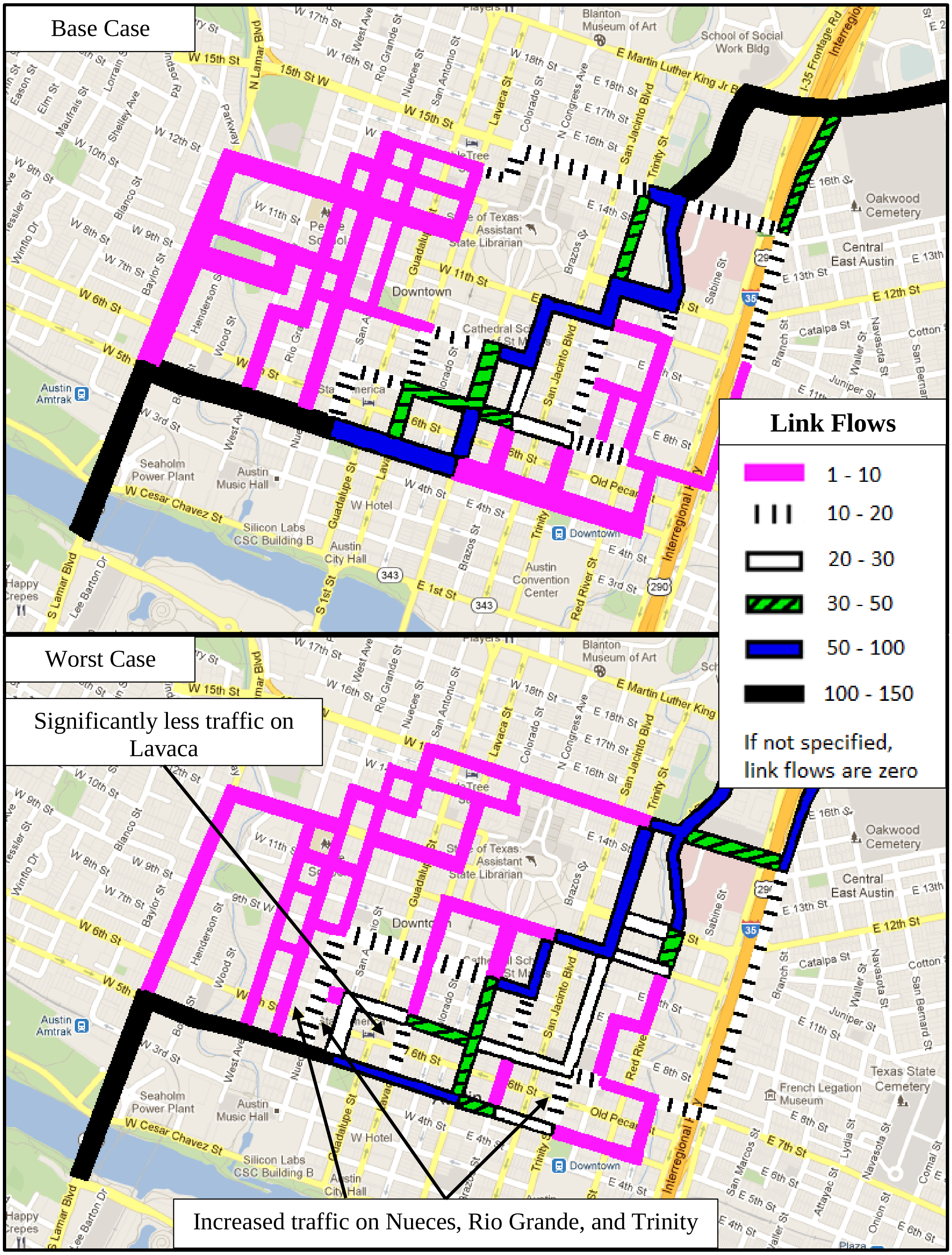

Figure 3.8: Comparison of travel patterns – the Base and Worst Case

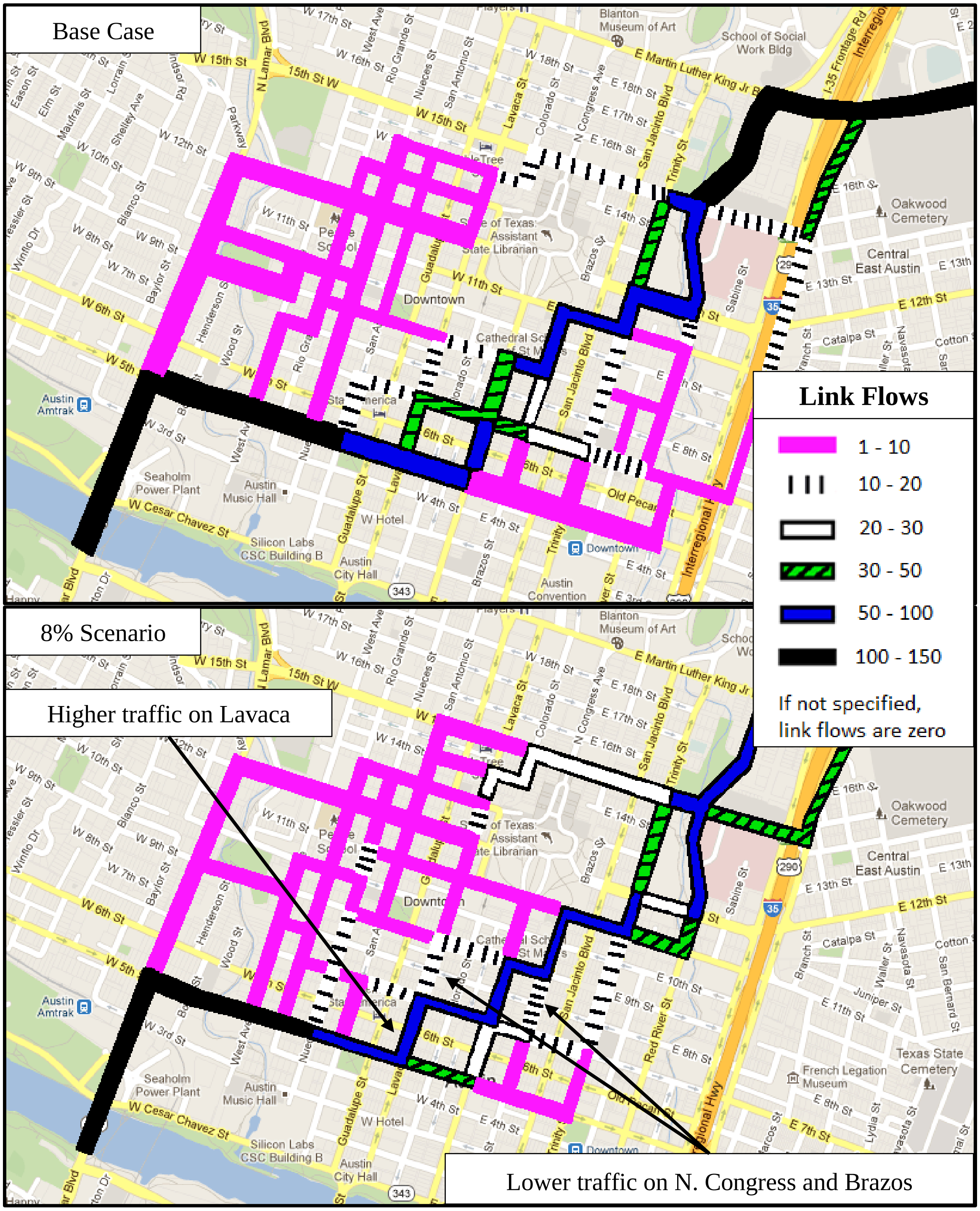

Figure 3.9: Comparison of travel patterns – the Base Case and 8% Scenario

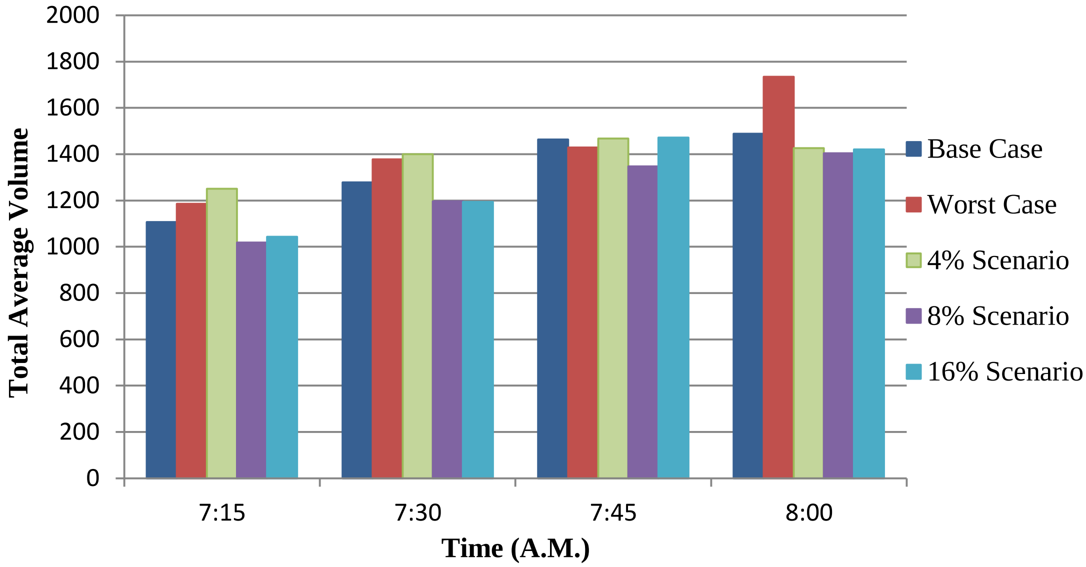

Figure 3.10: Total average volume on the N. Congress Avenue Corridor

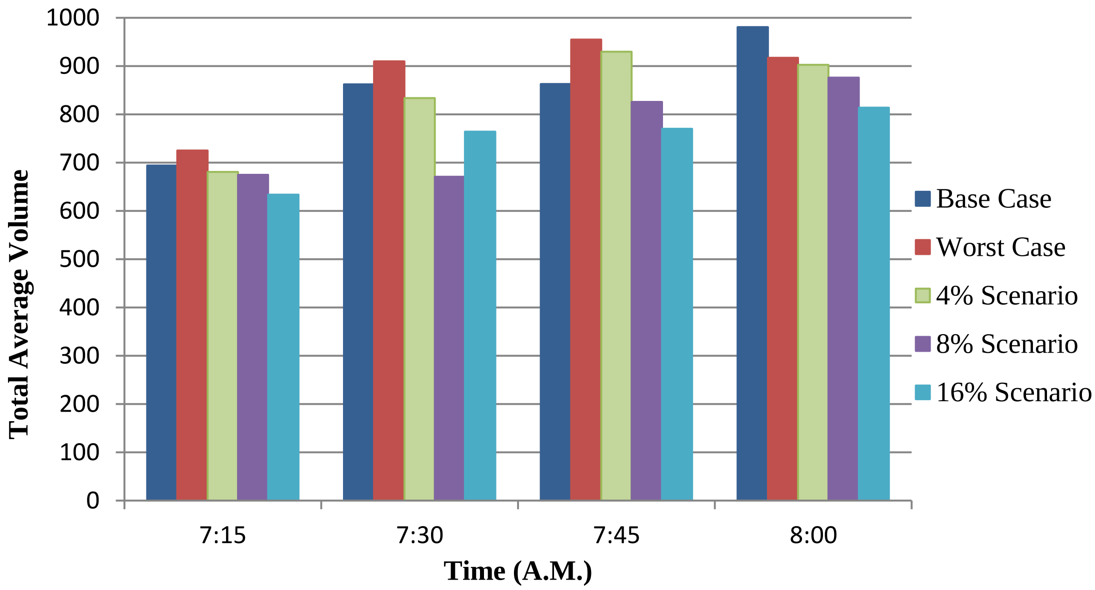

Figure 3.11: Total average volume on the Trinity Street Corridor

3.6 Conclusion

This chapter showcased the benefits of using dynamic traffic assignment to model traffic impacts caused by the implementation of new transit facilities. DTA bridges the two previously researched analysis methods (microsimulation and regional planning) by retaining their advantages while addressing their shortcomings; it can model route choice behavior using detailed inputs at a fine time scale across a region-wide spatial area. The proposed urban rail system in Austin, TX was used as a case study. The analysis showed that the urban rail has little impact on the overall transportation system since the network exhibits minimal congestion at the locations where major detrimental changes occur. The few locations where congestion exists along the railway path indicate that travel patterns do change; if low ridership occurs, traffic in these areas will switch to parallel roadways.

This chapter ends the discussion of dynamic traffic assignment as a tool to quantify traffic impacts and aid in the analysis of individual projects. The next chapter investigates dynamic traffic assignment in greater detail, focusing on the model itself and the properties of its output.

Chapter 4: Existence and Uniqueness Issues of Dynamic Traffic Assignment Equilibrium

4.1 Introduction

The exact nature of dynamic traffic assignment (DTA) equilibrium, including existence and uniqueness properties, is still not fully known, especially for simulation-based models. Peeta and Ziliaskopoulos (2001) warn that the surrounding theory of DTA is underdeveloped, especially regarding the major problem of not providing a universal solution for general networks. Multiple equilibria are possible, and equilibrium may not exist at all. This presents a problem to practitioners. Traffic flows from the model may be unrealistic and unrepresentative of actual conditions. If planners are using these inaccurate results, their policy decisions can dramatically and permanently worsen the transportation network.

This chapter presents several networks that demonstrate the chaotic behavior of DTA equilibrium. One network depicts a scenario with no equilibrium, another with multiple equilibria, and another with infinitely many equilibria. Two methods are presented to strengthen the theoretical foundation of DTA and to give planners more confidence in DTA results: (1) computational game theory – which will aid in interpreting multiple equilibria, guarantee that a certain type of equilibrium always exists, and limit the number of multiple equilibrium solutions and (2) piecewise linear fundamental diagrams – which will dramatically reduce the number of unlikely equilibria. The following section provides a high-level overview of dynamic traffic assignment models and their equilibrium properties.

4.2 Overview of Dynamic Traffic Assignment Models

Though simulation-based DTA models are the only model type suitable in practice, many analytical methods have been developed. Analytical models include mathematical programming, optimal control, and variational inequality formulations. Early DTA models, including the first developed by Merchant and Nemhauser (1978), were solved using mathematical programs and optimization techniques. Many of these and subsequent models guarantee equilibrium existence, uniqueness, and stability properties (Peeta and Ziliaskopoulos, 2001). However, in order to prove these properties, analytical models make restrictive assumptions that hinder the realism of traffic conditions and user behavior. For example, in virtually all mathematical programming methods only one destination can be modeled. If multiple destinations are modeled and the first-in-first-out (FIFO) rule is enforced, the solution constraint space becomes non-convex (Carey, 1992). FIFO (i.e., the first vehicle entering a link will be the first vehicle leaving) ensures realistic traffic flows. Other models restrict traffic realism by using static link performance functions (Janson, 1991; Boyce et al., 1995) or assuming that route cost functions are decay monotone with respect to route flow (Mounce, 2007). Overall, analytical models cannot realistically and efficiently solve large-scale networks and are not used in practice.

Simulation-based DTA models use simulation methods to determine the traffic flow propagation on the network; traffic flow dynamics are not solved analytically via a mathematical program. As detailed in Section 2.2.2, many DTA models use the LWR theory in the simulation process. However, simulation is only one component of the entire DTA procedure. Modern DTA models involve three linked steps: traffic simulation, path generation, and route flow adjustment as shown in Figure 4.1. The simulation process distributes traffic on the network based on fixed route flows from the previous step. Travel times are calculated, and shortest paths are determined. Path flows are adjusted by shifting some flow on previously used routes to the newly identified shortest paths as to move the solution closer to user equilibrium. Several techniques can be used: Method of Successive Averages (as in Section 3.4.2), gradient projection methods, and simplicial decomposition methods, among others. Due to the complexity and processes involved in simulation-based DTA models, equilibrium properties have not been proven.

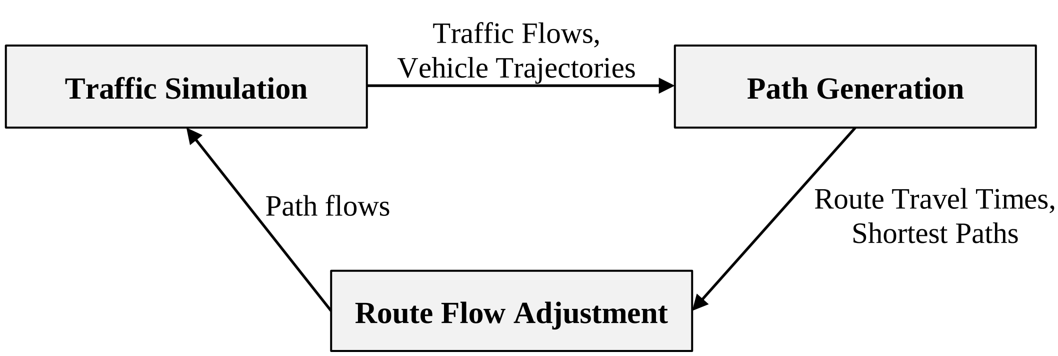

Figure 4.1: The simulation-based dynamic traffic assignment process

4.3 Game Theory and Dynamic Traffic Assignment

4.3.1 Background

Game theory mathematically models situations where one entities' action interacts or affects another entities' action. It is a broad method that has been applied to a variety of fields: economics, philosophy, resource allocation, biology, political science, military strategy, and network optimization. A game is characterized by three elements: (1) set I consisting of all entities/players, (2) a set of actions/decisions/strategies A_i for every $i \in I$, and (3) the utility/satisfaction u_i player i will expect from the given set of strategies, $u_i: A \to \mathbb{R}$.

The most widely used and recognized notion of an equilibrium state or solution of the game theory formulation was developed by John Nash (Nash, 1951). It is termed Nash equilibrium and can be categorized into two basic types of equilibrium: pure and mixed strategy. Pure strategy Nash equilibrium can be defined as the stable state where no player can improve his/her utility by changing strategies. It is expressed formally below, where (1) $a_i \in A_i$, (2) a_{-i} indicates the actions of all players except player i, and (3) a'_i indicates all other strategies available to player i besides a_i.

$$u_i(a_i, a_{-i}) \geq u_i(a'_i, a_{-i}) \qquad \forall\, i \in I$$

A game can have multiple pure strategy Nash equilibria or none at all. However, any game with a finite set of players and a finite set of actions is guaranteed to have a mixed strategy Nash equilibrium as proven by Nash (1951). In a mixed strategy solution, players are allowed to randomize among their various actions – to choose a probability distribution over their strategy set as to maximize their expected utility. Players are assumed to be rational individuals wishing to maximize their payoff and act independently from one another (i.e., players select strategies independently). It is important to note these assumptions are the same assumptions used in the principle of user equilibrium, which all traffic assignment processes are based upon.

The game theory concepts discussed above can be demonstrated using the classic Matching Pennies game. The game consists of two players, Player 1 and Player 2, who each have a penny. Every round, the players choose which side of the penny (heads or tails) to show the other player. If the pennies match sides, Player 1 wins a penny from Player 2. If the pennies do not match, Player 2 wins a penny from Player 1. The Matching Pennies game can be formulated as: (1) $I = \{1, 2\}$, (2) $A_1 = A_2 = \{H, T\}$, and (3) Table 4.1 shows the utilities of each player, where the first element can be interpreted as $\left(u_1(H, H), u_2(H, H)\right)$.

Table 4.1: Player utilities in the Matching Pennies game

		Player 2	
		Heads	Tails
Player 1	Heads	(+1, -1)	(-1, +1)
	Tails	(-1, +1)	(+1, -1)

As shown in Table 4.1, there is no pure strategy Nash equilibrium. If the pennies match, Player 2 can improve his utility by switching strategies so that the pennies do not match. However, once this is done, Player 1 can improve his payoff by switching strategies as well, leading to the previous scenario (i.e., Player 2 would switch strategies causing Player 1 to switch strategies, causing Player 2 to switch strategies, causing Player 1 to switch strategies, etc.). There is no stable pure strategy solution; the only equilibrium associated with this problem is a mixed strategy Nash equilibrium. Intuitively, the reader can reason that the equilibrium occurs when both players choose heads/tails 50% of the time. If this weren't the case, e.g., Player 2 shows tails 75% of the time, Player 2 can improve his payoff by lowering his tendency to show tails, eventually reaching the stable point of 50%.

Mixed strategy Nash equilibrium can alternatively be viewed as the solution where players have chosen their probability distributions to make all other players indifferent to their own set of strategies. Applying this methodology to the Matching Pennies game will yield the following set of equations which directly solve for p, the probability of Player 1 choosing heads, and q, the probability of Player 2 choosing heads. Substituting values from Table 4.1, $p = \frac{1}{2}$ and $q = \frac{1}{2}$.

$$p = \frac{u_2(T,T) - u_2(T,H)}{u_2(H,H) + u_2(T,T) - u_2(T,H) - u_2(H,T)}$$

$$q = \frac{u_1(T,T) - u_1(H,T)}{u_1(H,H) + u_1(T,T) - u_1(T,H) - u_1(H,T)}$$

4.3.2 Literature Review

To the author's knowledge, this is the first work to apply computational game theory literature, solution methods, and modeling techniques to dynamic traffic assignment equilibrium. Several research efforts have been conducted regarding static traffic assignment and game theory. These studies will be summarized briefly, since several results are not applicable to DTA or the research in this report. However, the formulation of traffic assignment and the relationship between user equilibrium and Nash equilibrium developed in these papers are crucial.

Charnes and Cooper (1958) were the first to discuss the relationship between static traffic assignment equilibrium and Nash equilibrium. They showed that their proposed traffic assignment model satisfied Nash equilibrium conditions. Dafermos (1971) proposed a static traffic assignment model for two-way roadways, where link travel time depended on the link volume as well as the volume on the opposite link. Dafermos discussed how a user-optimizing flow pattern in his model satisfied the user equilibrium conditions: all used routes connecting the same origin to the same destination have equal and minimal travel time. He then described how these conditions are equivalent to a Nash equilibrium point.

Charnes and Cooper (1958) and Dafermos (1971) Nash equilibrium analyses were explained at the introductory level. They did not formally define traffic assignment as a game and did not rigorously prove Nash equilibrium conditions. Devarajan (1981) was the first to completely characterize static traffic assignment with continuous flow as an economic game and to mathematically prove that the associated Nash equilibrium corresponds to user equilibrium. Devarajan defined the game as follows: (1) each player $i \in I$ represents a unique origin-destination pair and the set I encompasses all origin-destination pairs in the network, (2) the set of actions for each player includes all routes with feasible network flows connecting the analogous origin-destination pair, and (3) the utility of a player is the resulting total travel time of the sent demand, which the entity tries to minimize. Since the flows are continuous, the players choose from a continuum of pure strategies.

A more intuitive approach was taken by Rosenthal (1973). He formulated traffic assignment as a discrete game, where each player represents an individual driver. The strategies of each player are the set of routes connecting his/her origin to destination. The driver chooses a path to minimize the cost of sending one unit of flow (i.e., to minimize their individual travel time). Rosenthal proves the optimal user equilibrium solution for the discretized version of static traffic assignment corresponds to a pure strategy Nash equilibrium. He also demonstrates that every pure strategy Nash equilibrium does not necessarily satisfy user equilibrium. The following dynamic traffic assignment examples will utilize Rosenthal's game formulation (i.e., drivers are players who choose among their available routes as minimize their travel time) for several reasons: (1) the formulation is simple and intuitive, (2) vehicles are discrete at the fundamental level, and (3) modern dynamic traffic assignment models are discrete in nature, treating vehicles or packets of vehicles as discrete.

4.3.3 No Equilibrium Example Network

The game theory formulations presented in this thesis are meant to display the complicated nature of dynamic traffic assignment (i.e., equilibrium may not exist and multiple equilibria are possible) and propose methods from computational game theory that can address these potential issues. It is a stepping point for future DTA game theory research. Consider the network in Figure 4.2. One vehicle travels from A to B (Vehicle 1) and another from C to D (Vehicle 2). Vertical and horizontal links have a travel time of 1 minute, and diagonal links 1.5 minutes. If two vehicles arrive at a junction simultaneously, priority is determined as indicated in Figure 4.2. The vehicle yielding right-of-way is delayed by an additional minute.

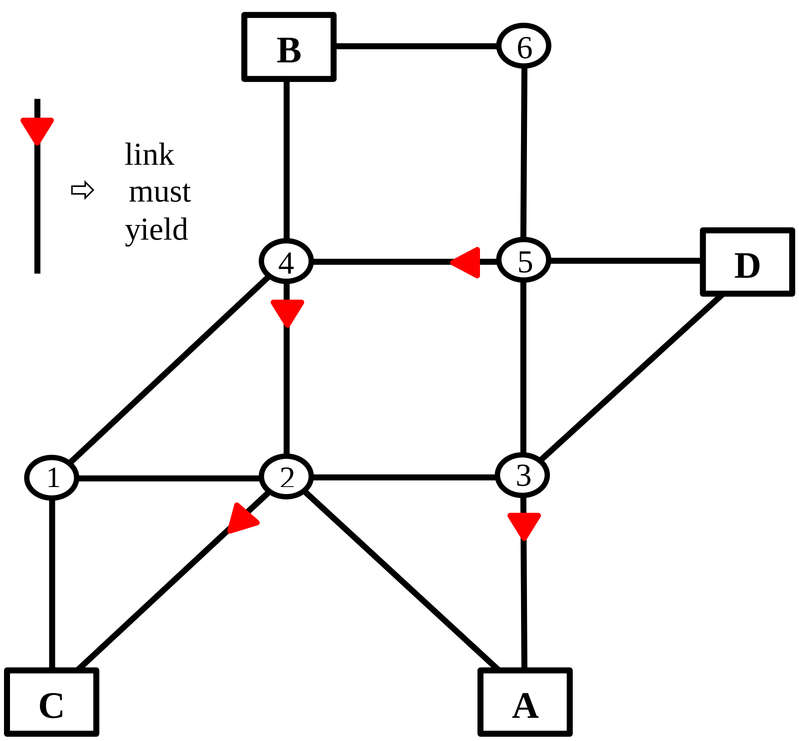

Figure 4.2: Network with no dynamic traffic assignment equilibrium

With no interfering traffic, the left path, A → 2 → 4 → B, is preferred for traveling from A to B. The bottom path, C → 2 → 3 → D, is preferred for traveling from C to D. However, if both travelers choose these paths, Vehicle 1 has priority at node 2 and Vehicle 2 is delayed by an additional minute. Thus, Vehicle 2 opts for the top path, C → 1 → 4 → 5 → D, with shorter travel time, gaining priority at node 4 and delaying Vehicle 1. Vehicle 1, in turn, opts for the rightmost path, A → 3 → 5 → 6 → B, which is faster since the second vehicle is on the top path. This frees the way for Vehicle 2 to return to the bottom path, allowing Vehicle 1 to return to the left path regaining priority, and so forth *ad infinitum*. It is clear that there is no deterministic assignment of vehicles to paths that satisfy the equilibrium principle: no matter what combination of routes is chosen one vehicle has a faster alternative.

The situation in Figure 4.2 can be defined as an economic game. $I = \{1,2\}$. $A_1 = \{left\,path\,(L),\ right\,path\,(R)\}$, and $A_2 = \{top\,path\,(T),\ bottom\,path\,(U)\}$. Table 4.2 shows the utilities (travel times) of the travelers.

Table 4.2: Vehicle travel times – no equilibrium example

		Vehicle 2	
		Top	Bottom
Vehicle 1	Left	(4.5, 4.5)	(3.5, 5.0)
	Right	(4.0, 4.5)	(4.0, 4.0)

Analyzing Table 4.2, the reader can confirm no pure strategy Nash equilibrium exists for the game. However, a mixed strategy Nash equilibrium is guaranteed to exist. The probability p of Vehicle 1 choosing the left path and probability q of Vehicle 2 choosing the top path can be determined directly from the equations discussed in Section 4.2.2.

$$p = \frac{u_2(R,U) - u_2(R,T)}{u_2(L,T) + u_2(R,U) - u_2(R,T) - u_2(L,U)} = \frac{4.0 - 4.5}{4.5 + 4.0 - 4.5 - 5.0} = \frac{1}{2}$$

$$q = \frac{u_1(R,U) - u_1(L,U)}{u_1(L,T) + u_1(R,U) - u_1(R,T) - u_1(L,U)} = \frac{4.0 - 3.5}{4.5 + 4.0 - 4.0 - 3.5} = \frac{1}{2}$$

Therefore, Vehicle 1 will choose the left path 50% of the time, and Vehicle 2 will choose the top path 50% of the time.

Since nearly all projects are evaluated at their equilibrium state, an instance that exhibits no equilibrium is an issue for transportation planners. There would be no equilibrium state to analyze, no point of comparison among other project alternatives. Also, if the stopping criterion of the DTA model is lax, project analysis will occur at a state of transition – which may be an inaccurate representation of the true traffic condition. However, by formulating the no equilibrium example as a game more information is provided to the practitioner. The p and q values can be interpreted as the fixed ratio of travelers choosing a certain path (among a set of paths) from their respective origin to destination. For example, 50% of users departing from A to B will use the left path, while 50% of users traveling from C to D will use the top path. With this information planners can approximate traffic flows on the network within a given time period, which can be used to estimate other indices of interest (e.g., crash rates, pavement deterioration, required maintenance, etc.).

4.3.4 Multiple Equilibria Example Network

Figure 4.3 depicts a network with multiple dynamic traffic assignment equilibrium. Vehicle 1 travels from A to B, and Vehicle 2 travels from C to D. The travel time on all links is one minute. As indicated in Figure 4.3, if Vehicle 1 and Vehicle 2 arrive at node 3 simultaneously, Vehicle 1 must yield and will incur one minute of extra delay. Links 2 → 1 and 1 → 4 are directed arcs (i.e., one-way streets).

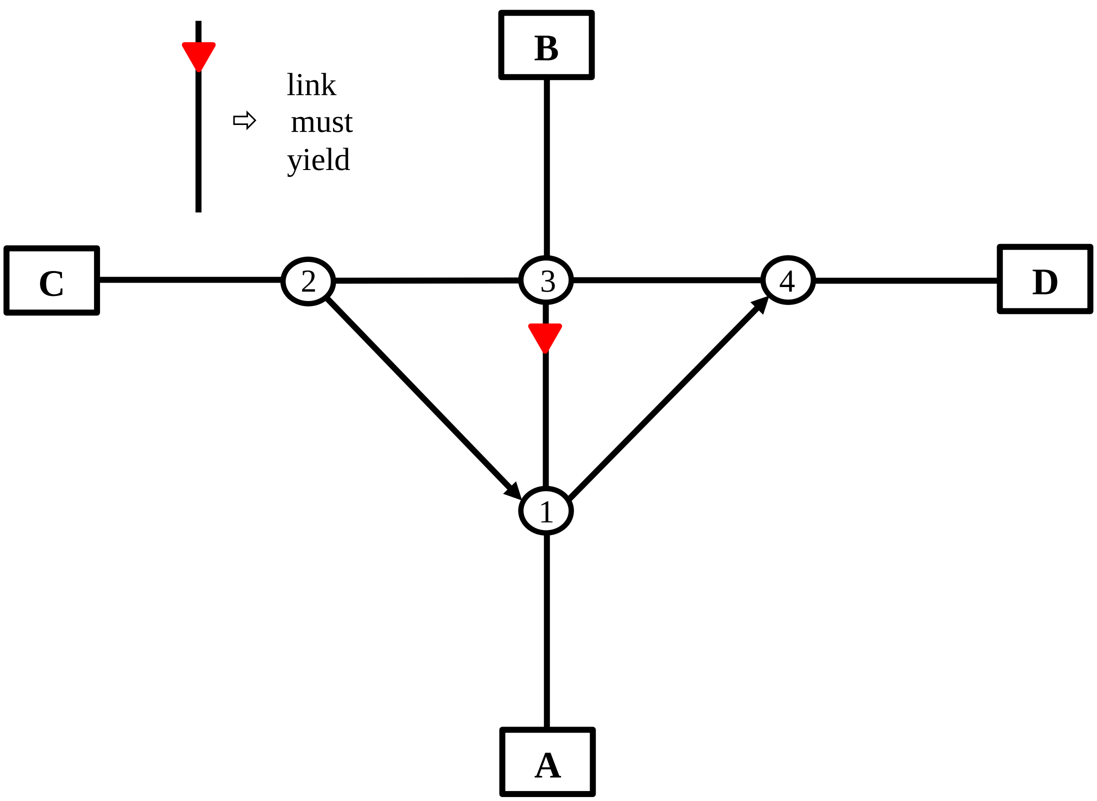

Figure 4.3: Network with multiple dynamic traffic assignment equilibria

Vehicle 2 has two shortest paths: (1) the top path, C → 2 → 3 → 4 → D and (2) the bottom path, C → 2 → 1 → 4 → D. Both paths have 4 minute travel times and are independent of Vehicle 1's actions. Vehicle 1 has two plausible paths: (1) the left path, A → 1 → 3 → B and (2) the rightmost path, A → 1 → 4 → 3 → B. The path travel times are influenced by Vehicle 2. The travel time of the left path will be 3 minutes if Vehicle 2 chooses the bottom path and 4 minutes if the top path. There are three user equilibrium solutions associated with this example: (1) Vehicle 1 chooses the left path and Vehicle 2 chooses the top path, (2) Vehicle 1 chooses the left path and Vehicle 2 chooses the bottom path, and (3) Vehicle 1 chooses the rightmost path and Vehicle 2 chooses the bottom path. They are bolded in Table 4.3.

Table 4.3: Vehicle travel times – multiple equilibria example

		Vehicle 2	
		Top	Bottom
Vehicle 1	Left	**(4, 4)**	**(3, 4)**
	Right	**(4, 4)**	(4, 4)

As shown in Table 4.3, the bolded elements also correspond to pure strategy Nash equilibrium points. However, applying equilibrium refinements from the game theory literature can reduce the number of unrealistic equilibrium solutions. One such refinement is the concept of trembling hand perfect equilibrium introduced by Selten (1975). The trembling hand property is based on the notion that players may, through "a slip of the hand", choose an incorrect strategy (i.e., players may not choose the strategy predicted by Nash equilibrium 100% of the time). Loosely, the definition of trembling hand perfect equilibrium is: a pair of pure strategies (i,j) is a trembling hand perfect equilibrium if and only if i is a best reaction for Player 1 not only to the pure strategy j of Player 2 (which it is in Nash equilibrium), but to mixed strategies in which Player 2 plays each of his pure strategies with some positive probability, and vice versa for Player 2.

Employing the trembling hand concept, it only makes sense for Vehicle 1 to choose the rightmost path if Vehicle 2 chooses the top path 100% of the time; the travel time on the left path will always be less than or equal to the travel time on the rightmost path. The pure strategy Nash equilibrium where Vehicle 1 chooses the rightmost path and Vehicle 2 chooses the top path is not a trembling hand perfect equilibrium. Therefore, the associated user equilibrium can be eliminated, and the amount of multiple DTA equilibria is reduced. This reduction is vitally beneficial to transportation engineers since any collected data, any comparison, any measured index must be made from a network model that represents realistic traffic conditions.

4.3.5 Project Selection Example with Multiple Equilibria

Assume a transportation planning agency is evaluating a set of project alternatives. Their goal is to minimize the total system travel time (TSTT) given a budget M. The base scenario is similar to the multiple equilibria network in Figure 4.3, except the travel times of the directed links are 1.5 minutes. These conditions yield a single user equilibrium: Vehicle 1 will use the left path, A → 1 → 3 → B, and Vehicle 2 will use the top path, C → 2 → 3 → 4 → D. The total travel time of the system is 8 vehicle-minutes. Project 1 is the same as the base case except the travel time on link 4 → D has been reduced to 0.5 minutes. The equilibrium path set of Project 1 will be the same as the base case (i.e., Vehicle 1 will use the left path, Vehicle 2 will use the top path); however, TSTT is 7.5 vehicle-minutes. The cost of implementing Project 1 is M. Project 2 is the multiple user equilibria network discussed in Section 4.3.4., the improvements being the directed arcs have a reduced travel time of 1.0 minutes. The cost of Project 2 is also M.

Using the refined pure strategy Nash equilibrium solutions via the trembling hand theory, there are two distinct user equilibria associated with Project 2: (1) Equilibrium I – Vehicle 1 will choose the left path, and Vehicle 2 will choose the top path, and (2) Equilibrium II – Vehicle 1 will choose the left path, and Vehicle 2 will choose the bottom path. Equilibrium I will have a TSTT of 8 vehicle-minutes, while Equilibrium II will have a TSTT of 7 vehicle-minutes. The project alternatives are summarized in Table 4.4.

Table 4.4: Project alternatives

Network	Link Travel Time Modifications	Cost	Equilibrium Paths	TSTT (veh-min)
Base Case	-	-	Vehicle 1: Left Vehicle 2: Top	8.0
Project 1	Link 4 → D: 0.5 minutes	M	Vehicle 1: Left Vehicle 2: Top	7.5
Project 2	Links 2 → 1, 1 → 4: 1.0 minutes	M	Vehicle 1: Left Vehicle 2: Top	8.0
			Vehicle 1: Left Vehicle 2: Bottom	7.0

It is nearly impossible to detect multiple dynamic traffic assignment equilibria in current software programs. Practitioners are simply given an approximate solution. Therefore, depending on the user equilibrium outputted from the DTA model, different projects will be selected. If the model converges to Equilibrium I, Project 1 will be selected. If Equilibrium II is generated by the model, Project 2 will be selected. As the example shows, multiple DTA equilibria can have a dramatic impact on planning decisions and thusly, major (and potentially permanently negative) impacts on the transportation system. For example, suppose Equilibrium II represents the actual traffic condition and Equilibrium I was outputted from the model. The budget M would be spent on implementing Project 1. Funds would be spent half as efficiently, and the travel time improvements would only be felt by Vehicle 2.

4.4 Current Modeling Techniques

This section continues to analyze DTA equilibrium but takes an alternative approach than the game theory formulations discussed previously. Equilibrium is investigated as a product of current modeling techniques and assumptions. Specifically, simplified fundamental diagrams are studied and a piecewise linear fundamental diagram is proposed to reduce multiple equilibria.

4.4.1 Background

As described in Section 2.2.2, modern DTA models use simplified fundamental diagrams: either triangular or trapezoidal. These diagrams are used to increase efficiency in the simulation process and are one of the main reasons DTA software programs can solve large-scale networks. However, they are approximate versions of flow (q) and density (k) relationships observed in the field. Several q-k relationships have been proposed beginning with Greenshields (1935). Based on data collected on a one-lane roadway, Greenshields proposed the quadratic diagram shown in Figure 4.4. Another well-known relationship was developed by Greenberg (1959), where $q \propto \frac{\log(k/k_j)}{k}$. The Greenberg fundamental diagram is shown in Figure 4.5. Many q-k relationships have been formulated over the years (Koshi et al., 1983; Kerner, 1998; Highway Capacity Manual, 2010). The main point of the discussion is simplified diagrams are linear approximations of models developed from empirical data.

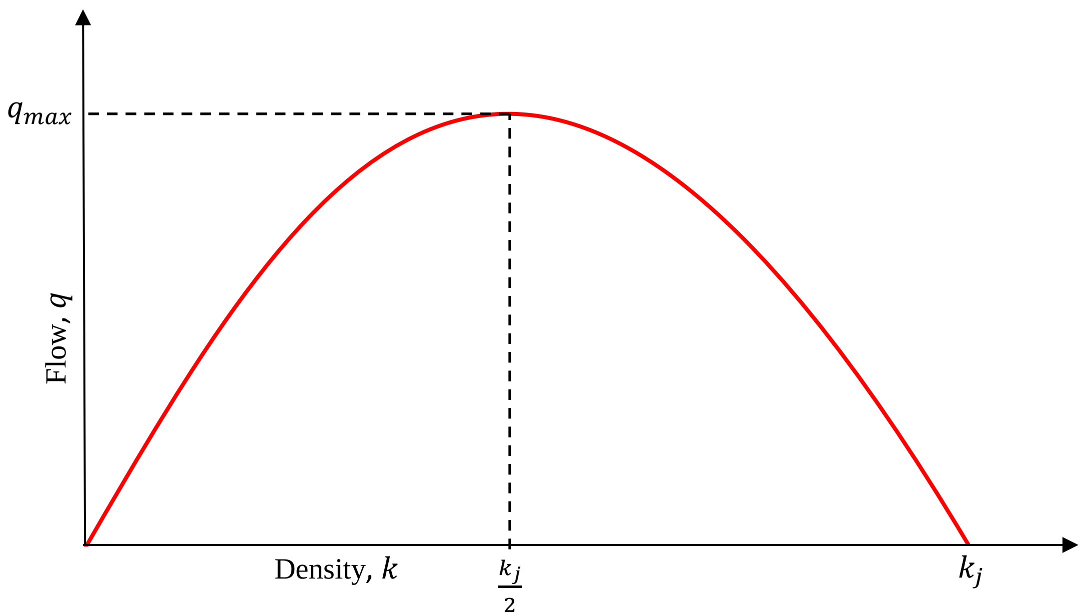

Figure 4.4: The Greenshields fundamental diagram

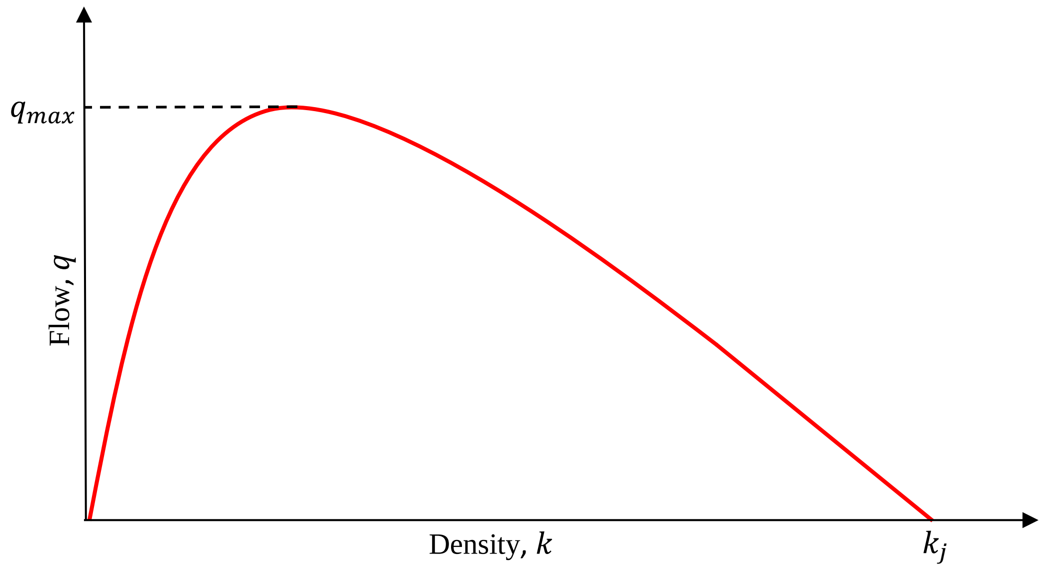

Figure 4.5: The Greenberg fundamental diagram

Recall the fundamental relationship: $q = uk$. Solving for speed: $u = \frac{q}{k}$. Thusly, aggregate link speed can be measured from the fundamental diagram as the slope of the line connecting the origin to the specified (k, q) coordinates. This is demonstrated in Figure 4.6 with a typical trapezoidal fundamental diagram. k_c is the critical density and is defined as the boundary between uncongested and congested conditions. As shown in Figure 4.6, the link will operate at free-flow speed at every point in the uncongested region (i.e., $k \in [0, k_c]$). This feature of simplified fundamental diagrams is crucial to understanding the infinitely many user equilibria network presented in Section 4.4.3.

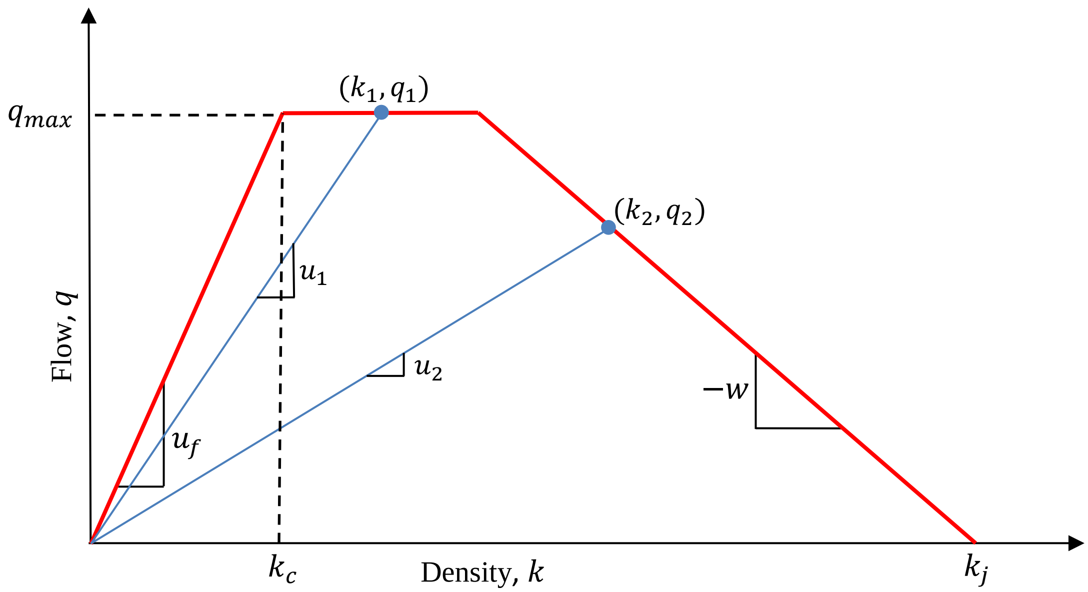

Figure 4.6: Obtaining speed from the fundamental diagram

4.4.2 Literature Review

There has been limited research on dynamic traffic assignment equilibrium. This section discusses the two most important and relevant studies. Furthermore, the networks analyzed in the following papers are similar to the infinitely many user equilibria network. Daganzo (1998) presented a simple diverge-merge network with two parallel paths. One path contains a bottleneck with a limiting capacity and a short free-flow travel time. The other path has a longer constant travel time. Increasing the capacity of the bottleneck can lead to queue spillover and a significantly different user equilibrium – a user equilibrium with a higher total system travel time and only half of the network being utilized. Essentially, if the number of travelers in the bottleneck queue yield a travel time greater than or equal to the longer path, both paths will be used. The inflow rate and outflow rate of the network will be equal. However, if this required amount of vehicles cannot fit on the bottleneck link, queue spillback will occur. All users will use the bottleneck path, and the total output of the system is restricted. Daganzo addressed the importance of queue spillback prevention and displayed the chaotic behavior of dynamic traffic assignment.

Nie (2010) studied a similar two-path diverge-merge network. The two parallel links have differing capacities and are represented using simplified fundamental diagrams (both links strictly operate at free-flow speed when uncongested). The capacity of the link downstream of the merge node is equal to one of the path's capacities, and the inflow rate is equal to the summation of the two parallel link capacities. Nie proved that three distinct user equilibria can form on this network: (1) the parallel link with capacity equal to the link downstream of the merge will operate at capacity while the other link is unused; congestion will form upstream of the diverge, (2) the parallel link with lower capacity will operate at capacity while the other link is underutilized such that the flow downstream of the merge is equal to the capacity of the downstream link; congestion will form upstream of the diverge, and (3) both parallel links operate at capacity; congestion will form on the links at the same rate. To determine the most realistic traffic condition, the equilibria were categorized using stability and efficiency properties.

4.4.3 Infinitely Many Equilibria Example Network

Figure 4.7 shows the infinitely many equilibria network. It is a diverge-merge network with identical paths and will be referred to as the DMIP network. Users traveling from origin O to destination D can choose either link 1 or link 2. Q_{in} is the inflow rate of the network, which occurs at node A. c_i represents the capacity of link *i*. As shown in the figure, links 1 and 2 have capacities one-half that of the inflow rate. Links 1 and 2 also have identical simplified fundamental diagrams (i.e., the same free-flow speed, backward wave speed, jam density, and length). The capacity of link s is sufficiently large so that there is no congestion or queue spillback occurring at node B. p_{r1} is the proportion of flow on link *r* that wishes to use link 1.

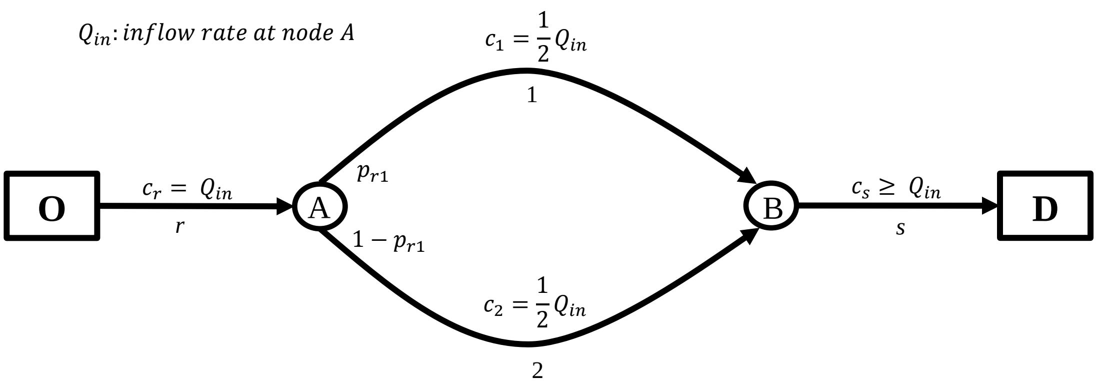

Figure 4.7: The infinity many equilibria network (the DMIP network)

The DMIP network has infinitely many solutions; p_{r1} can be any value subject to $p_{r1} \in [0,1]$. For example, assume that p_{r1} is sufficiently large so that $p_{r1}Q_{in} > c_1$. The flow on link 1 will be restricted by its capacity – will equal c_1 – and a queue will form on link r. All users (no matter which parallel link they choose) will experience the additional travel time of the queue. However, once they reach node A, links 1 and 2 will have the same travel time; links 1 and 2 will be uncongested, and because they were assumed to have simplified fundamental diagrams will operate at free-flow speed. Hence, the network will be in equilibrium: all used paths connecting O to D have equal and minimal travel time at every departure period.

It is important to note that the author does not believe that every equilibrium is realistic. Take the worst case user equilibrium for example, where one parallel link operates at capacity while the other is unused. It is unlikely that all users will choose the same path – since travelers make their route decisions independently of one another. Furthermore, users know that their likelihood of being impeded by other traffic corresponds to the number of vehicles on the roadway. Weaving movements and vehicular accidents are more likely to occur when more travelers are using the network. The author believes that users tend to distribute themselves equally given a choice of identical paths. Therefore, the only realistic user equilibrium corresponds to $p_{r1} = 1/2$. This is also the unique system optimal solution (i.e., the only solution where a queue does not form).

It is possible for current DTA software to output an unrealistic equilibrium for the network shown in Figure 4.7. Perhaps, an all-or-nothing assignment initializes the DTA process (p_{r1} is equal to 1 or 0). Then, because of the simplified fundamental diagram, the network is in user equilibrium, the stopping criteria is satisfied, the DTA process ends, and the planner is given the worst case user equilibrium.

4.4.4 Infinitely Many Equilibria Network in Series

Figure 4.8 shows two DMIP networks in sequence. Links 1, 2, 3, and 4 are identical. Turning movements at node B are assumed to be separated and do not impede other turning movement flows as shown in Figure 4.9. p_{13} and p_{23} represent the proportion of flow on link 1 and link 2, respectively, that wish to use link 3. The set of dynamic user equilibrium solutions in this scenario is even more complicated than the previously presented network. For example, assume that a value of p_{r1} is given. Because the capacities are defined as one-half the inflow rate, one parallel link downstream of node B (link 1 or 2) will operate at capacity no matter the assumed value of p_{r1}.

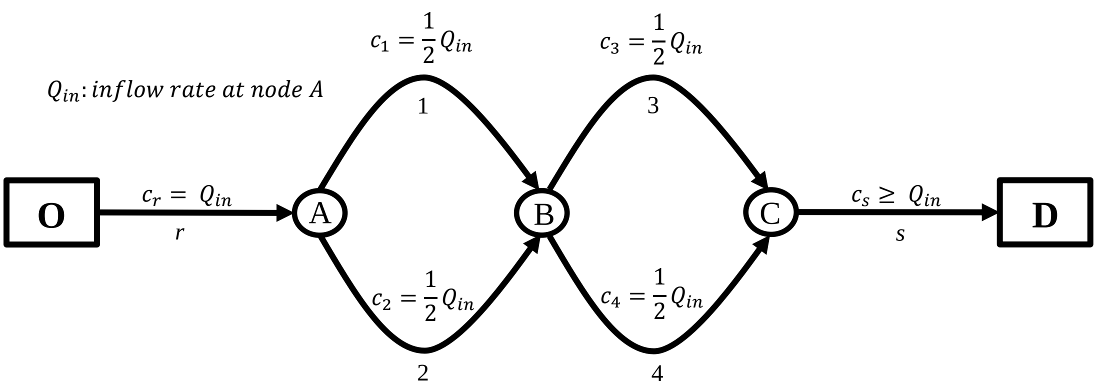

Figure 4.8: Two DMIP networks in series

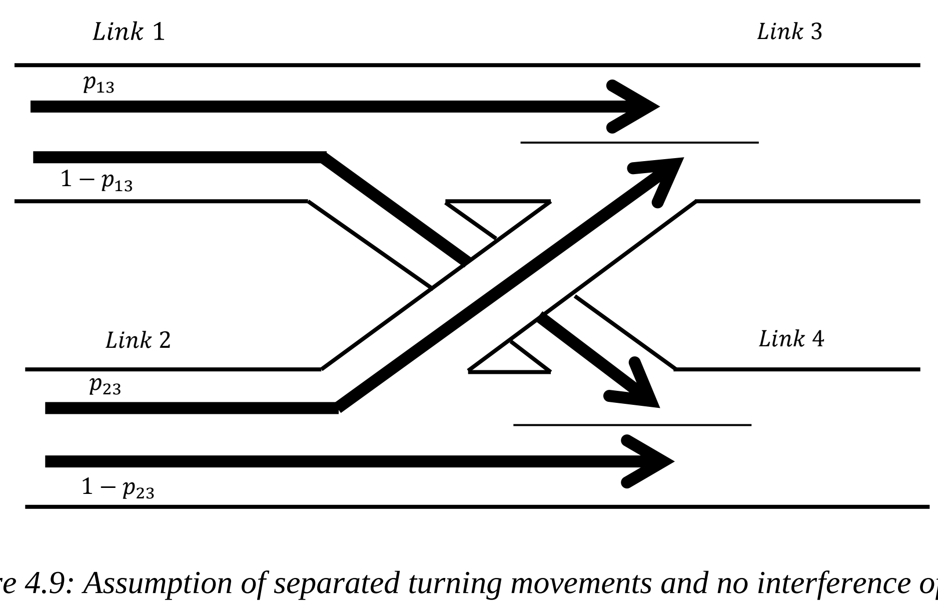

Figure 4.9: Assumption of separated turning movements and no interference of flows

For each p_{r1} value there is infinitely many user equilibria, and, as discussed in the previous section, there can be infinitely many values of p_{r1} that will yield a user equilibrium solution. To demonstrate, the author will show there are infinitely many p_{13} and p_{23} values that will yield an uncongested state at node B. If there is no spillback at node B, all parallel links (links 1, 2, 3, and 4) will operate at free-flow speed. Therefore, all four paths after the node A junction will experience free-flow travel time, and since all users experience the queue spillback on link r, the network will be in user equilibrium. There are additional equilibria corresponding to cases where there is congestion at node B. However, these equilibria are more difficult to analyze and visualize.

In order for node B to operate without congestion, the following constraints must be satisfied. It is assumed that link 1 will operate at capacity. Link 2 could have been chosen to operate at capacity without affecting the analysis; the constraint space will be identical. q_2 is the total flow on link 2. The constraints enforce the respective sending flow of links downstream of node B to be less than or equal to the receiving flow of the upstream links.

$$p_{13}c_1 + p_{23}q_2 \leq c_3$$

$$(1 - p_{13})c_1 + (1 - p_{23})q_2 \leq c_4$$

$$p_{13}, p_{23} \in [0,1]$$

Figure 4.10 plots the above constraint space with varying values of q_2 (i.e., varying values of p_{r1}). q_2 can range from 0 to its capacity, c_2. As shown in the figure, the constraint space decreases as q_2 increases; however, there are always infinitely many values of p_{13} and p_{23} for each q_2 value. Therefore, for every p_{r1} value there are infinitely many user equilibria.

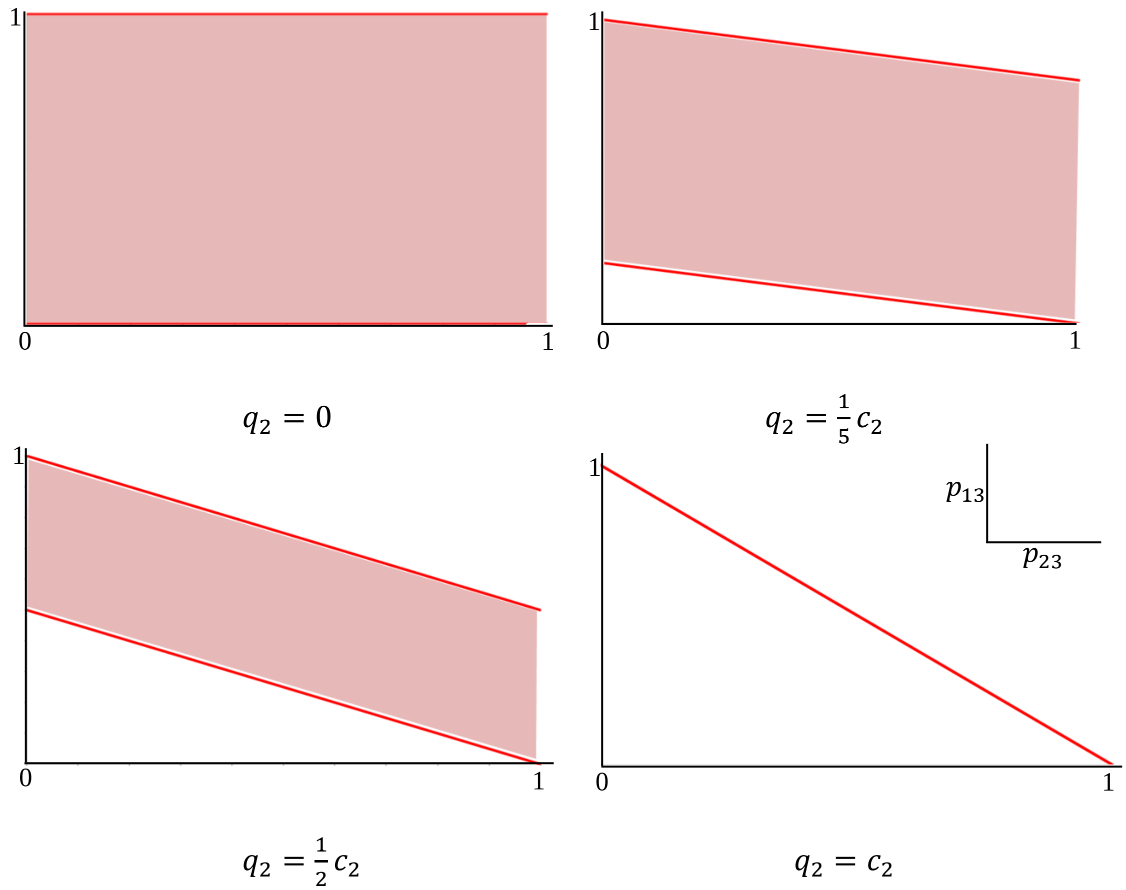

Figure 4.10: Plotted constraint space with varying values of link 2 flows

The network's optimal solution occurs when links 1, 2, 3, and 4 operate at capacity. As shown in Figure 4.10 ($q_2 = c_2$ is the system optimal solution), there are infinitely many system optimal solutions. Likewise, there are infinitely many worst case user equilibria ($q_2 = 0$).

4.4.5 The Piecewise Linear Fundamental Diagram

Figure 4.11 depicts a piecewise linear fundamental diagram, with an added line segment in the uncongested region. u_r is the slope of the additional line segment. As shown in Figure 4.11, link speed will decrease for densities larger than k_r. In fact, there will be a unique link speed for each density value in the range (k_r, k_j). The piecewise linear fundamental diagram will not significantly impact the operational efficiency of DTA models. The following numerical example will demonstrate how the piecewise linear fundamental diagram eliminates unrealistic user equilibria.

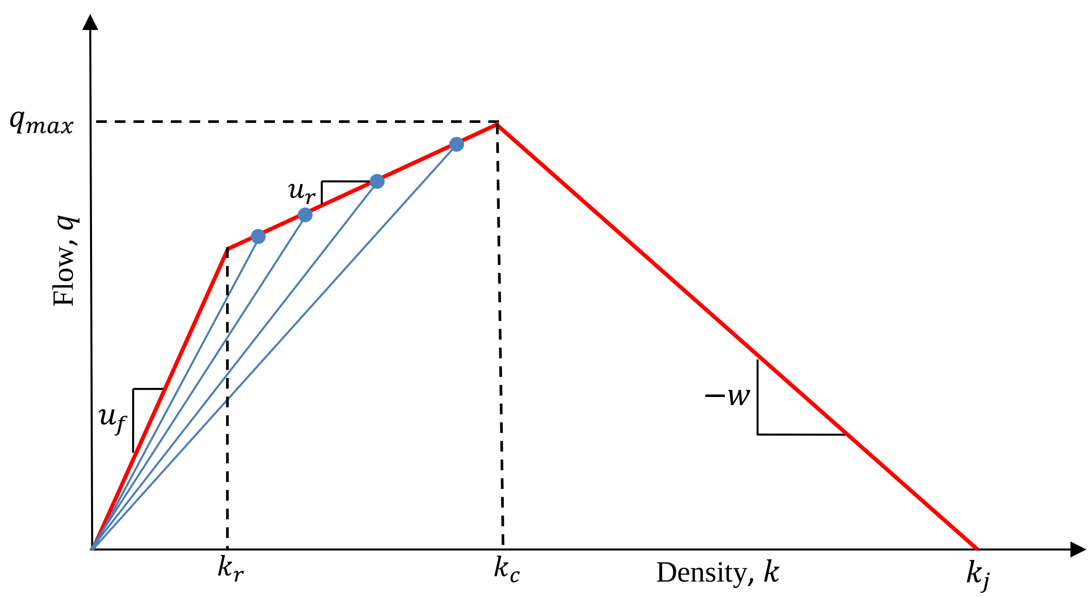

Figure 4.11: A piecewise linear fundamental diagram

4.4.6 Numerical Example

Using the link transmission model (see Yperman (2007) for more detail), traffic flow was simulated on the network shown in Figure 4.12 under two cases: (1) links 1 and 2 are represented by triangular fundamental diagrams and (2) links 1 and 2 have piecewise linear diagrams. p_{r1} is assumed to equal one (the worst case user equilibrium). The free-flow speed and capacity of the parallel links in both cases are 1 mile per minute and 50 vehicles per minute, respectively. $u_r = \frac{1}{2}\frac{mi}{min}$, $k_r = 40\ \frac{veh}{mi}$, and $w = \frac{5}{9}\frac{mi}{min}$ in the piecewise linear case. The simulation time step was assumed to be one minute.

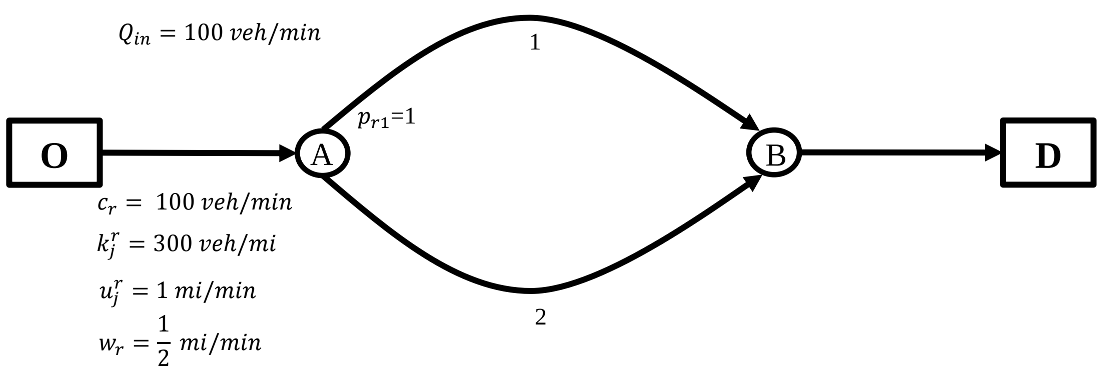

Figure 4.12: Numerical example using the DMIP network

Figure 4.13 shows the upstream and downstream cumulative count curves for link 1 in the triangular fundamental diagram case. Figure 4.14 shows the cumulative count curves for the piecewise linear case. The cumulative count curves for link r are not shown since both available paths share the link and, therefore, will experience the same travel time. τ_f is link 1's free-flow travel time, which is equal to one minute. The horizontal distance between the upstream and downstream curves represents the experienced travel time at the corresponding cumulative count value.

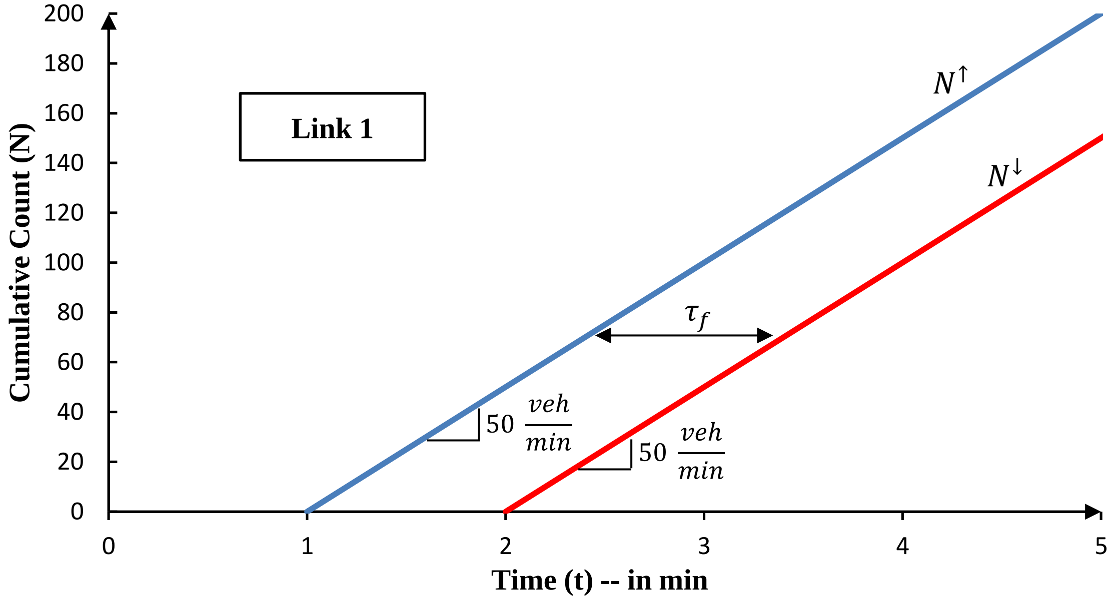

Figure 4.13: Cumulative count curves – triangular fundamental diagram case

As shown in Figure 4.13 the upstream and downstream curves are parallel, each with a slope of 50 vehicles per minute. The travel time is constant along link 1 and is equal to the free-flow travel time. Therefore, since link 2 also operates at free-flow speed, the network is in user equilibrium. However, the network is not in equilibrium when using the piecewise linear fundamental diagram. As shown in Figure 4.14, the travel time on link 1 after the first simulation period is greater than the free-flow travel time; it is equal to 1.2 minutes. Link 2 now has a shorter travel time. In fact, the network in Figure

4.12 will have a single user equilibrium (p_{r1}= 1/2) when links 1 and 2 are represented by piecewise linear fundamental diagrams. As mentioned before, no matter the value of p_{r1} at least one parallel link will be operating at capacity. In order for the other link to have the same travel time it must also operate at capacity – since each link will have a unique speed and travel time at k_c.

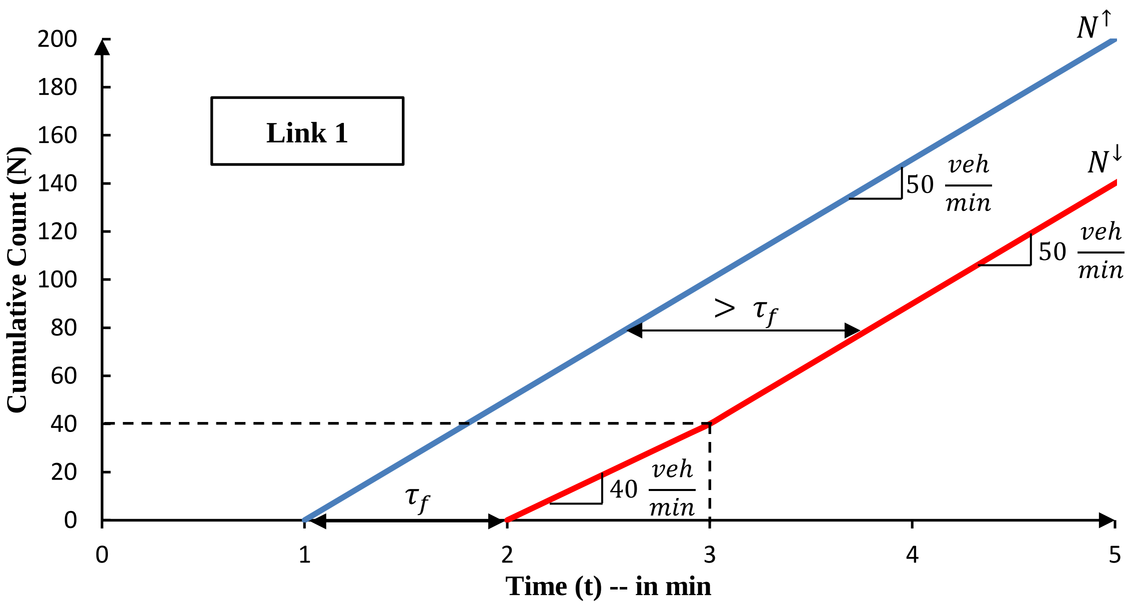

Figure 4.14: Cumulative count curves – piecewise linear fundamental diagram case

4.4.7 Required Shape of the Piecewise Linear Diagram

In the previous example, placement of the additional line segment in the piecewise linear fundamental diagram was relatively arbitrary. Travel speeds were assumed to decrease at $k_r = 40\ \frac{veh}{mi}$. However, in order for the network to maintain a single user equilibrium, k_r could have been chosen as any value in the range $\Big(0,\ {}_{u_f}c =$

$50\ \frac{veh}{mi}\big)$. The closer k_r is to the origin, the wider the density range corresponding to a unique travel speed. However, if k_r is near the origin, the fundamental diagram may not be representative of reality (i.e., travel times will start to decrease at low density levels).

Essentially, to achieve a single, realistic user equilibrium in a simple diverge-merge network, the determination of k_r is dependent on the inflow rate. By realistic, the author means that users will split themselves proportional to their capacities in diverge scenarios. For example, given the DMIP network, assume that c_1 and c_2 are differing values and are independent of Q_{in}. Also assume that the free-flow speeds of links 1 and 2 are equal. If $Q_{in} < c_1 + c_2$, $k_r = Q_{in}\frac{c_1}{c_1+c_2}u_f$ for link 1 and $k_r = Q_{in}\frac{c_2}{c_1+c_2}u_f$ for link 2 will yield a single, realistic user equilibrium where travelers split themselves on the parallel links proportional to their capacities. Likewise, if $Q_{in} \geq c_1 + c_2$, $k_r < u_f c_1$ for link 1 and $k_r < u_f c_2$ for link 2 will yield a realistic user equilibrium where both links operate at capacity.

4.5 Conclusion

Simulation-based DTA models do not guarantee a universal solution for general networks and may exhibit multiple equilibria. This is problematic for transportation practitioners since projects are evaluated at a unique equilibrium state. This chapter investigated two methods to limit the number of user equilibria and to gain fundamental knowledge regarding DTA equilibrium properties. Two network examples were presented in the context of game theory: one displayed a scenario with no DTA equilibrium and the other showcased a scenario with multiple equilibria. The first network was shown to have a mixed strategy Nash equilibrium, providing additional information that could be used to estimate traffic flows. In the second network, the amount of equilibria was reduced by applying the trembling hand refinement from game theory.

This chapter also presented a simple diverge-merge network with two identical parallel paths and showed that this network can have infinitely many user equilibria. The large number of equilibria is due to simplifying assumptions used in modern DTA models – specifically, the simplified fundamental diagram. A solution method was proposed to eliminate unlikely traffic conditions by introducing piecewise linear fundamental diagrams. Using these diagrams, the network exhibits a unique, intuitive user equilibrium. Improving DTA models and exploring the nature of its equilibrium will give planners more confidence in the accuracy of results and will further the growth of DTA in previously unexplored sectors.

Chapter 5: Conclusion

5.1 Implications of Work

This thesis presents several improvements to dynamic traffic assignment (DTA) and extends its role in transportation planning. A comprehensive view is taken, investigating DTA in three distinct planning functions: (1) high-order transportation planning, (2) project evaluation and impact analysis, and (3) traffic assignment, specifically exploring fundamental properties of DTA equilibrium.

5.1.1 High-level Transportation Planning

This work is the first to present a method of integrating DTA and the traditional four-step model such that traffic assignment is conducted at the subnetwork level while the feedback system occurs at the regional level. This structure is beneficial and allows subnetwork and regional network interaction (i.e., planners can model regional impacts without running the regional DTA model). Based on an illustrative example, the proposed method more accurately modeled traveler behavior than previous implementations. Furthermore, the method is generic and can easily be applied and customized to an agency's unique travel demand model; it offers an efficient, intuitive, and cost-effective approach to add temporal dynamics to high-order transportation planning.

5.1.2 Individual Project Analysis

The proposed urban rail system in downtown Austin, TX is modeled using a modern DTA software program. Typically, in order to estimate traffic impacts due to rail transit facilities, agencies will either conduct a corridor-specific analysis or regional planning method. The case study demonstrates that DTA is a perfect tool for this type of project evaluation, especially when investigating route changing behavior. Also, the analysis encourages agencies to expand the role of DTA by administering it to a new application.

5.1.3 The Dynamic Traffic Assignment Process

This thesis examines the chaotic behavior of dynamic equilibrium by presenting three simple and intuitive network examples: one network showcases a scenario with no equilibrium, another showcases multiple equilibria, another infinitely many equilibria. The large amount of equilibrium solutions in the latter network is due to the simplified fundamental diagram used in many modern simulation-based models. By replacing the simplified diagram with a piecewise linear diagram, unintuitive equilibria are reduced (they were completed eliminated on the example network).

Game theory is applied to dynamic traffic assignment to further investigate equilibria properties. A mixed strategy Nash equilibrium is shown to exist on a network with no user equilibrium – providing additional, beneficial information to planners. In the scenario with multiple equilibria, the trembling hand equilibrium refinement reduces the number of unrealistic equilibrium solutions. Both of these approaches further the fundamental knowledge of DTA equilibrium. In addition, they reduce the amount of unrealistic equilibria, which is beneficial to practitioners since projects are evaluated at a single unique state.

5.2 Future Work

Much of the research presented in this thesis laid the first stepping stones for future advancement. There are extensive possibilities for further improvement and contribution. The future direction of the work is described below.

5.2.1 High-level Transportation Planning

The feedback metric from dynamic traffic assignment to trip distribution and the mode choice steps is crucial to developing an efficient integrated system. For example, take the dynamic path travel times for an origin-destination pair in the downtown Austin, TX network (see Section 2.2.3) shown in Figure 5.1.

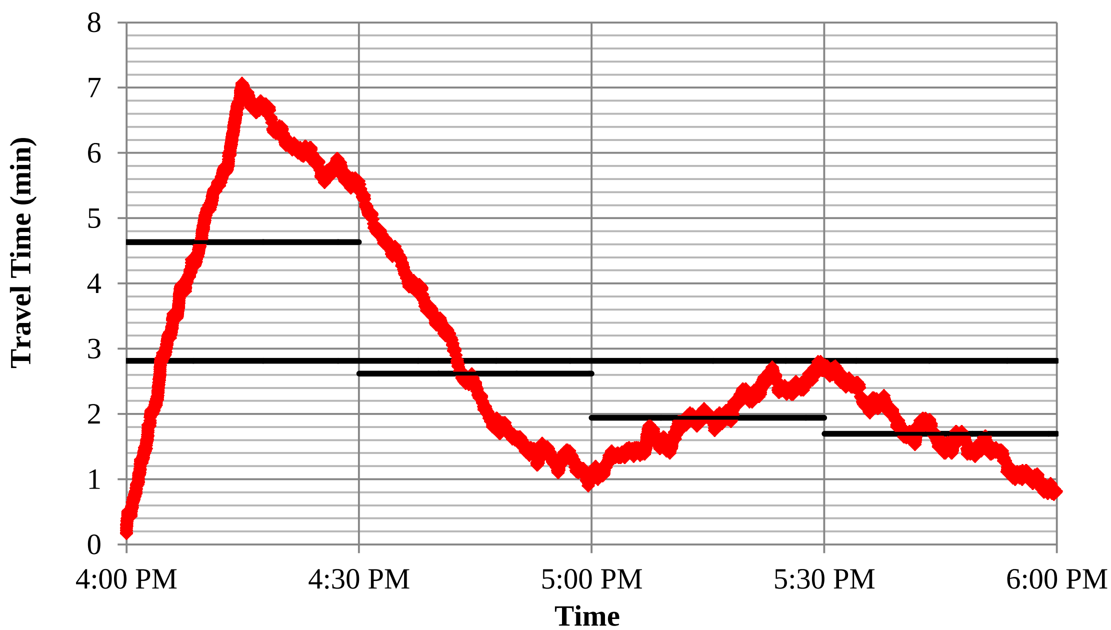

Figure 5.1: Dynamic path travel times of example origin-destination pair

As shown in Figure 5.1, using the average travel time over the entire simulation period as feedback does not accurately represent temporal dynamics. Much of the benefit from the detailed outputs of DTA is lost. One option of capturing travel time variation is to further divide the trip distribution and mode choice processes into finer time intervals. For example, one can divide the modeling period into 30 minute intervals and use the average travel time within these intervals as the feedback metric. As shown in Figure 5.1, this better captures the travel time dynamics within the simulation period. Another option is to introduce a departure time choice model, which will relax the assumption of a fixed demand profile. A natural extension of the work presented in this thesis involves testing various feedback structures and the proposed integrated method on a large-scale network.

5.2.2 Individual Project Analysis

DTA can not only be used to evaluate large-scale transit alternatives, it can be a useful tool in the design process. For example, DTA can be used to determine the location of transit stops so that the overall total system travel time is minimized. DTA can also be used to conduct a sensitivity analysis of several estimated demands. Perhaps a certain configuration is attractive at a particular ridership level, but may be detrimental to the system if ridership is low. Another transit design may cause fewer negative effects when ridership is low but is not as attractive at the predicted ridership level. The expansion of DTA as a useful tool in the design of large-scale transit systems (i.e., not only as a tool evaluating a select group of alternatives with fixed input data) provides numerous possibilities and the potential for more efficient systems.

5.2.3 The Dynamic Traffic Assignment Process

Game theory is a well-researched and vast field consisting of many equilibrium concepts, solution methodologies, and game types. Extensions of the work presented in this thesis include analyzing: (1) more complicated forms to model traffic assignment, such as *n*-player games, extensive-form games, and sequential games, (2) additional equilibrium refinements and concepts, such as subgame perfect equilibrium, Mertens stability, proper equilibrium, etc., and (3) game theory algorithms to determine if game theory can be applied to large-scale traffic networks.

The presented research regarding the infinitely many user equilibria network can be expanded in several ways as well. Further research is needed to quantify the extent to which this phenomenon occurs in larger networks and to investigate other means of resolving or mitigating this modeling issue. The calibration and appropriate geometric shape of the piecewise linear fundamental diagram can be explored in greater detail. Furthermore, existence and uniqueness properties of DTA equilibrium can be studied on additional intersection configurations.

Hopefully, the work in this thesis inspires additional improvements to the practically and attractiveness of DTA – so that DTA can be utilized in transportation planning processes as to maximize the benefits of its unique capabilities. This thesis presents several promising methods that provide an important step in accomplishing this goal.

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