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16. Abstract <p>Both public and private entities are concerned with the impacts of future toll revenue, and the effects of tolled facilities on system congestion. Due to the inherent complexity of transportation systems, it is impossible to predict travel demand and congestion conditions exactly, and simplistic attempts to account for this consistently underestimate true levels of congestion. Thus, in the context of roadway pricing, there is a need to develop mathematical models which explicitly account for both demand and supply uncertainty in both the short-term and long-term time scales. This project will develop these models, which will be suitable either to determine the best pricing policies to maximize revenue or minimize congestion, or to evaluate alternative toll policies according to these metrics. Thus, these models will produce more accurate predictions of toll revenues and congestion levels than are available using current methods.</p>					
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**ROBUST PRICING OF TRANSPORTATION
NETWORKS UNDER UNCERTAINTY**

by

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Research Report SWUTC/10/169206-1

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ABSTRACT

Both public and private entities are concerned with the impacts of future toll revenue, and the effects of tolled facilities on system congestion. Due to the inherent complexity of transportation systems, it is impossible to predict travel demand and congestion conditions exactly, and simplistic attempts to account for this consistently underestimate true levels of congestion. Thus, in the context of roadway pricing, there is a need to develop mathematical models which explicitly account for both demand and supply uncertainty in both the short-term and long-term time scales. This project will develop these models, which will be suitable either to determine the best pricing policies to maximize revenue or minimize congestion, or to evaluate alternative toll policies according to these metrics. Thus, these models will produce more accurate predictions of toll revenues and congestion levels than are available using current methods.

EXECUTIVE SUMMARY

Transportation systems are highly complex, since their operation depends both on human behavior as well as events such as incidents and weather which are themselves unpredictable, or even chaotic. Thus, it is impossible to predict system conditions even in the short-term, let alone for longer time horizons. At the same time, it is incorrect to base predictions on nominally “average” or “expected” system conditions – due to the nonlinearity of transportation systems, this approach consistently underestimates congestion levels. (Waller et al., 2001) Instead, one must fully incorporate this uncertainty into the modeling process, accounting for it at each step.

Uncertainty plays an integral role in pricing transportation networks. Transportation agencies have commonly used tolls to generate revenue to offset infrastructure construction and maintenance costs. More recently, private firms have been allowed to own and operate transportation facilities as well. However, most of these tolls and revenue forecasts are based on a single, deterministic point estimate of roadway demand and supply. Failure to account for the uncertainties inherent in travel patterns when establishing toll policies leads to inaccurate predictions of both congestion levels and revenue.

Therefore, there is a need to develop methodologies to support toll pricing decisions which account for the uncertainties in future demand and road usage. This problem is especially relevant in recent times with an increase in the number of highway Build-Operate-Transfer (BOT) projects, in which private firms attempt to recover and profit on their investment through tolling. Failing to account for future uncertainty can result in overestimation of the future cash flow causing significant losses for investors, or unnecessary congestion throughout the network.

This report includes a review of existing methodologies related to modeling supply- and demand-side uncertainty mathematically, incorporates these techniques into pricing models, allowing both optimal toll policies to be found (satisfying goals such as revenue maximization or congestion minimization), and allowing alternative toll policies to be evaluated according to a variety of measures of effectiveness, and examines the suitability of these models for a variety of applications, such as prediction of toll revenues, transportation planning, and alternative analysis involving BOT projects. While considering both demand and supply side uncertainty, the framework developed in this report also allows the benefits of real-time travel information to be compared directly against the benefits of responsive pricing, allowing planning agencies to

identify the value of these policy options or contract terms in publicly- or privately-operated toll roads. Specifically, six scenarios reflect different combinations of policy options, and correspond to different solution methods for optimal tolls.

Background

Broadly speaking, the sources of uncertainty in transportation systems can be classified in several ways, according to the time scale (short-term/operational vs. long-term/planning), and to whether travel supply or demand is affected. For instance, incidents affect roadway supply (capacity) in the short-term, while uncertainty in future economic conditions affects travel demand in the long-term. These are discussed in more detail in the following subsections.

Demand-Side Uncertainty

Demand-side uncertainty occurs both in the short-term (operational) and the long-term (planning) time scales, and these two effects must be considered. In the short-term, daily demand fluctuations contribute to unreliability of travel times, which has demonstrable effects on route choice behavior. In the long-term, changes in land use or travel demand patterns may lead to a facility which is either overused or underused, as compared to the design scenario. When tolls are involved, both of these also have significant implications for revenue streams, in addition to the usual effects on congestion levels.

Numerous works have focused on developing bi-level mathematical programming formulations and solution algorithms for the network design problem under uncertain demand (Karooonsoontawong and Waller, 2006; Ukkusuri, Tom, and Waller, 2007; Waller and Ziliaskopoulos, 2007). Gardner, Unnikrishnan, and Waller (2008) show that marginal social cost prices obtained using the expected value of demand can significantly deteriorate system performance especially when the actual system state deviates from the planned forecasted conditions. Chen et. al. (2007) studied the problem of setting optimal tolls and capacity on a subset of links in a highway B-O-T project under demand uncertainty. In this study, private investors set tolls to maximize profit and to offset the construction cost under numerous regulations imposed by the state and other stakeholders. More recently, Li. et. al. (2007) developed a bilevel mathematical programming formulation to determine the optimal tolls to improve travel time reliability under uncertain demand and capacity.

Supply-Side Uncertainty

Supply-side uncertainty occurs from nonrecurring changes in roadway capacity: incidents are the most significant source of this type of uncertainty, although weather, special events, and construction also contribute to this. This affects toll revenue in three distinct ways: first, motorists prefer more reliable facilities to less reliable ones, which has an impact on habitual route choice even when no disruption occurs. Second, when roadway capacity is reduced, toll revenue is adversely affected because fewer vehicles can use the facility. Finally, if tolls can be varied in response to supply uncertainty, the operator must choose how to adjust prices in order to reflect realized conditions, according to their particular objective and constraints.

This type of uncertainty has several impacts on the effectiveness of congestion pricing, primarily depending on what information travelers have, and on the ability of the system operator(s) to vary tolls in response to observed conditions. These two affects can work synergistically: Yang (1999) showed that responsive pricing and travel information “complement each other and that their joint implementation can reduce travel time more efficiently.” Similar results were shown by de Palma and Lindsey (1998). More recently, Lindsey (2008) show that marginal-cost congestion pricing still generates enough revenue to construct the socially-optimal amount of capacity, as long as drivers are informed and tolls are responsive; this generalizes an earlier result of Mohring and Harwitz (1962).

Aside from the impact on system congestion, supply-side uncertainty also impacts toll revenues directly, because travel time reliability plays a key role in users’ travel choices, often being nearly as significant as average travel time (Small et al., 2005). This has been revealed by many researchers using a variety of econometric techniques, regardless of how reliability is measured: for instance, Small et al. (2005) and Liu et al. (2007) use the difference between the 80th- and 50th-percentile travel times, while Pinjari and Bhat (2006) used the maximum additional travel time that might be required. For additional details, Bates et al. (2001) and Noland and Polak (2002) provide overviews of theoretical and empirical research in valuation of travel time reliability. Without using methods such as these to determine user route choice, it is impossible to accurately predict either toll revenue or to produce the tolling policies which maximize revenue or minimize congestion levels.

Demonstrations are provided on both the Sioux Falls and Anaheim networks. Under conditions of both supply and demand uncertainty, results indicate that providing information to drivers implemented alongside responsive tolling may reduce expected total system travel time by over 9%, though more than 8% of the improvement is due to providing information, with the remaining 1% improvement gained from responsive tolling.

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INTRODUCTION

Roadway pricing is of great interest today as state and federal governments seek funding for building and maintaining transportation infrastructure. This interest is demonstrated by the increasing number of regions implementing roadway tolling, and by the growing number of public-private partnerships (PPPs) and other innovative financing structures. In addition to serving as an alternate revenue stream, tolling has demonstrated potential to reduce travel delay by providing incentives to choose a travel route, mode, or departure time which is less burdensome to society.

In the case of PPPs, tolls are often collected from the public to pay back the cost of the project, and to generate future revenue for the private investor. Such investors are highly concerned with the risk of this investment, in addition to the expected rate of return over the project lifetime (Nijkamp and Rienstra, 1995). This risk arises from deviations in the predicted revenue which accrue both in the short-run (due to daily fluctuations in demand or capacity disruptions such as incidents) and in the long-run (due to prediction errors in forecasting land use, travel demand, or the future roadway system). Including these events in pricing models is critical for accurately evaluating the investment success of a toll project.

Even for publicly-operated facilities, these uncertain events wield major influence over the revenue generated from tolls, as well as the degree to which congestion can be managed. In terms of congestion management, resilience in the face of short-term capacity disruptions (such as incidents or poor weather) plays a central role even in long-term planning; it is well-demonstrated in the mode and route choice literature that short-term reliability wields considerable influence on habitual decision-making (see, for instance, Small et al., 2005; Pinjari and Bhat, 2006; or Liu et al., 2007), often of the same order of magnitude as average travel time. Furthermore, innovative technologies now allow tolls to be varied dynamically, and methods for developing and analyzing responsive pricing strategies must be applied.

In this context, the contribution of this paper is the development of a framework for considering different toll policy options or contract terms, accounting for uncertainty both on the supply-side and on the demand-side. Methods are given for quantifying the benefits of responsive tolling and real-time travel information, such as advanced traveler information systems (ATIS). The former is directly related to toll implementation, while the latter naturally

complements decision making under uncertainty, as shown by the long history of information and pricing models in the economics literature (described in more detail in the background section). As a result, (i) the effects of information and toll flexibility in pricing problems can be quantified, allowing both public agencies and private entities to know the value of these options when setting a toll policy or negotiating PPP contracts; (ii) revenue variability due *both* to short-term supply disruption and long-term forecasting error can be measured; and (iii) the error introduced by assuming known future conditions can be quantified.

BACKGROUND

Demand Uncertainty

As with all traffic flow modeling, travel demand is a key factor in determining network performance, yet inherent uncertainty in the prediction process makes it difficult to properly account for. Changes in land use such as suburban sprawl, increasing population, and changing gas prices, are just a few of the reasons accurately predicting long term travel demand is virtually impossible. Given the long-range planning setting of this work, we only consider long-term demand uncertainty. The actual (future) demand realization is unknown when the tolls are being set; after the tolls are set (and after some time) the actual demand is realized and the users equilibrate deterministically. This assumption is motivated by the idea that users gain knowledge of the actual demand level through their own driving experience, and over time have learned the optimal route minimizing their travel time. Short-term demand fluctuations are not considered in this work because neither the drivers nor the network manager could possibly know the demand realization on a specific day until after it is past, and therefore it cannot affect either the routes chosen or the tolls set on a specific day. The impact of demand elasticity is not considered here, as the focus of this work is to specifically isolate the effects of uncertainty. To a limited degree, the effect of elasticity can be incorporated in the probability distributions chosen for travel demand, or by creating artificial links between each origin and destination, as described in Sheffi (1985).

Capacity Uncertainty

Network capacity is another major factor in network performance, and like demand, capacity is subject to change on a regular basis. In contrast to demand, short term capacity changes (rather than long term) are of greater concern because they are more subject to uncertainty. Long term capacity changes, such as added and removed lanes, are more controlled factors, and can be accounted for more appropriately through multi-scenario analysis. Furthermore, it has been repeatedly demonstrated in the mode and route choice literature that short-term reliability plays a critical role in long-term, habitual traveler decisions, as described in the literature review. Thus, in this paper, capacity uncertainty is only considered in the short

term, as the result of day-to-day fluctuations due to accidents, temporary lane closures, construction, variations in driver behavior, weather, and similar factors. This type of uncertainty directly leads to unreliability in travel times, which has a clear impact on long-term user behavior, especially when traveler information is provided and users can switch routes in response to system conditions.

Even with knowledge of supply and demand uncertainty, the common practice is still to determine toll prices on traffic networks based on a single, expected demand value and “typical” roadway capacity, or to represent a deterministic demand-supply relationship through elasticity. This is partially due to the computational cost in determining the optimal prices under uncertainty due to the numerous possible stochastic scenarios, but for certain traveler information and toll flexibility scenarios, tractable numerical (or even analytical) procedures are available.

Role of Information and Responsive Tolling

Real-time information is a natural mitigation strategy in stochastic networks: if what is uncertain is made known, the effects of stochasticity are reduced, and indeed information only has value because of uncertainty. The most important issues in stochastic network modeling concern the information available to each of the parties (in this case, the network manager and the drivers) when they make their decisions (toll prices and travel routes, respectively). In this work, travelers are assumed to be either fully informed about network conditions before embarking on travel, or having no information except the probability distributions based on experience; likewise, the network manager is either able to vary the tolls in response to realized network conditions, or must levy the same tolls every day. The options for pricing (either to vary tolls in response to incidents and short-term disruption, or to levy static tolls regardless of the network state) reflect either technological options or legal ones, allowing the value of flexible pricing to be quantified for the purposes of cost-benefit analysis; and the options for traveler behavior allow a similar analysis for ATIS implementation. These assumptions generate six possible scenarios relating to the information available to motorists when choosing a travel route, and to the ability of the network manager to adjust the toll in response to network conditions, as discussed more precisely in the following section.

Research Objectives

Considering the above mentioned issues of uncertainty, pricing and information, this research aims to answer the following questions:

1) *What role does uncertainty play in network behavior?*

How does the system performance and revenue generated vary under conditions of supply and demand uncertainty when the network is subject to tolling? What is the difference in network performance and toll revenue collected when (a) not considering any uncertainty at all, (b) only considering supply uncertainty, (c) only considering demand uncertainty, and (d) considering both supply and demand uncertainty.

2) *What is the benefit of providing information to the public?*

What is the impact of providing information to users on current network conditions? How does the expected system performance change when this information is available to users? What is the effect on the revenue generated? Does there appear to be incentive for private operators to provide information to roadway users?

3) *What is the benefit of responsive pricing?*

With responsive pricing the tolls are allowed to vary, or respond to the actual network conditions, rather than remain fixed day-to-day. What is the quantitative benefit of responsive tolling capabilities? What impact does responsive tolling have on the expected system performance? What impact does it have on expected revenue? How does the system performance vary under responsive tolling when information is/is not provided to users? When should network controllers implement responsive tolling techniques? For private investors, what is the value of negotiating contract terms which allow responsive pricing?

LITERATURE REVIEW

There has been much research on how users respond to prices, and how to set prices to achieve specific objectives including minimizing the total toll collected, minimizing the maximum toll collected, minimizing the number of toll booths and so forth. (Hearn and Ramana, 1998; Yildirim and Hearn, 2005) One of the more common pricing objectives is to maximize social welfare (or system performance) by setting the link tolls to be equal to the difference between the private cost experienced by the user on a link and the total cost experienced by the link users due to the individual's decision to travel. These resulting tolls are known as marginal social cost prices, and are in accord with the theory developed by Pigou (1920). One common assumption in most of the previously mentioned works is that the marginal cost tolls are based on a single value of travel demand or a deterministic elastic demand relationship, and a deterministic level of network capacity.

Studying the impact of demand uncertainty on the traffic assignment problem has received increasing attention in recent years. Waller *et al.* (2001) showed that neglecting the impact of long term demand uncertainty by using a single fixed estimate of future demand can result in significant underestimation of the future system performance, which could further result in sub-optimal network design decisions (Duthie *et al.*, 2009). Whether or not this same effect applies to network pricing problems is still an active research question, but it is obvious that demand will play a major role in the success of a tolled transportation system, at least from the perspective of revenue and investment risk. Lam and Tam (1998) determined probability distributions for the future toll revenue and traffic flow when numerous input parameters such as population and probable toll charges were assumed to follow a normal distribution. Chen and Subprasom (2007) studied the problem of setting optimal tolls and capacity on a subset of links under demand uncertainty by private investors to maximize profit in a highway build-operate-transfer project under numerous regulations imposed by the state and other stakeholders. However, this study does not directly address robustness and focuses on the implications of regulations and second best pricing strategies. In the area of robust first best tolling under uncertain demand, previous work by Gardner *et al.* (2008) demonstrated the potential problem associated with using the expected demand to deterministically calculate link tolls. Nagae and Akamatsu (2006) formulated the problem of choosing the optimal toll level from two discrete

values as a stochastic singular control problem where the demand was assumed to vary following a stochastic differential equation. More recently, Li. et. al. (2007) developed a bi-level mathematical programming formulation to determine the optimal tolls to improve travel time reliability under uncertain demand and capacity where users are assumed to make route choices based on the multinomial logit model. Numerical results are provided on a small network although no solution methodology is presented for large networks.

Most of the research into the effect of capacity disruptions on roadway pricing has been descriptive, (that is, studying how tolls affect reliability), rather than prescriptive (that is, studying how tolls should be set under uncertainty). Examples of the latter include studies of the I-15 FasTrak project in San Diego, California (Supernak *et al.*, 2003) or California State Route 91 (Liu *et al.*, 2004), estimating travelers' valuation of reliability by comparing performance on tolled lanes to parallel free lanes.

Even though the focus of the models in this paper is long-term, short-term capacity disruptions have been demonstrated to play a substantial role in habitual routing and mode choice decisions. The importance of travel reliability has been measured to be of the same order of magnitude as the importance of typical travel costs. For instance, in an analysis of SR-91 data in California, Small et al. (2005) estimated a \$19.56/hr value of reliability, as compared to a \$21.46/hr value of travel time. "Reliability" can be defined in different ways; Small et al. (2005) used the difference between the 80th- and 50th-percentile travel times (approximately one standard deviation in many probability distributions), as did Liu et al. (2007). Pinjari and Bhat (2006), on the other hand, used the maximum additional time that might be required for a trip (and also identified reliability as being nearly as important as average conditions in mode choice), while de Palma and Picard (2005) proposed four different utility functions to specify preferences towards short-term travel time reliability. More comprehensive overview of theoretical and empirical issues in valuation of travel time reliability can be found in Bates et al. (2001) and Noland and Polak (2002).

Regarding analytical models of pricing under uncertainty, the majority of the research has been conducted in idealized settings and small networks. For instance, Verhoef *et al.* (1996) showed that information provision and *unresponsive* tolling was nearly as effective as a perfectly responsive toll, at least in a small two-link network. de Palma and Lindsey (1998) show that improved information always improves welfare, whether in un-tolled networks, networks with

fixed (unresponsive) tolls, or in networks with responsive tolls. When information is provided and tolls are perfectly responsive, Kobayashi and Do (2005) show that marginal cost prices maximize social welfare in networks with a single origin-destination pair and no overlapping routes. In general, when information provision is costly, Emmerink et al. (1996) shows that no subsidy or tax is needed to yield the socially optimal proportion of informed users. Yang (1999) also found that information provision and responsive tolling exhibit complementary, synergistic effects in numerical tests on a small network. Previous work by Boyles *et al.* (2010) studied the problem of tolling under stochastic supply conditions, and found that attempts to incorporate uncertainty into nonresponsive tolls involve significantly higher prices, under the assumption of deterministic demand.

These past works on pricing under uncertainty have considered a variety of different price flexibility and traveler information scenarios, with and heuristic or sampling-based techniques developed for larger networks. In this light, the major contribution of this paper is common framework for quantifying the effect of different assumptions on toll flexibility and traveler information. This comparison is performed on the basis of benefits to society, toll revenue, the variability in these quantities due to uncertainty, and can be applied by agencies considering different options for addressing uncertainty (for instance, by quantifying the effects of ATIS alone, responsive pricing alone, and both together). Solution methods for each of these scenarios for large-scale networks also had to be created, adapting these techniques from the past literature to fit into the common assumptions described in the following section.

MODEL FORMULATION

Notation

Consider a stochastic transportation network $G = (N, A, \mathbf{D}, \Omega_1, P_1, \Omega_2, P_2)$ consisting of a set of nodes N ; a set of directed arcs A ; a demand matrix \mathbf{D} with $|N|$ rows and columns, mapping the demand for travel from every node to every other node; a set of demand realizations Ω_1 with probability distribution P_1 ; and a set of roadway supply realizations Ω_2 with probability distribution P_2 .

Each demand realization corresponds to a possible value of the demand matrix \mathbf{D} . Each arc (i,j) can exist in one or more states S_{ij} (different states may represent, for instance, “typical conditions,” “mild incident”, “severe incident”, “thunderstorm”, and so on) with a corresponding delay function $t_{ij}^s(x_{ij})$ mapping the demand for travel x_{ij} on this link to its delay in state $s \in S_{ij}$. The supply realizations $\omega_2 \in \Omega_2$ thus associate each arc with one of its possible states. The supply and demand realizations may be either dependent or independent, and in this respect are only limited by the ability to generate or sample appropriately correlated realizations.

Drivers are rational self-optimizers, whose perception of travel times, the demand realization Ω_1 , and the distribution of supply realizations (Ω_2, P_2) are accurate, and who seek to minimize their generalized travel cost, defined as the sum of their travel time and any (nonnegative) arc tolls τ_{ij} levied by the network manager, assumed to be measured in time units for notational convenience. More precise definitions of user and network manager behavior depend on the specific information scenarios, discussed in the next subsection. The network manager, in turn, sets prices so as to maximize social welfare. This represents the perspective of a public agency, or a private entity whose contractual arrangement also favors the public welfare.

Information Scenarios

Two types of agent exist in this model: the network manager, responsible for setting tolls; and the drivers, who choose routes such that no individual can improve his or her travel time by switching routes. We consider three possibilities for the information the network manager has when making their decisions, and two possibilities for the information drivers have, leading to a total of six information scenarios. Each of these corresponds to a particular pricing strategy or

framework: if the network manager is unable to vary tolls flexibly in response to the demand or supply realizations (either for technological, legal, or contractual reasons), the prices are the same each day, exactly as if the network manager had no access to information on day-to-day conditions. For drivers, access to ATIS provides information on the supply realization on a given day, whereas if such information is not provided, the travel decision will be the same on each day (simply because there is no reason for it to change in our static equilibrium framework).

More specifically, for the network manager, we assume that they must either levy completely unresponsive tolls (UT); that they may vary tolls in response to the demand realization, but not the supply realization (DRT) (long-term flexibility, but short-term constraints), or that they have the ability to use fully responsive tolls (FRT) to adapt to the demand and supply realizations. In the UT case, the same toll must be levied regardless of the demand and supply realizations ω_1 and ω_2 , perhaps representing a contractual agreement or legal framework in which tolls must remain fixed over an extended period. In the DRT case, tolls can depend on the demand realization, and are expressed $\tau_{ij}(\omega_1)$. In the FRT case, the tolls do in fact depend on both of these; this is denoted $\tau_{ij}(\omega_1, \omega_2)$.

For drivers, we assume that they either have no information on travel conditions (NI) before choosing routes, or that they are fully informed (FI). In the NI case, travelers minimize the *expected* travel cost, based on expected travel times and the expected tolls. That is, the state-dependent delay functions $t_{ij}^s(x_{ij})$ are replaced with a single delay function $t_{ij}(x_{ij}) = E_{\omega_2} [t_{ij}^s(x_{ij})]$ representing the expectation over all supply realizations, with any state-dependent tolls handled similarly. Therefore, the link flows do not depend on the supply realization, and are simply denoted $x_{ij}(\omega_1)$. In the FI case, travelers know the supply realization ω_2 and the current value of the toll vector τ exactly before choosing their routes; their route choices thus form a user equilibrium with respect to the cost functions corresponding to link states ω_2 and tolls τ , and the link flows are state-dependent and denoted $x_{ij}(\omega_1, \omega_2)$. In all cases, drivers are aware of the demand realization ω_1 , as this is learned through experience in the long run. Unlike the network

manager, a scenario in which drivers are ignorant of both the demand and supply realization is inconsistent with the assumptions of experienced drivers.¹

The six information scenarios corresponding to possible combinations of NI, FI, UT, DRT, and FRT are now discussed in more detail. Figure 1 provides a graphical interpretation of the sequencing of the four events which define these scenarios. Although there are twenty-four permutations of these events, the demand realization always precedes the supply realization (as the former is long-term, and the latter short-term), and the demand realization always precedes route choice (as drivers are always aware of the demand matrix). Furthermore, whenever the network manager and drivers have the same information available to them, the network manager “moves first” without loss of generality due to the equilibrium assumption. These precedence relations limit the set of allowable scenarios to six. As it turns out, one of these is a special case of another, leading to a total of five distinct scenarios, as demonstrated in the following section.

¹ The case of short-term demand uncertainty would admit drivers uninformed of the demand matrix, as in the equilibrium models Clark and Watling, 2005 or Unnikrishnan et al., 2009. Although an interesting variation, this requires a fundamentally different equilibrium concept which is beyond the scope of this paper.

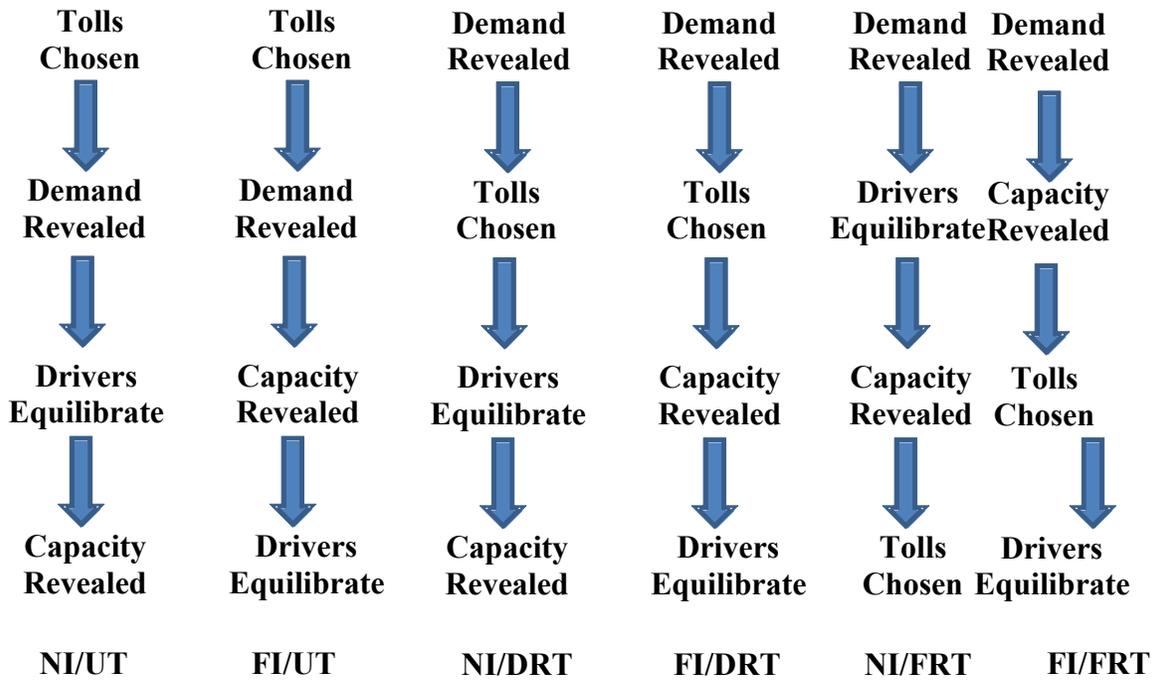


Figure 1. Six different information scenarios

Evaluation Criteria

A major issue that arises with privately managed transportation projects results from the conflicting objectives of private and public entities. Generally a private firm is interested in maximizing profit, while a public agency strives to maximize public welfare. These two objectives often lead to different project design parameters (such as toll levels and capacity). With this in mind this paper evaluates vehicular delay, measured as total system travel time ($TSTT$), and calculate the corresponding expected revenue generated, denoted as $E[TSTT]$ and $E[R]$ respectively. An additional criterion results from the stochastic nature of this work wherein the variability of system performance is also of interest. Under conditions of uncertainty “optimal” tolls should be robust across changes in demand and supply levels. Therefore the standard deviations of the $E[TSTT]$ and $E[R]$ are also calculated and compared for the different scenarios, denoted as $\sigma[TSTT]$ and $\sigma[R]$ respectively. Collectively, these four metrics are the measures of effectiveness (MOEs) for these models. Computation times are also noted for the different cases.

SOLUTION METHODS

This section presents methods to determine tolls and link flows for each of the six scenarios described above. These methods bear some resemblance to those of Gardner *et al.* (2010) and Boyles *et al.* (2010), where demand and supply uncertainty was considered separately. However, considering both sources of uncertainty together introduces additional complexity: solution methods must be developed for five scenarios, rather than three as in Boyles *et al.* (2010); and the proliferation of realizations (up to $|\Omega_1||\Omega_2|$) demands the use of efficient sampling techniques rather than complete enumeration even in medium-sized networks.

No Information/Unresponsive Tolls (NI/UT)

For this case, we seek a single vector of tolls τ independent of the supply and demand realization, and a collection of flow vectors $\mathbf{x}(\omega)$ which depend on the demand realization but not the supply realization. If the number of demand realizations is large or infinite, it one can estimate the four MOEs using sampling techniques; a comparison of sampling techniques for transportation networks with uncertain demand can be found in Duthie *et al.* (2009) We seek a solution to the mathematical program

$$\begin{aligned} \min_{\tau} \quad & \sum_{(i,j)} \iint x_{ij}(\omega_1) t_{ij}^{\omega_2}(x_{ij}(\omega_1)) d\omega_2 d\omega_1 \\ \text{s.t.} \quad & \mathbf{x}(\omega_1) \in \text{Eq}(\omega_1, E_{\omega_2}[\mathbf{t}^{\omega_2}] + \tau) \quad \forall \omega_1 \end{aligned}$$

where $\text{Eq}(\omega_1, E_{\omega_2}[\mathbf{t}^{\omega_2}] + \tau)$ represents the set of equilibrium link flows given demand realization ω_1 and cost functions $E_{\omega_2}[\mathbf{t}^{\omega_2}] + \tau$. Since the tolls cannot depend on the demand realization, this is a nonseparable mathematical program with equilibrium constraints (MPEC) which is difficult to solve exactly. Thus, heuristics are needed.

The most direct method is to use a single point estimate \mathbf{D}^0 of demand, and determine the marginal cost tolls when the demand is \mathbf{D}^0 and the cost functions are $E_{\omega_2}[\mathbf{t}^{\omega_2}]$. A simple point estimate is $\mathbf{D}^0 = kE_{\omega_1}[\mathbf{D}]$ for some constant k . For instance, using $k = 1$ would set prices according to the mean demand, while $k > 1$ or $k < 1$ would set prices according to an inflated or deflated demand value, respectively. Previous research by Gardner *et al.* (2010) found that deflating the mean demand by 20% resulted in the most robust tolls under demand uncertainty

when roadway capacity was deterministic. In the following section, different k values are explored.

Full Information/Unresponsive Tolls (FI/UT)

For this case, we seek a single vector of tolls $\boldsymbol{\tau}$ independent of the supply and demand realization, and a collection of flow vectors $\mathbf{x}(\boldsymbol{\omega}_1, \boldsymbol{\omega}_2)$ which depend on both the supply and the demand realization. That is, we seek a solution to

$$\begin{aligned} \min_{\boldsymbol{\tau}} \quad & \sum_{(i,j)} \iint x_{ij}(\boldsymbol{\omega}_1, \boldsymbol{\omega}_2) t_{ij}^{\omega_2}(x_{ij}(\boldsymbol{\omega}_1, \boldsymbol{\omega}_2)) d\boldsymbol{\omega}_2 d\boldsymbol{\omega}_1 \\ \text{s.t.} \quad & \mathbf{x}(\boldsymbol{\omega}_1, \boldsymbol{\omega}_2) \in \text{Eq}(\boldsymbol{\omega}_1, \mathbf{t}^{\omega_2} + \boldsymbol{\tau}) \quad \forall \boldsymbol{\omega}_1, \boldsymbol{\omega}_2 \end{aligned}$$

As with the NI/UT case, this is a nonlinear MPEC which, in general, cannot be solved to optimality. Simple averaging is a tractable heuristic which performs well in practice. For each demand and supply realization (or a sample), the set of first-best tolls is determined, and the arithmetic average of these is used as a solution to the FI/UT problem. That is, multiple instances of FI/RT (described in the following subsection) are solved and averaged. With deterministic demand, and in a two-link, single OD network, Lindsey (2009) showed that this method is indeed exact. In more complicated networks with multiple OD pairs, Boyles *et al.* (2010) showed that this method still produces high-quality solutions.

No Information/Demand Responsive Tolls (NI/DRT)

For this case, we seek toll vectors $\boldsymbol{\tau}(\boldsymbol{\omega}_1)$ and flow vectors $\mathbf{x}(\boldsymbol{\omega}_1)$ which depend only on the demand realization, not the supply realization. In this case, the problem decomposes by demand realization, that is, we seek solutions to

$$\begin{aligned} \min_{\boldsymbol{\tau}(\boldsymbol{\omega}_1)} \quad & \sum_{(i,j)} \iint x_{ij}(\boldsymbol{\omega}_1) t_{ij}^{\omega_2}(x_{ij}(\boldsymbol{\omega}_1)) d\boldsymbol{\omega}_2 d\boldsymbol{\omega}_1 \\ \text{s.t.} \quad & \mathbf{x}(\boldsymbol{\omega}_1) \in \text{Eq}(\boldsymbol{\omega}_1, E_{\omega_2}[\mathbf{t}^{\omega_2}] + \boldsymbol{\tau}(\boldsymbol{\omega}_1)) \quad \forall \boldsymbol{\omega}_1 \end{aligned}$$

This is the well-known deterministic first-best toll setting problem with the demand table corresponding to $\boldsymbol{\omega}_1$ and the cost functions $E_{\omega_2}[\mathbf{t}^{\omega_2}]$, which can be solved by setting tolls equal to the marginal link cost multiplied by the link's flow.

Full Information/Demand Responsive Tolls (FI/DRT)

In this case, the tolls depend only on the demand realization, but the flows depend on both the supply and demand realizations, that is, we seek vectors $\boldsymbol{\tau}(\omega_1)$ and $\mathbf{x}(\omega_1, \omega_2)$ which solve

$$\begin{aligned} \min_{\boldsymbol{\tau}(\omega_1)} \quad & \sum_{(i,j)} \iint x_{ij}(\omega_1, \omega_2) t_{ij}^{\omega_2}(x_{ij}(\omega_1, \omega_2)) d\omega_2 d\omega_1 \\ \text{s.t.} \quad & \mathbf{x}(\omega_1, \omega_2) \in \text{Eq}(\omega_1, \mathbf{t}^{\omega_2} + \boldsymbol{\tau}(\omega_1)) \quad \forall \omega_1, \omega_2 \end{aligned}$$

As with the NI/DRT scenario, the problem decomposes by demand realization; and as in the FI/UT scenario, the optimal tolls for each demand realization can be approximated through sampling supply realizations and averaging.

No Information/Fully Responsive Tolls (NI/FRT)

If drivers receive no information on the supply realization, neither is there any value in varying the toll accordingly, as drivers cannot adapt their route choice to the toll on that day. Thus, the link flows will be constant from day-to-day regardless of any toll variation attempting to account for supply uncertainty, and this scenario is in fact a special case of NI/DRT and can be solved as such.

Full Information/Fully Responsive Tolls (FI/FRT)

This case is the simplest to solve. We seek a collection of toll and flow vectors $\mathbf{t}^{\omega_2}(\omega_1)$ and $\mathbf{x}^{\omega_2}(\omega_1)$ which depend on both the supply and demand realizations, that is,

$$\begin{aligned} \min_{\boldsymbol{\tau}^{\omega_2}(\omega_1)} \quad & \sum_{(i,j)} \iint x_{ij}^{\omega_2}(\omega_1) t_{ij}^{\omega_2}(x_{ij}^{\omega_2}(\omega_1)) d\omega_2 d\omega_1 \\ \text{s.t.} \quad & \mathbf{x}^{\omega_2}(\omega_1) \in \text{Eq}(\omega_1, \mathbf{t}^{\omega_2} + \boldsymbol{\tau}^{\omega_2}(\omega_1)) \end{aligned}$$

In this case, we simply solve a deterministic first-best pricing problem for each network realization (ω_1, ω_2) and calculate the four MOEs.

Untolled Scenarios (NI/No Toll; FI/No Toll)

As an additional basis for comparison the user equilibrium assignment solution without tolls is computed for two cases, without information (NI/No Toll) and with full information (FI/No Toll). When pricing a deterministic network to maximize social welfare, the difference

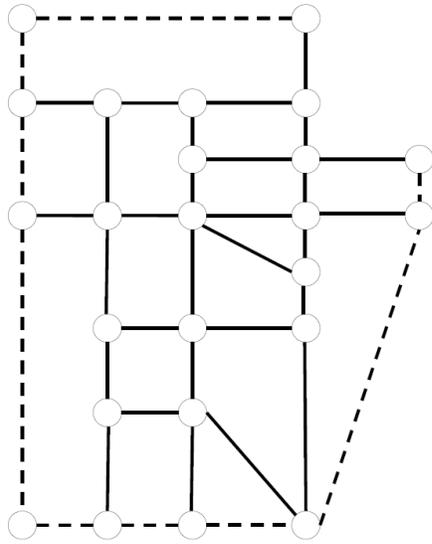
between $TSTT$ in the user equilibrium and system optimal objective values without tolls provides an upper bound on the improvement in system performance which tolling can produce. In this paper the network is stochastic, and the new objective value is an expected system performance. The expected system performance for the NI/No Toll and FI/No Toll scenarios can be used in a similar manner. The larger the difference in $E[TSTT]$ for untolled (NI/No Toll or FI/No Toll) and the tolled scenarios, the closer the tolls are to bringing the network to a system optimal state, and thus more optimal. Comparing the evaluation criteria without tolls to the five information/tolling scenarios provides valuable information on the role of information and dynamic pricing on network system performance and revenue generated under conditions of uncertainty.

DEMONSTRATIONS

Experiment Design

The network used in the analysis of this work is the well-known Sioux Falls network, and can be seen in Figure 2. Network data, including travel demand, are obtained from Bar-Gera (2009). This network contains 24 nodes (all of which are origins and destinations) and 76 links. Demand is assumed to be normally distributed, with a specified coefficient of variation; demand is also truncated at zero to ensure non-negativity. Arterial arcs are assumed to exist in only one possible state, while freeway arcs exist in one of two states (“no incident” and “incident present”). Travel times are given by the well-known Bureau of Public Roads cost function, with shape parameters 0.15 and 4; capacity during an incident is a fixed proportion of the “no incident” capacity. This proportion is varied parametrically in the analysis that follows. Supply and demand realizations are assumed independent.

For each of the scenarios described above (five information/tolling and two un-tolled) we compute the four MOEs. By adjusting various parameters such as the number of realizations, planning demand, level of demand uncertainty, level of supply uncertainty, and incident severity, we can gain insights into the impact of uncertainty in tolled transportation networks when information and responsive tolling are/are not implemented. A list of the parameters and the values used in the base case are shown in Figure 2.



Parameter values for Base Case	
Realizations drawn for each scenario	100
Planning Demand Multiplier	1
Demand Coefficient of Variation	0.4
Value of travel time (\$/hr)	10
Freeway Incident probability	0.1
Freeway Incident Capacity Multiplier	0.333

Figure 2. Sioux Falls network and base parameter values (dashed lines represent freeway links)

The *realizations drawn for each scenario* parameter specifies the number of demand realizations sampled from a normal distribution using antithetic sampling. Multiple realizations are necessary to evaluate expected system behavior when conditions are uncertain. The *planning demand multiplier* is multiplied by the expected demand to determine the planning demand value used to compute prices for the NI/UT scenario. (that is, the *k*-value) This parameter is disregarded in the remaining three scenarios where the tolls are set using the true demand. The *demand coefficient of variation* gives the ratio of the standard deviation and mean value for demand. In the base case it is set at 0.4, or 40% of the mean demand. The *value of travel time* remains constant at \$10/hr for all evaluations. The *freeway incident probability* specifies the probability that an incident occurs independently on each link in the network, and set at 0.1 in the base case. Finally the *freeway incident capacity multiplier* specifies the percentage of link capacity that remains if an incident does occur. In the base case this is set to .333, meaning 2/3 of the capacity is removed from a link if an incident does occur. The effect of varying these parameters is discussed in the remainder of this section.

The base case scenarios and computation time are also evaluated for the Anaheim planning network, containing 416 nodes (38 of which are origins and destinations) and 914 links. Sensitivity analysis with all parameters is omitted for this network for reasons of brevity.

Sample Size

The base case was evaluated with the number of realizations ranging from 10 to 1000. The system results stabilized at near 100 realizations for supply and demand each, a total of 10,000 realizations. for all five scenarios and four MOEs. This suggests that 10,000 realizations provide a reasonable tradeoff between solution stability and computational effort, at least in this network. This number of realizations was used in all of the following experiments.

Planning demand

As previously stated, the planning demand parameter k only arises in the NI/UT case where it is used to determine the tolls. We evaluated each of the scenarios for planning demands varying between 50% and 150% of the expected demand value and found the results to be robust across all planning demands. Variations did occur (for the NI/UT scenario only) among the computational times, which increased for lower planning demands, whereas the expected value and standard deviation of the revenue increased for higher planning demands. Because there was not sufficient evidence to use a planning demand other than the expected demand, k is set to 1 in all of the scenario evaluations.

Level of Uncertainty

As stated previously one of the main focuses of this work is the impact on network performance when both the supply and demand are random variables. As part of the evaluation we will consider various cases such as isolated demand uncertainty, isolated supply uncertainty, and when both demand and supply may vary. The *demand coefficient of variation* and *freeway incident probability* are both evaluated within the range of 0 to 0.9. When the *demand variance* is 0 the demand is deterministic, when the *freeway incident probability* is 0 no incidents will occur, therefore the supply is deterministic. Each of the five information/tolling combination scenarios is evaluated for each level of uncertainty to evaluate the role of information and responsive tolling under varying stochastic conditions.

Isolated Demand/Supply Uncertainty

For the isolated cases of uncertainty where only demand is uncertain or only supply is uncertain, all the criteria evaluated are higher under conditions of supply uncertainty relative to

demand uncertainty for all information/tolling scenarios. This suggests that when the two types of uncertainty occur in isolation, supply uncertainty has a greater impact on network performance than demand uncertainty, resulting in increased travel times on average. In addition both cases of isolated uncertainty result in higher $E[TSTT]$ and $E[Revenue]$ than the fully deterministic case, suggesting that the fully deterministic case underestimates system performance and expected revenue.

Role of Demand Uncertainty

To evaluate the role of demand uncertainty on a network where supply is also stochastic, we fix the *freeway incident probability*, (set to the base case, 10% likelihood of an incident occurring per link), and vary the *demand coefficient of variation*, while the remaining parameters remain constant at their base case values. The results are summarized in Figure 3, where the x-axis is the *demand coefficient of variation*, and the y-axis is in units of either *TSTT* (min) or *revenue* (dollars). Each graph represents one of the MOEs. Legends are not included in the remaining Figures (4,5 and 6), but the reader can refer to that in Figure 3.

From the $E[TSTT]$ results there are a few conclusions to draw. Most obvious is the benefit associated with providing information. There is a significant difference between the four scenarios where information is provided and the remaining three where information is not provided. This information-based difference also appears in the $\sigma[TSTT]$, where the three cases with “full information” have very similar $\sigma[TSTT]$ values which are all lower than the “no information” scenarios. The $E[TSTT]$ is highest for the NI/NO TOLL case, followed by the NI/UT and NI/DRT, demonstrating that the tolls (even under stochastic conditions) improve system performance over the un-tolled state. The fact that it is lower under NI/DRT than under NI/UT is expected as well, since the responsive tolling has the additional benefit of determining tolls based on the actual demand. Again for the case of “full information”, all scenarios where tolls are applied, FI/UT, FI/DRT and FI/FRT improve the system performance over the no-tolls scenario, FI/NO TOLL. Additionally the FI/FRT case has the lowest $E[TSTT]$, again suggesting responsive tolling offers improvement over unresponsive tolling. There is negligible improvement from the FI/UT to the FI/DRT scenario, which suggests it is not beneficial to set tolls responsive to only demand levels when full information is provided to the users. And

surprisingly, the improvement from FI/UT to the FI/DRT scenario decreases as the level of demand uncertainty increases.

The revenue results do not display quite the same consistent behavior as the system performance. Again there is a disparity between the cases of “no information” and “full information”. The two “no information” scenarios resulted in higher expected revenues than the “full information” scenarios. And for each information scenario, the responsive tolling resulted in higher $E[R]$. Independent of information, the two responsive scenarios resulted in higher $\sigma[R]$. This is possibly because in the responsive tolling scenarios fluctuations in both link flows and toll levels occur, where as in the unresponsive tolling, the tolls do not change once they are set, only the flows vary due to changes in the demand. FI/DRT has the lowest $E[R]$, performing worse than the UT cases.

With respect to demand uncertainty, both the $E[TSTT]$ and $E[R]$ vary minimally as the level of uncertainty increases up until the *demand coefficient of variation* reaches 0.4. Once the parameter surpasses 0.4 the $E[TSTT]$ and $E[R]$ increase quickly. In particular the $E[R]$ for the NI/DRT case increases and diverges quickly from the other scenarios. The reasoning for such behavior is that a higher variance results in random demands realizations that greatly exceed the expected demand value, which result in high levels of congestion, and significantly higher travel times, regardless of the tolls that are set. In addition, the expected revenue will increase because the tolls will be higher due to the increased marginal costs which reflect this overloading of the network. The $\sigma[TSTT]$ increases more consistently with the level of uncertainty, while the $\sigma[R]$ increases at varying rates dependent on the scenario.

Role of Supply Uncertainty

Similar to above, the role of supply side uncertainty is explored independent of demand uncertainty by varying the *freeway incident probability* and fixing the *demand coefficient of variation* at 0.4. Again the remaining parameters remain constant at their base case values. The results for each of the scenarios and each of the criteria evaluated are provided in Figure 4.

The results for the $E[TSTT]$ and $E[R]$ resemble the results from those of demand uncertainty with regards to the relative order of the information/tolling scenarios, with the exception that for higher levels of supply uncertainty the FI/DRT scenario actually performs slightly worse than the FI/UT scenario (with a higher $E[TSTT]$), and the $E[R]$ behaves more

radically. The same reasoning applies here as well with regards to the benefits of information and responsive tolling. Additionally, as supply uncertainty increases both $E[TSSTT]$ and $E[R]$ increase, and all scenarios appear to converge in value, while in the case of demand uncertainty; the different scenarios diverged. The “convergence” may be a result of drivers choosing to travel on more arterial links which are deterministic, and the “convergence” occurs under the deterministic arterial-only situation. In contrast there is no “deterministic” alternative for uncertain demand, and increased levels of uncertainty will only results in more extreme behaviors. As supply uncertainty increases there is also a more consistent increase in $E[TSSTT]$ and $E[R]$, contrasting the stable behavior we saw with low levels of demand uncertainty. Under supply uncertainty the $\sigma[TSSTT]$ behaves much more sporadically than in the case of demand uncertainty, especially for the “no information” cases, and is on average much lower. These “no information” scenarios result in significantly higher $\sigma[TSSTT]$ values at lower levels of uncertainty, but seem to stabilize after the probability of an incident exceeds 0.3. The “full information” scenarios result in very similar $\sigma[TSSTT]$ values across all levels of uncertainty, which increase until the probability of an incident reaches 0.5, at which point the $\sigma[TSSTT]$ actually begins decreasing. This may be due to similar reasoning as before, where more people travel on the deterministic arterials, resulting in less variation in flows, and therefore less variation in revenue. With the exception of the NI/RT case the $\sigma[R]$ is relatively stable and similar for all scenarios. As in the case of demand uncertainty the NI/DRT case results in a much higher $\sigma[R]$, again likely a result of changes in both flow patterns and tolls.

Demand and Supply Uncertainty

The following analysis is useful for gaining insight into more realistic network behavior, when both the supply and demand levels vary. In this evaluation the parameters are increased proportionally from the deterministic case (0 for both parameters) to 1 for *demand coefficient of variation* and 0.6 the *freeway incident probability* (moving right along the x-axis), and the results are shown in Figure 5. The results are consistent with those we have seen previously in the individual uncertainty cases. The same disparity exists between “information” and “no information” cases, as well as the relative ranking between the various scenarios in the expected behavior. The $E[TSSTT]$ and $E[R]$ are higher when both supply and demand are variable, compared with the two previous uncertainty studies, as well as the deterministic case. The main

difference in the case of full uncertainty is in the standard deviations. The $\sigma[TSTT]$ for the “no information” cases are again much higher than the “full information” cases for lower levels of uncertainty. However after the level of uncertainty increases beyond a certain point, the $\sigma[TSTT]$ for all seven cases converges to the same value and continue to increase almost linearly. The $\sigma[R]$ behaves the same as it does for when just the demand uncertainty level varies.

Severity of Incident

Another parameter that is of interest is the *freeway incident capacity multiplier*. This parameter dictates the amount of link capacity that remains if an incident occurs, and is varied between 0 and 0.8 to explore different levels of incident severity on network performance. In this analysis the uncertainty levels remain constant and the additional parameters remain fixed at their base case values. The results can be seen in Figure 6. The x-axis represents the percentage of link that is closed if an incident occurs, so the level of severity increases as you move right along the x-axis. The y-axis’ are same as before. The case for entire link closure was also considered however the results are not shown in the Figure because they are beyond the scale of the graph.

From the results it is clear that increased severity levels negatively affect network performance, as expected. Additionally, as the severity increases, or percentage of link closure exceeds half, the role of information becomes more vital, and the $E[TSTT]$ diverges quickly for all the “no information” scenarios. This implies that information will be more beneficial where incidents are more severe rather than more likely to occur. For similar reasoning, where incidents are minor and result in less than 40% of the link to be closed, the role of information, as well as responsive tolling appears almost negligible. The same divergent behavior between “full information” and “no information” scenarios is apparent in the $\sigma[TSTT]$. The $E[R]$ is again higher for the “no information” cases, and the $\sigma[R]$ is much higher for cases of responsive tolling. This is likely an exaggerated case of what we saw previously with responsive tolling; the flows will be changing dramatically in cases of severe incidents, on top of the varying tolls, further increasing the variability of network performance.

Base Case Analysis

Tables 1 and 2 summarize results of the four MOEs for the scenarios considered, all evaluated using the base case parameters. The results reveal expected network behavior under moderate levels of both demand and supply uncertainty, and provide a basis for comparative analysis regarding the value of information and responsive tolling. The values in the last column quantify the expected improvement in system performance when information is provided, responsive tolling is implemented, or both. The values are the percentage decrease in $E[TSST]$ for each scenario relative to the “no information” un-tolled case (NI/No Toll). It is clear that the maximum system improvement occurs by providing both information and fully responsive tolling (FI/FRT), where the expected total system cost is decreased by 11.15%, as opposed to only 2.11% when no information or responsive tolling is available (NI/UT). The *benefit of providing information* is captured by the difference in expected system performance between the NI/UT and FI/UT scenarios, equal to 8.37% for the base case. Similarly the *benefit of responsive tolling* is represented by the difference in expected system performance between the FI/UT and FI/FRT scenarios, which is only 0.67% for the base case. These results indicate that the impact of providing information is significantly higher than the impact of responsive tolling for the Sioux Falls network used here. Additionally, demand responsive tolling just marginally improves expected system performance over unresponsive tolling, indicating responsive tolling should account for both supply and demand realizations..

Responsive tolling does however have a large impact on the expected revenue under “no information” (NI/DRT), where the revenue is the highest of the five pricing scenarios. In the FI/UT scenario the expected revenue is increased as well, more than twice that of the NI/UT scenario, and the total system cost and both standard deviations are lower, suggesting information better serves both objectives previously mentioned, improving social welfare and increasing profit.

Table 1. Comparison of MOE's for all scenarios for Sioux Falls network, base case

Scenario	$E[TSTT]$ (min)	$\sigma[TSTT]$ (min)	$E[R]$ (dollars)	$\sigma[R]$ (dollars)	Decrease in $E[TSTT]$ vs. NI/No Toll
NI/No Toll	8538139	838426	0	0	–
FI/No Toll	7846987	493686	0	0	8.09%
NI/UT	8357858	769090	1277977	19739	2.11%
NI/DRT	8347075	757485	3177593	239055	2.24%
FI/UT	7643559	515138	2650235	41070	10.48%
FI/DRT	7631709	512605	2442940	39705	10.62%
FI/FRT	7586454	503678	2670099	305063	11.15%

Table 2. Comparison of MOE's for all scenarios for Anaheim network, base case

Scenario	$E[TSTT]$ (min)	$\sigma[TSTT]$ (min)	$E[R]$ (dollars)	$\sigma[R]$ (dollars)	Decrease in $E[TSTT]$ vs. NI/No Toll
NI/No Toll	2120646	796254	0	0	–
FI/No Toll	2032213	785619	0	0	4.17%
NI/UT	2079640	790124	238937	7986	1.93%
NI/DRT	2076945	788354	597226	100750	2.06%
FI/UT	2008060	785263	516560	17543	5.31%
FI/DRT	2013593	786220	1147137	70154	5.05%
FI/FRT	1997288	785049	520572	520123	5.82%

Computation Time

All models were solved on a 2.83 GHz, dual quad-core Q9550 machine running Windows XP with 3.25 GB memory. When needed, user-equilibrium traffic assignment was solved using the Frank-Wolfe algorithm to a relative gap of 10^{-4} . Although a relative gap of 10^{-5} is recommended in Boyce et al. (2004), our judgment is that evaluating additional samples to a looser relative gap is preferred to evaluating fewer realizations to a tighter gap. Computation times for both the Sioux Falls network and the Anaheim network are reported in Table 3, in terms of the average time needed for each sample. The total computation time for a given analysis can be approximated by multiplying the figures in this table by the number of samples required (100 in the results presented in this section).

Note that these computation times include *both* the time needed to find the optimal tolls, and to evaluate their performance, both of which involve repeated use of antithetic sampling.

Table 3. Computation time needed for samples different scenarios, in seconds

Scenario	Sioux Falls	Anaheim
NI/No Tolls	1.09	1.11
FI/No Tolls	0.49	0.55
NI/UT	0.71	1.18
NI/DRT	2.17	3.80
FI/UT	1.41	2.12
FI/DRT	1.51	2.36
FI/FRT	1.22	1.88

CONCLUSION

This report presented a modeling framework for representing uncertainty in long-term travel demand, and in day-to-day network capacity, in the context of pricing problems. By considering five different scenarios on user information and flexibility in pricing, the value of these options can be quantified for a given network, providing guidance both to public agencies and private entities in developing toll policies and in influencing legal structures regarding toll regulation.

Tractable solution methods were presented for all of these information scenarios; when tolls can be varied flexibly, they are exact. When tolls must be independent of the demand and supply realization, heuristics were provided. These methods were all applied to the well-known Sioux Falls network, to demonstrate the relative benefits of providing information and/or responsive tolling under conditions of uncertainty. These methods were also implemented on the Anaheim network, and comparable results were obtained. Although both improve the system performance, in this experiment information was more valuable than responsive tolling. Clearly this result is highly dependent on the network and modeling parameters, but serves to illustrate the type of benefit quantification which can guide policy implementation alongside a cost analysis. Again, for Sioux Falls, providing information alone with unresponsive tolls seemed to provide the most robust results, with both lower system cost and increased revenue over the no-information scenario, and with relatively low variance. Finally, it is clear that both supply and demand uncertainty should be considered simultaneously, as congestion levels are underestimated when they are evaluated independently, or not at all.

It would be both interesting and useful to extend this analysis to other scenarios such as limited information, imperfect information, or costly information for travelers; constraints on the links which can be tolled (or on the maximum toll); or when users can switch routes due to information received while traveling. Another significant limitation of all of the models discussed thus far is their static nature. While introducing a host of methodological and computational difficulties, a dynamic model of congestion would improve the fidelity of these models greatly.

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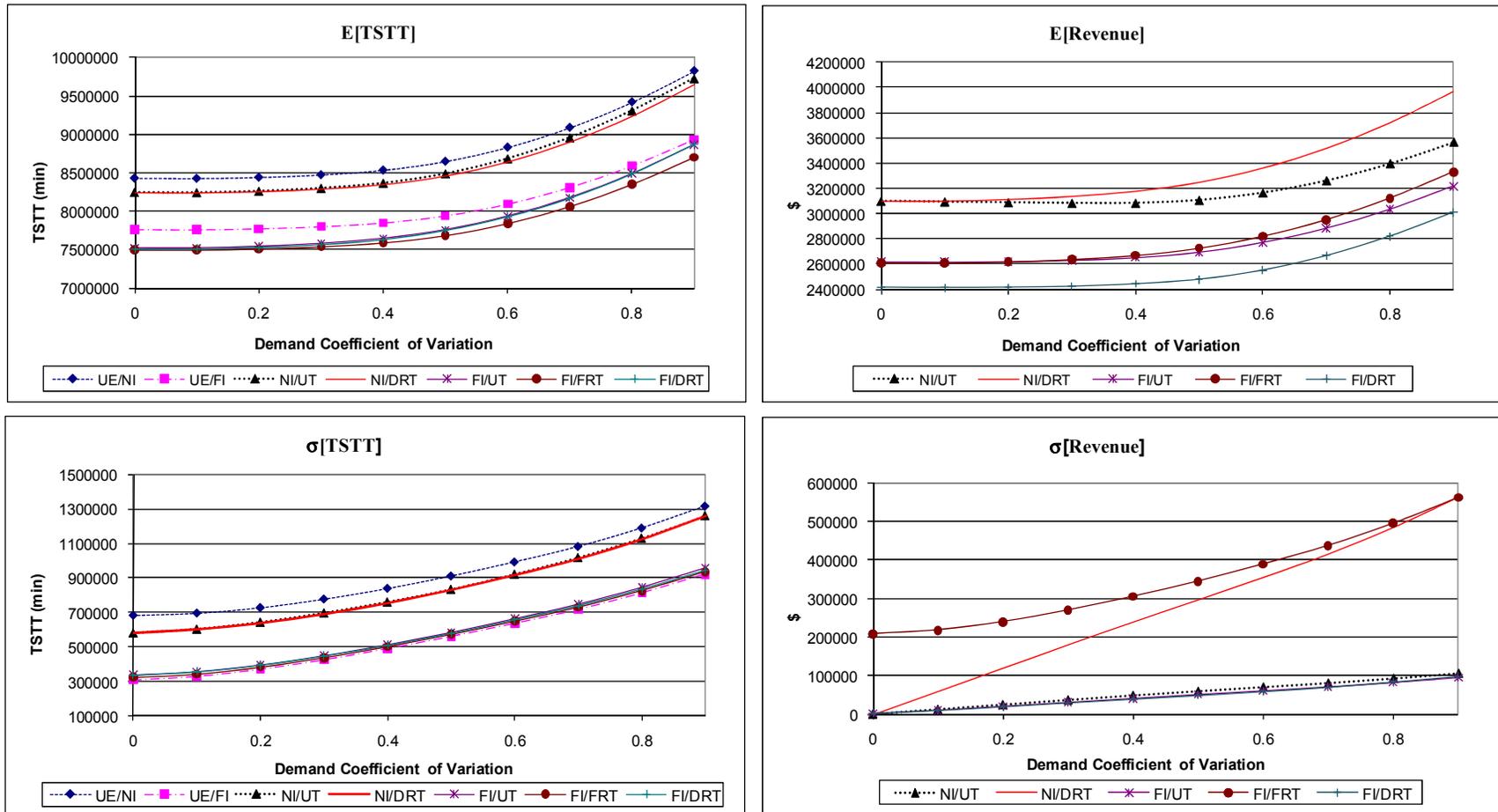


Figure 3. Impact of Demand Uncertainty on System Performance and Revenue

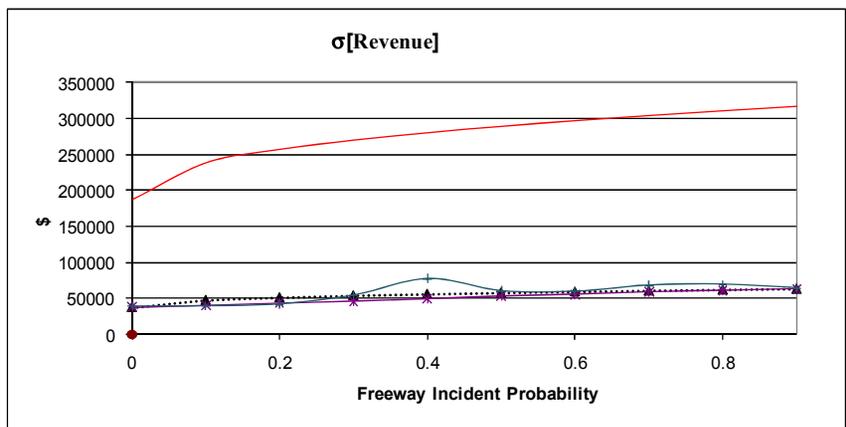
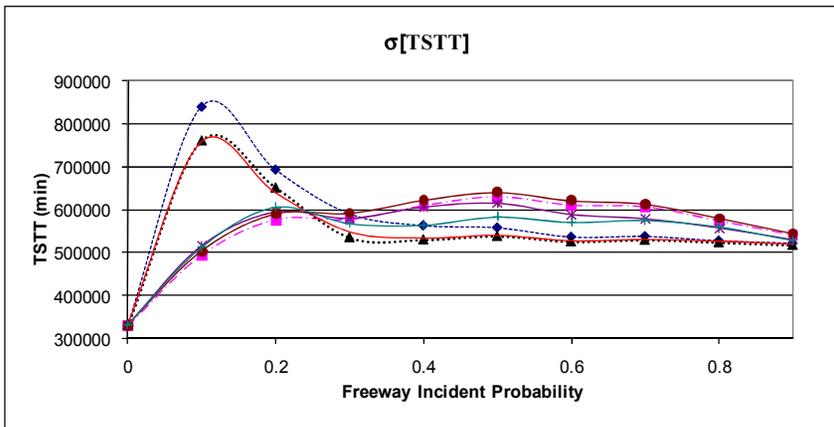
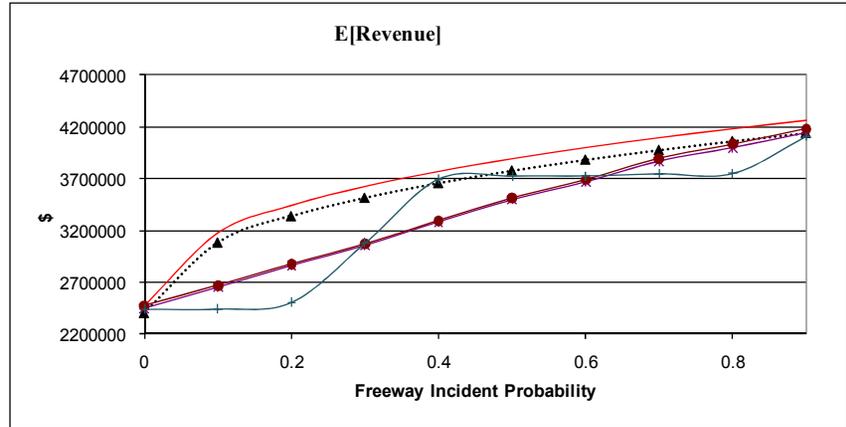
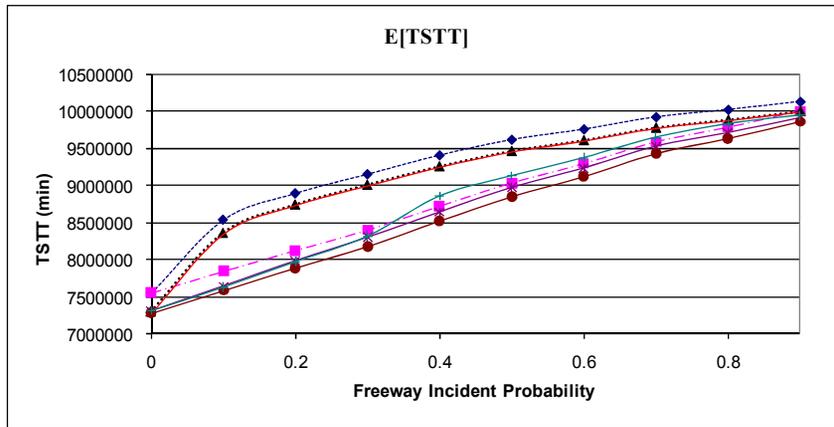


Figure 4. Impact of Supply Uncertainty on System Performance and Revenue

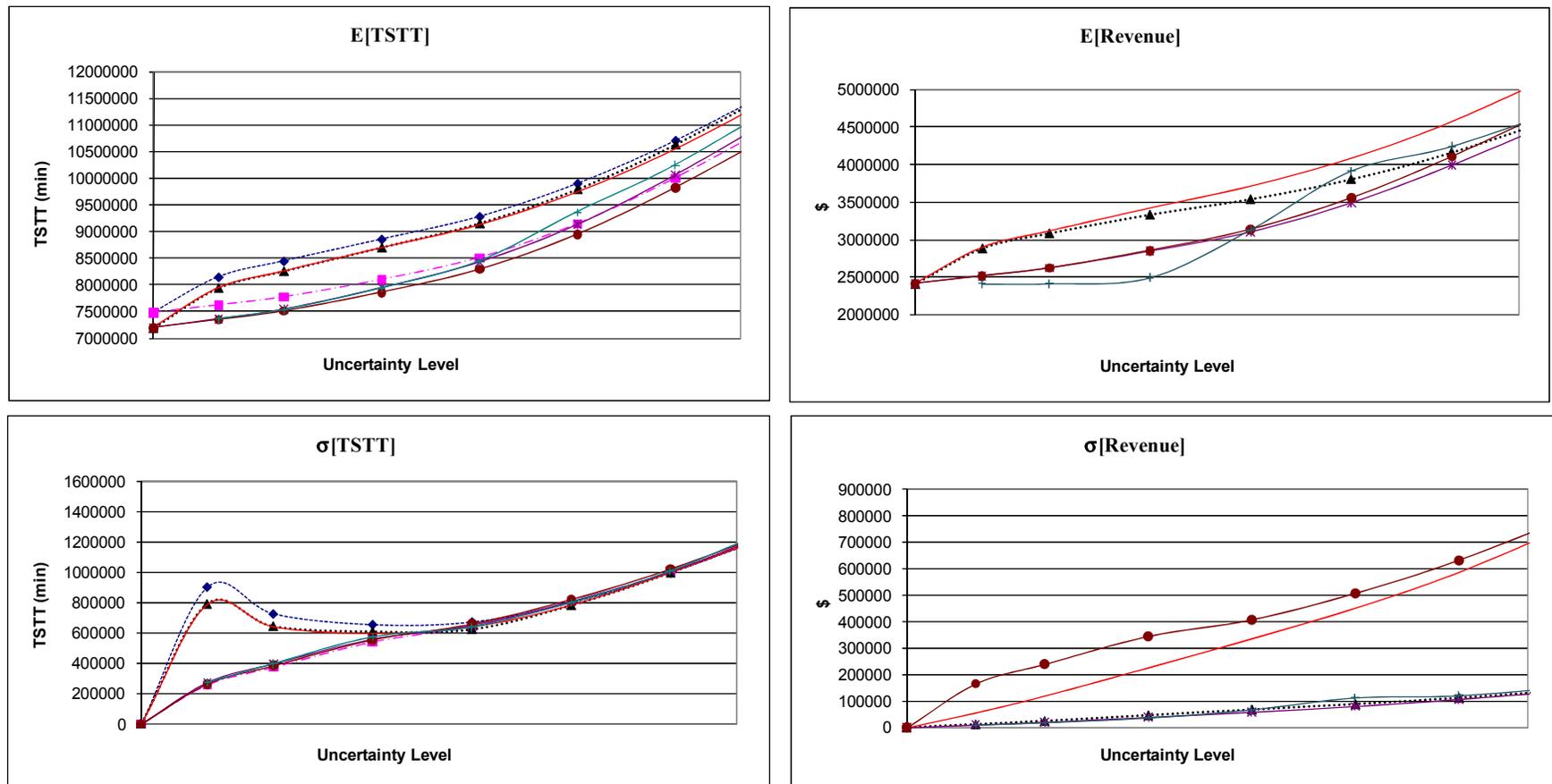


Figure 5. Impact of Supply and Demand Uncertainty on System Performance and Revenue

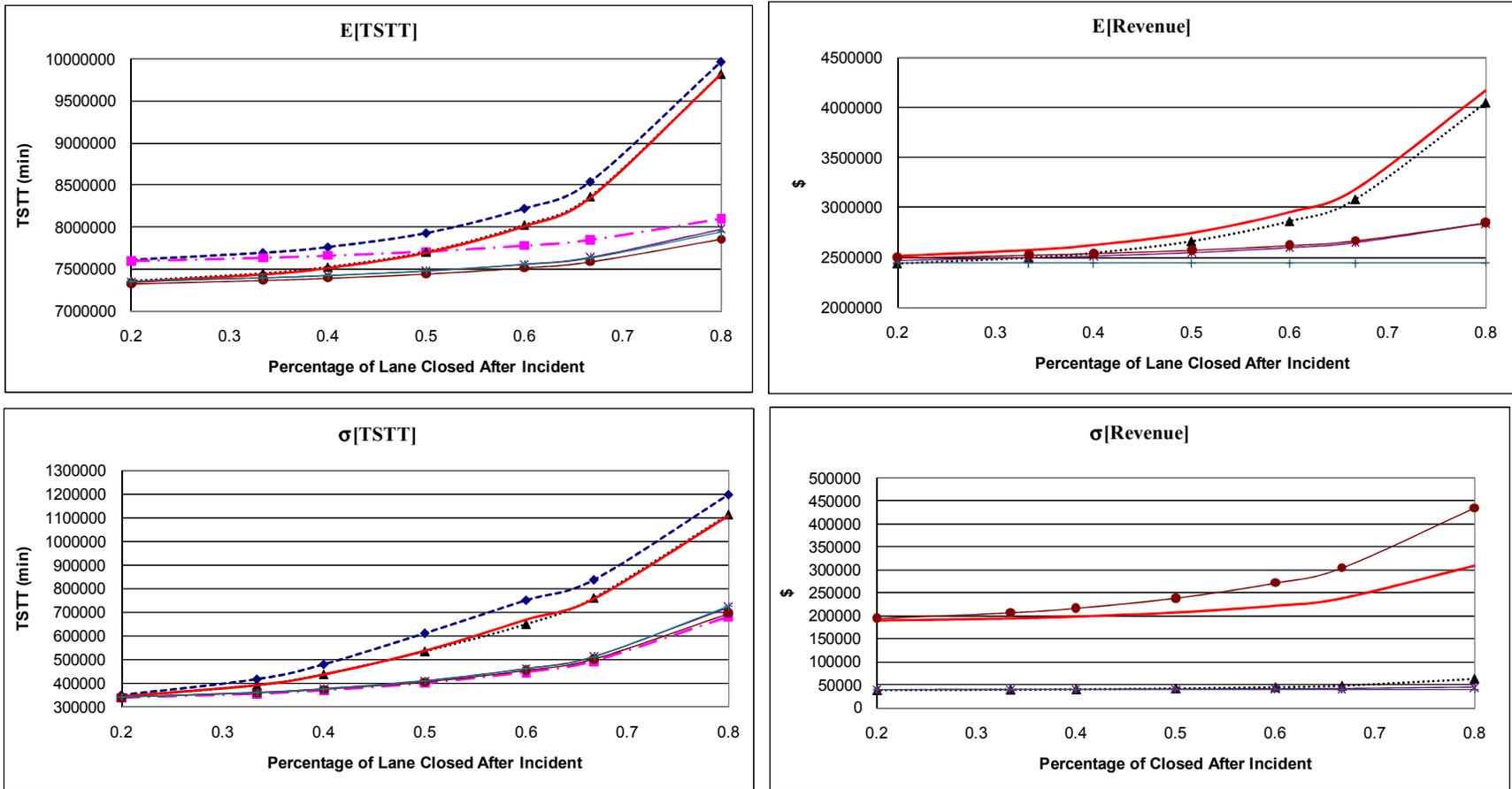


Figure 6. Impact of Incident Severity on System Performance and Revenue