How unbalanced can a barbell be before it tips over?

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In weight training, a barbell is loaded with plates to produce the desired loading for an exercise. A typical Olympic barbell weighs 45 lbs, and typical plates range in size from 2.5 to 45 lbs each. It's easier to load and unload a barbell if it's resting on a rack. The rack also makes for a convenient starting position for certain exercises. Figures 1 and 2 show two power racks, one from the campus gym and another one I built in my garage as a woodworking project.

Can you load or unload multiple plates from the same side of the barbell before moving to the other, or do you need to do this one plate at a time to maintain balance and keep the barbell from tipping? Many lifters (including me) are a bit compulsive about keeping the barbell as balanced as possible, even if it means going back and forth from one end to the other a few times. A partially-loaded barbell tipping over can be really dangerous (and also really embarassing), so we err on the side of caution.

But is this *actually* necessary? As an engineer, I'll use principles from statics to derive an expression for how much imbalance a barbell can bear before tipping. As a professor, I'll also walk you through how I came up with this expression, including some false starts and steps that didn't make it into the final derivation. I hope this is useful to engineering students, especially graduate students who often see polished derivations in textbooks or technical papers, but not all of the scaffolding or intermediate steps that led there.

I imagine most readers are just interested in the answer. If you just want to see the baby without hearing about the labor pains, the next two sections are for you. I'll derive a simple formula for the maximum imbalance in weight before the barbell tips, and use some specific numbers to demonstrate. After that, I show the (much lengthier) way I initially derived the formula, and some alternate mathematical notation, before I discovered and settled on the cleaner main derivation. I'll conclude by discussing some generalizations, showing how to relax some of the assumptions made earlier.

A standard disclaimer applies. I am presenting this analysis as an academic exercise, not as practical advice, so use these results at your own risk. To paraphrase Don Knuth, I have only proved my result true, I haven't tested it yet. Despite what I compute below, I still load and unload barbells symmetrically. I am not responsible for any injuries (or embarassment) from how you apply this analysis.

1 Solution

We represent the barbell by the free-body diagram in Figure 3, and indicate the relevant forces at play. The weight of the unloaded barbell is denoted by B, the weight loaded onto the lighter side of the barbell is W, and the weight loaded onto the heavier side of the barbell is $W + \Delta$, so Δ gives the excess weight on the heavier side. The reaction forces R_1 and R_2 are supporting the barbell on either side of the rack; R_1 on the lighter side, and R_2 on the heavier side. We will denote the width of the rack by L, and the distance

Figure 1: Power rack in Gregory Gymnasium.

Figure 2: Power rack in my garage.

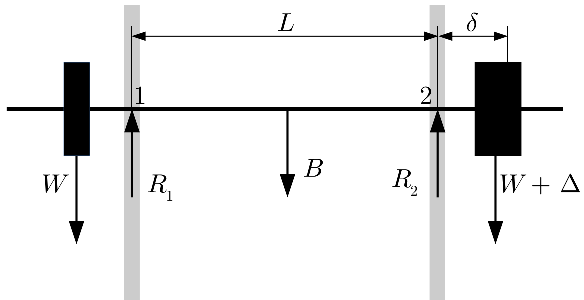


Figure 3: Free-body diagram labeling forces on the barbell.

between the rack and the additional weight plates by δ. To be specific, δ should be measured from the rack to the *center of mass* of the additional weight plates, which we'll discuss more below.

This is a statics problem, meaning all of the forces should balance: there is no net force or torque on the barbell that would cause it to move. As we show below, as Δ increases, the reaction force R_2 will increase, but R_1 will decrease. At some point, $R_1 = 0$, and the barbell is just barely stable, and any additional force on the heavier side will cause it to tip. So, we will obtain an expression for R_1 in terms of Δ, equate it to zero, and thereby determine how large Δ can be before the barbell tips.

The principle of superposition applies in statics, so the reactions are the sum of what the reactions would be if we consider each applied force individually. If the barbell were evenly loaded, with a weight of W on each end, the reaction forces are determined immediately by symmetry: $R_1 = R_2 = W + B/2$. (See Figure 4a.) If the only weight on the barbell was the excess load Δ on one side (Figure 4b), the reaction force R_1 is found by calculating the moment about point 2 (where reaction R_2 is located) and equating it to zero:

$$R_1 L + \Delta\delta = 0 \qquad \Rightarrow \qquad R_1 = -\Delta\frac{\delta}{L}\,.$$

Therefore, when considering all the loading on the barbell, we have

$$R_1 = W + \frac{B}{2} - \Delta\frac{\delta}{L}\,. \tag{1}$$

As Δ increases, R_1 decreases until it reaches zero, at which point it is no longer possible to balance the barbell. This happens when

$$\Delta = \frac{L}{\delta}\left(\frac{B}{2} + W\right)\,. \tag{2}$$

The maximum imbalance a barbell can withstand thus depends on the rack width L, the distance between the rack and the plates δ, the empty bar weight B, and the weight of the plates on the lighter end W.

2 Practical implications

Equation (2) is all well and good, but what does it actually mean? Here I compute the values of Δ using two real-world examples: the power rack and barbell in one of the UT campus gyms, and the ones in my garage. In all these examples, I assume a standard Olympic bar with $B = 45$ lbs.

The power racks used on campus at the Bellmont Hall weight room have an upright width of $L = 46$ inches, and the collars protrude by 4 inches on each side. As discussed in Section 4.1 below, we need to account for the plate width in computing δ, which is measured at the center of mass of the additional weight Δ. If all n plates have the same weight and thickness, this is easy to compute: take the distance to the inside

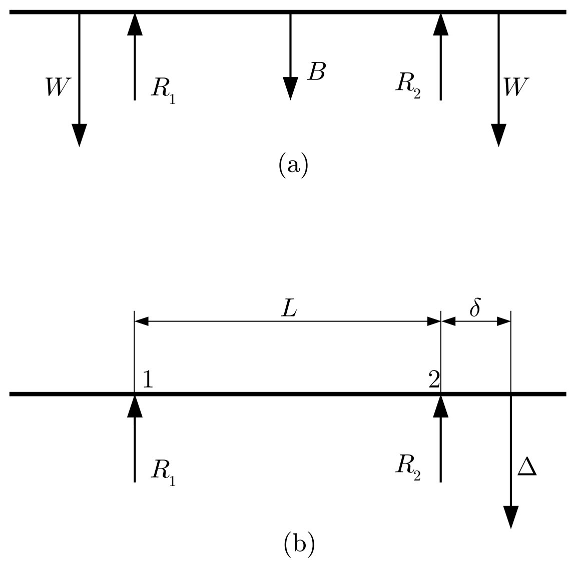


Figure 4: Free-body diagrams showing even and uneven loadings separately, to determine R_1 and R_2 by superposition.

edge of the first additional plate, and add half the plate thickness n times. Each 45-pound plate in Bellmont is 1.5 inches wide, so we will use $\delta = 4 + 1.5(W/45) + \frac{1.5}{2}(\Delta/45)$ inches in computation. Substituting this expression into (2) and re-solving for Δ would involve some messy algebra to solve a quadratic equation. I will adopt the simpler approach of iterative guess-and-check, making sure that the solution in Δ is consistent with the assumed value of δ.

The bar is least stable if there is no weight on the left end ($W = 0$). Substituting $L = 46$, $\delta = 4$, $B = 45$, and $W = 0$ into equation (2) gives a maximum inbalance of $\Delta = 258$ pounds! That sounds like a lot, and is in fact more than five plates. Rounding up to six for safety, this means that the actual width of the added load occurs further away from the uprights than originally assumed, at $\delta = 4 + 1.5 \times 6/2 = 8.5$ inches. This new value of δ gives a maximum imbalance of $\Delta = 122$ pounds — the value of δ really matters quite a bit. This is three plates' width, so trying once more with $\delta = 4 + 1.5 \times 3/2 = 6.25$ inches gives $\Delta = 166$ pounds, or four plates. One last iteration with $\delta = 7$ inches gives $\Delta = 147$ pounds, which is consistent with the final result.

The conclusion: you can have an empty bar on one side, and 147 pounds (more than three plates!) on the other, and the barbell will not tip. Adding even one plate to the left end increases stability greatly: repeating the calculation with $W = 45$ gives $\Delta = 310$ pounds, or nearly seven plates!

My garage gym has slightly different dimensions. I measured $L = 42$ inches, a collar protrusion of 6.5 inches on each side, and a plate width of 1 inch. Repeating the analysis with these dimensions indicates stability with an empty bar on one side, and 118 pounds on the other, or more than two plates.

I repeat my disclaimer from above. This derivation assumes a perfectly uniform bar which is perfectly centered in the uprights. A barbell loaded exactly to the Δ values given here is "stable" but even brushing against it slightly can cause it to tip. Please be cautious about how you apply this result.

3 How the sausage was made

The solution given in Section 1 is much shorter than the first one I found. In the interest of academics, I will show my first derivation of the result, and then how I went back and simplified it to what appears above. I hope this is useful for students to see: your first solution does not have to be the one that actually appears in a submitted assignment or research paper, and it's worth taking some time to consider alternative approaches. Once you know what the answer is, you can often find a better way to derive it.

The preliminary steps are to first realize that the question can be answered by static mechanics; to draw a free-body diagram (never a bad idea in statics); and to realize that the "almost tipping" condition occurs when one of the reaction forces is zero. From here the solution is standard, but this doesn't mean it's clean!

When I first solved the problem, I also defined M as the distance between the collars. This value is related to the others through $\delta = (M - L)/2$. My first attempt was to solve for R_1 and R_2 by solving the following system of equations representing static equilibrium conditions: the first requiring the sum of vertical forces to equal zero, the second requiring that the moment about point 1 equal zero:

$$\sum F_y = 0 \Rightarrow R_1 + R_2 = B + 2W + \Delta \tag{3}$$

$$M_1 = 0 \Rightarrow W\delta + LR_2 = \frac{L}{2}B + (M - \delta)(W + \Delta)\,. \tag{4}$$

After some routine but mildly tedious algebra — solve equation (4) for R_2, and substitute into equation (3) to solve for R_1, we find

$$R_1 = \frac{B}{2} + W\left(2 - \frac{M - 2\delta}{L}\right) + \Delta\left(1 - \frac{M - \delta}{L}\right) \tag{5}$$

$$R_2 = \frac{B}{2} + \frac{M - \delta}{L}(W + \Delta) - \frac{W\delta}{L}\,. \tag{6}$$

Note: At this point, I wanted to check that my derivation was correct. I created a spreadsheet where I could input values of B, W, etc. and compute the reactions R_1 and R_2 using the formulas above. I then added cells to check the conditions (3) and (4), to confirm that I solved the system of equations correctly; and added another cell to compute the moment around point 2 and confirm that it is also zero. This is technically redundant, but is another check that the equations (3) and (4) were correct themselves. It is very useful to check your work before getting too far in; in fact, I found and fixed a small mistake in my algebra, which would have been much more painful later on.

These formulas are correct, but a bit unwieldy. I looked for ways to simplify them. An immediate simplification comes from the relation $\delta = (M - L)/2$, because

$$2 - \frac{M - 2\delta}{L} = 2 - \frac{M - 2(M - L)/2}{L} = 1\,,$$

so the first factor in parentheses in equation (5) can be deleted. Likewise, $(M - \delta)/L$ can be simplified to $(M + 1)/2L = 1/2 + M/2L$. I also noticed that the ratio M/L appears multiple times in the equations, so I defined $\rho = M/L$ and looked for additional simplifications. In the end, I had these slightly cleaner — and definitely more aesthetically appealing — formulas for the reaction forces:

$$R_1 = \frac{B}{2} + W + \frac{\Delta}{2}(1 - \rho) \tag{7}$$

$$R_2 = \frac{B}{2} + W + \frac{\Delta}{2}(1 + \rho)\,. \tag{8}$$

Note: I also checked each of these simplifications as I went, using my spreadsheet to confirm numerically that I hadn't made any algebra errors.

From here, I set equation (7) to zero to solve for the critical Δ value, giving

$$\Delta = \frac{2}{\rho - 1}\left(\frac{B}{2} + W\right).$$

One last algebra check shows that $2/(\rho - 1) = L/\delta$, yielding the formula given in the section above:

$$\Delta = \frac{L}{\delta}\left(\frac{B}{2} + W\right).$$

At this point, I had obtained the formula that I wanted. But looking at (7) and (8), I was struck by the $B/2 + W$ terms appearing in both. That would be the reaction forces if the barbell were perfectly balanced ($\Delta = 0$), and I thought it was neat that the reaction forces can be divided into a "balanced" component and an "unbalanced" component (with the latter proportional to Δ.) This was my clue that the superposition principle could be applied to simplify the derivation.

I redid the derivation along similar lines using superposition, and I realized I could simplify the argument yet again by computing the moment around point 2, rather than point 1. I only need the value of R_1, and I can solve for that directly by computing the moment around point 2, without having to solve a system of equations. At this point I had the argument given in Section 1.

I was slightly embarassed I hadn't thought of the superposition trick, or of computing the moment around point 2, earlier on, since these are fairly standard techniques. I'll blame this on my statics being rusty, having taken the course more than 20 years ago, but I think it's actually more instructive for me to write everything out both ways. If you are a student in a statics course, this shows the value of solving problems carefully when studying, then taking the time to review your solution and look for faster ways. You'll be surprised how many tricks you discover when you look over solved problems and ask if there was a faster or easier way to come up with the answer. On exams or other time-sensitive situations, having a repertoire of these "tricks" can be really helpful.

As one final note, my notation changed once or twice as I was working on the problem. My original handwritten derivation is in Figure 5, you can see a few changes I made midstream. The lesson from this is that it's OK to change your notation from a rough draft to a final version, just make sure you proofread carefully!

4 Generalizations

This section explores two generalizations of the model presented above. Our analysis assumed all the plates were uniformly-sized 45-pound plates; Section 4.1 shows what changes in the more general case. We also assumed that the bar was placed symmetrically in the rack; Section 4.2 shows what happens if it's not.

4.1 Nonuniform plates

What if you use plates of different weights and thickness?

Assuming the load W is symmetric on both sides of the barbell, the only thing which changes in the above analysis is how δ is computed.

In the general case, imagine the excess weight Δ is composed of n plates; the i-th plate has weight w_i and thickness t_i. Imagine first that the innermost edge of the first plate of excess weight is located right at the

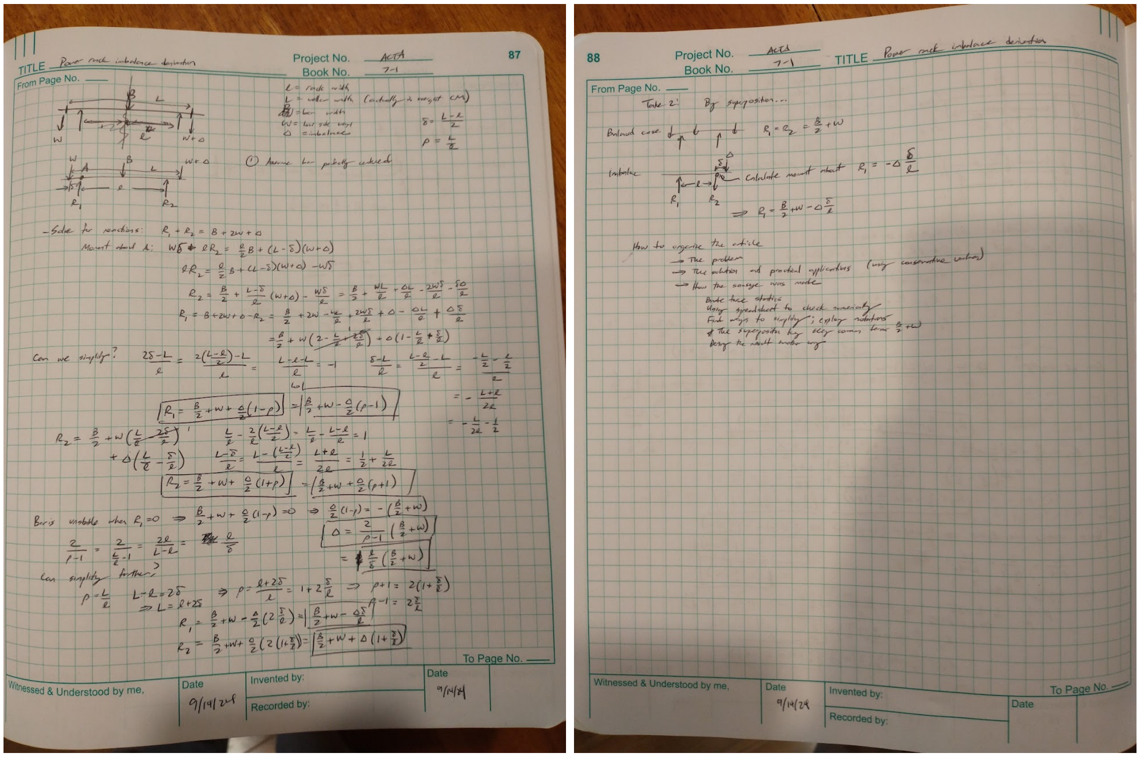


Figure 5: Original hand derivation, before cleaning it up.

rack. This isn't actually the case (the collars protude some, and there may be a symmetric load W), so we'll call this fictitious value δ' and compute the real value of δ at the end.

The general formula for the center of mass is

$$\delta' = \frac{\sum_{i=1}^{n} w_i \left((t_i/2) + \sum_{j=1}^{i-1} t_j \right)}{\sum_{i=1}^{n} w_i} \,. \tag{9}$$

This formula simplifies if all of the plates have the same thickness t. In this case we have

$$\delta' = \frac{\sum_i t w_i (i - 1/2)}{\sum_i w_i} = t \left(\frac{\sum_i w_i i}{\sum_i w_i} - \frac{1}{2} \right) \,. \tag{10}$$

(If all the plates have uniform width w, then some additional algebra and the identity $\sum_{i=1}^{n} i = n(n+1)/2$ gives the further simplification $\delta' = tn/2$, as we used in the main analysis.)

We finally correct for the fact that δ is measured from the rack, not from the innermost edge of the first additional plate. This is nothing more than a coordinate shift. If the distance between these two points is ℓ, then $\delta = \ell + \delta'$.

We can imagine an even more general case where the symmetric loading W may not be symmetrically placed on the barbell, or where the plates are not all flush. This generalization is left as an exercise for the reader.

4.2 What if the barbell isn't perfectly centered?

The assumption of a symmetrically-placed barbell made the earlier derivation much easier. We could go through the entire analysis again with the barbell shifted to one side, but I'm too lazy to do that. Instead, we can use superposition a second time to simplify this case.

Assume the barbell is shifted by a distance S to the side with additional weight. (If it's shifted in the other direction, just use a negative value for S.) As shown in the free-body diagrams in Figure 6, we can treat this as the sum of two loadings: the symmetric loading we analyzed in the main document, and a second loading which has the asymmetric loads, *and the original loads oriented in the opposite direction.* We compute the reaction R_1 in these two cases separately, and sum them. The symmetric loading has already been done in equation (1). For the asymmetric load, notice that the forces all sum to zero, but there is a net clockwise moment of $(B + 2W + \Delta)S$. Therefore, the reactions R_1 and R_2 also form a couple, oriented in the opposite direction. To cancel the moment from the loading, we must therefore have $R_1 = -S(B + 2W + \Delta)/L$.

Summing the two components, we have the asymmetric reaction R_1 as

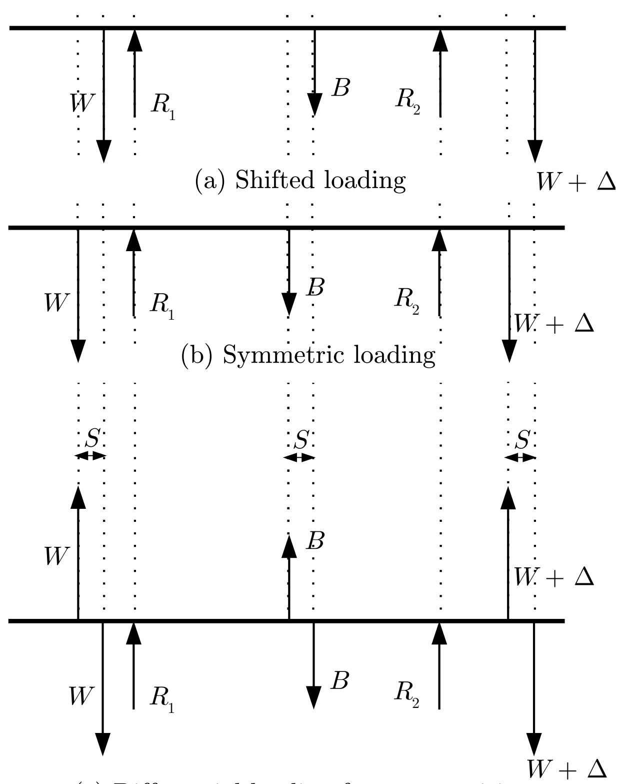
$$R_1 = W + \frac{B}{2} - S\left(\frac{B + 2W}{L}\right) - \Delta\left(\frac{\delta + S}{L}\right) , \tag{11}$$

and the barbell is unstable at the point when this reaction vanishes, that is, when

$$\Delta = \frac{L}{\delta + S}\left(W + \frac{B}{2} - S\left(\frac{B + 2W}{L}\right)\right) . \tag{12}$$

(As a check, notice that this formula reduces to equation (2) when $S = 0$, as it must.)

Repeating the analysis from Section 2, we consider the worst case when the barbell is shifted as far to the right as possible given the collar width. At Bellmont, that means $S = 3$ inches, which means a maximum imbalance of $\Delta = 97$ pounds, quite a bit less than the 147 pounds computed when placed symmetrically. In my garage, the maximum possible shift is 5 inches, which gives a maximum of imbalance of 84 pounds, quite a bit lower than the 118 pounds computed above. If you're going to push stability to its limits (and you really shouldn't), at least make sure the barbell is symmetric on the rack!



(a) Shifted loading

(b) Symmetric loading

(c) Differential loading for superposition

Figure 6: Decomposing the loading (a) when the barbell is shifted into the symmetric loading (b) and a decomposition loading (c) which reduces to a couple.