Adaptive transit routing in stochastic time-dependent networks

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ABSTRACT

An adaptive transit routing (ATR) problem in a stochastic time-dependent transit network is defined and is formulated as a finite horizon Markov Decision Process (MDP). Routing strategies are defined to be conditional on the arrival times at intermediate nodes, and real time information on the location and arrival times of other buses in the network. The objective is to find a strategy that minimizes the expected travel time, subject to constraints that guarantee that the destination is reached within a certain threshold. The problem inherits *the curse of dimensionality* and state space reduction through pre-processing is achieved by solving variants of the time dependent shortest path problem. An interesting analogy between the state space reduction techniques and the concept of light cones is discussed. A dynamic program framework to solve the problem is developed and numerical results on a small instance of the Austin transit network are presented to investigate the extent of state space reduction using the proposed methods.

1 INTRODUCTION

Transit networks are often subject to uncertainty in link travel times. Variability in the time taken by a bus to traverse a link could result from several factors such as congested road conditions, presence of traffic signals, inclement weather, etc. Randomness in transit networks plays an important role in route choice for trips that involve transfers and in cities with a large number of alternate transit options for traveling between an origin-destination (OD) pair. However, most transit routing applications seldom take this into account while providing routing a priori strategies.

A priori strategies in stochastic networks are generally sub-optimal and by making use of additional information, it is possible to construct adaptive strategies that are optimal. Advances in Intelligent Transportation Systems (ITS) let us gather a large amount of real time data related to location and travel time of buses in a network which may be used for this purpose. Consider the following example to illustrates the adaptive nature of the problem. Suppose, buses A and B start at nodes 1 and 4 at $t = 0$ respectively. Assume that the travel time on the transit arc $(4, 5)$ is either be 1 or 10 with equal probability. Also, let the walking travel time on arcs shown in the network be 5.

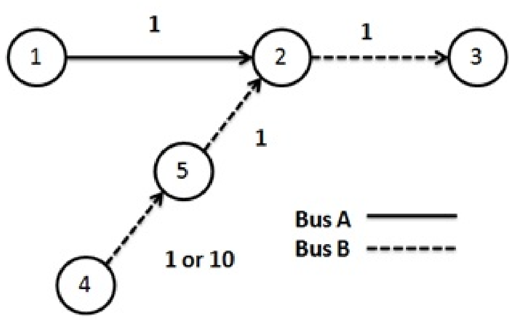


FIGURE 1: Adaptive routing in transit networks

Consider the OD pair $(1, 3)$. The expected cost of the optimal a priori strategy (i.e. board bus A and walk) is 6. But if bus B reaches node 5 at $t = 1$, it is guaranteed to arrive at node 2 at $t = 2$ and hence waiting is optimal. However, if we receive information that it failed to reach node 5 at $t = 1$, walking from node 2 is optimal. Such an adaptive strategy has an expected cost of 4.5, which is lower than the optimal a priori solution.

Shortest adaptive paths in stochastic networks have been widely researched in literature. Hall *(1)*, in his seminal paper on this subject noted that the standard shortest path algorithms cannot be used in stochastic time varying networks and the least expected time path is not a simple path, but a strategy or a hyperpath in which links are chosen based on the arrival time at intermediate nodes. Miller-Hooks and Mahmassani*(2)*, and Miller-Hooks *(3)* developed efficient labeling algorithms to solve the problem of finding the adaptive least expected path in networks, in which the travel time distributions are assumed to vary with time. Polychronopoulos and Tsitsiklis *(4)* formulated stochastic shortest path problems with recourse using a dynamic programming framework, in which arc costs are random and the uncertainty is revealed as the network is traversed. However, finding adaptive paths in transit networks is relatively difficult due to the possibility of waiting and the issue of *common bus lines* as noted by Chriqui and Robillard *(5)*. A traveler in a transit network is often faced with the option of boarding multiple buses on different lines to traverse a link or a section of a route.

Spiess and Florian *(6)*, and Nguyen and Pallottino *(7)* were among the first to make notable contributions to adaptive route choice in transit networks which was primarily studied as a sub-problem in frequency based transit assignment. The latter is credited for the development of a graph theoretical framework, in which travelers are assumed to choose hyperpaths instead of transit arcs. Cea and Fernandez *(8)* and Wu et al. *(9)* examined strategies in a broader class of transit assignment problems which incorporate effects of congestion. In these approaches, the headway between buses arrivals are treated to be random with known distributions (typically exponential). In the presence of common bus lines, travelers are assumed to choose a subset of available lines, also called as the *attractive set*, such that the total expected travel time to the destination in minimized. A strategy is defined by the line chosen at each stop (or a probability distribution over the attractive set) and the alighting point given that a particular line was chosen.

Route choice in stochastic time dependent transit networks in the presence of online information was studied by Hickman *(10)*, Hickman and Wilson *(11)*, and Hickman and Bernstein *(12)* using a dynamic path choice model which analyzes if a traveler at the origin has to board a bus or wait for a bus that arrives later based on the information gained wile waiting. Conceptually, their models can be extended to handle situations involving transfers in which strategies are not only dependent on the arrival time at a node, but also on the information of buses serving the node, received until that node is reached. Yet, their adaptive framework is still myopic in nature as the proposed extension does not utilize the information of other buses in the network. A more detailed summary of literature on adaptive routing in transit systems can be found in Rambha *(13)*.

A major goal of this paper is to develop a method to find an optimal strategy using the notion of system states (which are defined by the spatial and temporal locations of buses and the traveler in the network). For each of the choices available to a traveler at a particular state, the expected time to reach the the destination from possible future states can be used to determine the optimal decision at that state. The rest of this paper is organized as follows: In section 2, we describe the problem and present the notation used. Section 3 contains algorithms used for preprocessing and a discussion of the elimination of individual state space. In section 4, we develop a framework for solving the Adaptive Transit Routing(ATR) problem as a Markov Decision Process(MDP). Section 5 contains the computational results on a small instance of the Austin transit network. Finally, in section 6, we summarize the findings and limitations of this study, and discuss possible directions for future research.

2 PROBLEM DESCRIPTION

Let $G(N, A)$ be a directed network, where N represents the set of nodes/bus stops. A node which is the destination of a route and the origin of another is replicated. The set of arcs A is defined as $A_w \cup A_r \cup A_d$, where A_w represents the set of shortest walking arcs between every pair of nodes in N, A_r consists of links between bus stops along routes in the network and A_d represents a set of dummy arcs used to connect node copies to model the slack in schedules. The set of routes in the network is similar to those defined by transit agencies, except that routes in opposite directions (for instance northbound(NB) and southbound(SB)) are treated as different routes. Assume the time period of interest is divided into unit intervals $T = \{0, 1, 2, .., t, ...|T| - 1\}$ each of which denotes the time elapsed from a fixed time (say the start of the first trip of first bus). For example, if the first bus enters the network at say $6:00$ AM, then $7:20$ AM is represented as $t = 120$. Let t_O be the time at which a traveler departs at the origin node.

2.1 Notation

TABLE 1: List of symbols

Symbol	*Description*
$r \in R$	Set of Routes
$b \in B$	Set of Buses
$\rho_r \in P_r$	Set of trips along route r. The k^{th} trip on a route is denoted by ρ_r^k
$\mathbb{M}_r : P_r \to B$	A mapping which assigns a bus to each trip on route r
$\beta_b \in I_b$	Itinerary of a bus b which is the set of trips made by the bus. A trip is assumed to contain information related to the stops and the scheduled arrival times. We reference the k^{th} trip in I_b by β_b^k
$n_{\beta_b} \in N_{\beta_b}$	Set of nodes visited by bus b in trip β_b. We reference the k^{th} node in trip β_b by $n_{\beta_b}^k$
$t_{n_{\beta_b}} \in T_{n_{\beta_b}}$	Set of times at which a bus reaches node n_{β_b}
$f_{n_{\beta_b}}$	Earliest possible time a bus b can reach node n_{β_b}
$l_{n_{\beta_b}}$	Latest possible time a bus b can reach node n_{β_b}
$a_{\beta_b} \in A_{\beta_b}$	Set of arcs included in a trip β_b
$\Omega_{a_{\beta_b}}$	Distribution of time on arc a_{β_b} (Without loss of generality assume the pmf is arranged in increasing order of time)
$C^{\omega}_{a_{\beta_b}}$	A particular realization of the cost on arc a_{β_b}
w_{ij}	Walking arc cost between nodes i and j, where $i, j \in N$
$order(s_b)$	The number of nodes visited by bus b from the current state before reaching the state s_b
$\tilde{n}(s_b)$	Node associated with state s_b, where $b \in B$
$\tilde{t}(s_b)$	Time associated with state s_b, where $b \in B$

We define the set of *Individual States* of a bus $s_b \in S_b$ using the ordered pair $(n_{\beta_b}, t_{n_{\beta_b}})$, where n_{β_b} denotes the most recently visited bus stop and $t_{n_{\beta_b}}$ is the time at which the bus b arrived at node n_{β_b}. The individual state of buses, in practice may be determined by observing the actual arrival times of buses at bus stops in the network. The *System State* space S is defined as the subset of the cartesian product of the individual states, the set of nodes N and time periods T, i.e. $S \subseteq \mathbb{S}$ where $\mathbb{S} = S_{b_1} \times S_{b_2} \times S_{b_3} ... \times S_{b_{|B|}} \times N \times T$. The construction of the individual states S_b and the state space S will be discussed later. The node n and time t in a state vector $(s_{b_1}, s_{b_2}, ...s_{b_{|B|}}, n, t)$ denotes the node and time where a traveler is present in the network (also referred as *individual state of the traveler*).

The ATR problem is defined as follows. Given a stochastic transit network, the initial state of the system and a destination, we intend to find an optimal policy conditional on the current state of the system that minimizes the total expected travel time subject to the constraint $\mathbb{P}(t_O + X \leq \textit{cutoff}) = 1$, where X represents the cost of the adaptive strategy and value of the *cutoff* reflects the risk averse attitude of the traveler.

2.2 Assumptions

Given below is a list of assumptions used in the ATR problem.

1. Each bus can serve multiple routes. However, a bus can serve route r_1 followed by route r_2, only if the final destination of r_1 is the origin of r_2.

2. Printed schedules for each trip ρ_r and the $\mathbb{M}_r$ are assumed to be known for all $r \in R$.

3. All buses have infinite capacity.

4. Travel times on all arcs are assumed to be integer-valued.

5. The time taken by buses during boarding and egress of passengers is neglected.

6. Buses begin and end trips/runs at a garage. This assumption is not restrictive as a bus that goes out of the network and comes back at a later point of time may be considered as a new bus.

7. Travel times on transit arcs are assumed to be independent random variables with finite probability distributions. It is also assumed that the lowest possible travel time realization on a transit arc (i, j) is the difference of the scheduled arrival times at j and i (i.e. buses never arrive ahead of the scheduled time).

8. We assume that travel time pmfs on transit arcs vary only across trips of a route. However, the methods developed can be extended to cases in which pmfs vary with time.

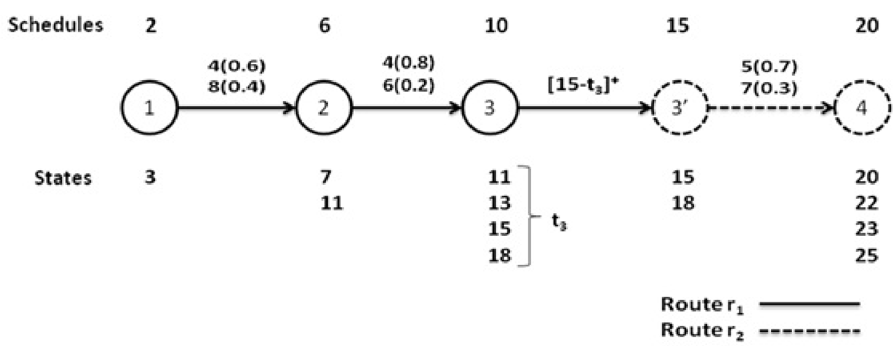
9. The first trip made by a bus starts on time and buses begin new trips on schedule if they can, by adjusting the slack provided in schedules between trips. Hence, if a bus arrives at or after the scheduled departure time at the origin of the next trip, the bus is assumed to proceed along the next trip immediately.

10. Bus bunching and overtaking is permitted, i.e. FIFO order need not be preserved.

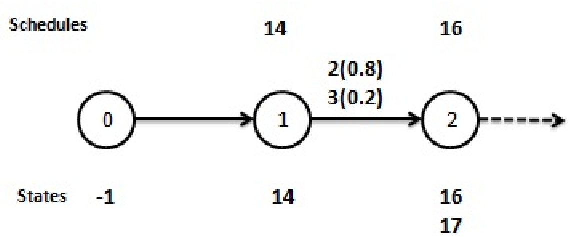
2.3 Constructing individual states of a bus:

In order to construct the individual states of a bus, we make use of the current state, the set of trips I_b and the pmfs on arcs for a particular trip $\Omega_{a_{\beta_b}}$. By convention, we assume that the state of a bus that hasn't yet been in the network is represented by the node time pair $(0, -1)$ and a bus in the garage that left the network is characterized by the state $(0, -2)$.

If a bus is present in the network, the current state vector provides information about the most recently visited stop and the time at which the bus visited it. This gives us the first state of the bus and the states at subsequent nodes are obtained using the travel time distributions. When route changes occur, the arc that connects the destination of one route and the origin of the other is modeled to reflect the assumptions made about the slack in schedules.



(a)



(b)

FIGURE 2: Individual states of a bus

Consider a bus b, which was scheduled to arrive at stop 1 at time $t = 2$ (see figure 2a). Suppose, the current system state vector indicates that the bus was last seen at stop 1 at time $t = 3$. Since the travel time on the first link is either 4 or 8, the possible arrival time at the next stop is either 7 or 11. Proceeding in a similar manner, we find the states at node 3, for each possible arrival time at node 2. Let node 3 be the destination node of route r_1 and the origin node of route r_2. For the sake of brevity, let t_3 represent the arrival time of the bus at node 3. In order to model the slack, cost of the arc $(3, 3')$ is defined as $(15 - t_3)^+$. This construct ensures that if the bus arrives at node 3 at $t_3 = 11$ or 13, it waits for 4 or 2 min respectively but if it arrives at $t_3 = 15$ or 18, it proceeds immediately to serve the next route. The states of a bus are assumed to spatially ordered based on the number of nodes visited by the bus from the current state, in order to reach a particular state. For instance, $order((1, 3)) = 1$ as it is the first node visited by the bus from the current state. Likewise, order of states $(2,7)$ and $(2,11)$ is 2 and so on.

If a bus is found to be present in the garage in the current state, we append a node 0 (used to represent the garage) before the start of its first trip. For example, suppose a traveler departs from his/her origin node at $t = 3$ and a bus b is scheduled to begin its trips from node 1 at $t = 14$. Since the bus isn't in the network at $t = 3$, we add a node 0 and an arc $(0,1)$ as shown in figure 2b. This network transformation is necessary for the construction of the system state space.

Clearly the number of individual states grows exponentially with increase in the number of trips made by the bus, which in turn results in an exponential number of system states and thus the ATR problem exhibits *the curse of dimensionality*. Solving the ATR problem as an MDP by application of the Bellman's principle or other approximation techniques thus becomes extremely difficult unless we find ways to reduce the size of the state space.

3 PREPROCESSING

The algorithms used for the preprocessing methods comprise of variants of the TDSP problem. In this section, we define and solve these algorithms using labeling approaches. A comprehensive summary of TDSP algorithms can be found in Chabini*(14)*. More specifically, the following set of problems are considered.

(a) *Earliest Origin-to-All TDSP (EOA):*
Given the departure time at the origin, the EOA problem involves computation of shortest path labels, where the label of a node represents the earliest possible time at which we can reach the node with positive probability ($w.p. > 0$).

(b) *Earliest All-to-Destination TDSP (EAD):*
This involves finding shortest path labels which specify the earliest we can reach the destination from all nodes $w.p. > 0$, for all possible departure times.

(c) *Latest Origin-to-All TDSP (LOA):*
In this problem, given the departure time at the origin, we find labels which represent the earliest we can reach a node with probability 1 ($wp1$).

Let us now briefly review the applications of these labels in the preprocessing procedure. First, the LOA label of the destination is used as the risk averse measure or the *cutoff*. We reason that if a traveler is guaranteed a strategy that ensures that the destination is reached within *cutoff*, he/she might not be inclined to follow a strategy which could potentially take longer to reach the destination. An elimination procedure is then developed to discard states of buses that do not affect the optimal strategy. Since the traveler has to reach the destination within a threshold $wp1$, the EAD labels may be used to verify if boarding a bus at a specific individual state violates the risk aversion constraint. Additionally, we use the EOA labels to check if we can catch a bus at an individual state.

Let O and D be the origin and destination node respectively and let SE represent a scan eligible list. In addition, let $\Gamma(i)$ denote the set of nodes adjacent to node i and let $\Gamma^{-1}(i)$ denote the set of nodes from which we can reach node i directly (i.e., $j \in \Gamma^{-1}(i) \Leftrightarrow (j,i) \in A$).

3.1 Earliest Origin-to-All TDSP (EOA)

Suppose that μ_n represents the label of a node n, where $n \in N$. In order to find the earliest time by which a node in the network can be reached $w.p. > 0$, we define a time dependent cost parameter $\zeta_{ij}(t) \; \forall \; t \geq t_O$ as follows:

$$\zeta_{ij}(t) = \min \left[\min_{\substack{b \in B,\, \beta_b \in I_b,\, a_{\beta_b} \in A_{\beta_b} \\ :\, t \in T_{i_{\beta_b}} \,\&\, a_{\beta_b} = (i,j)}} C^{[1]}_{a_{\beta_b}},\; w_{ij} \right]$$

The cost of arc (i, j) for a departure time t is set to the minimum of the walking travel time between i and j and the lowest possible transit cost on the arc (i, j) among all buses that reach node $i \; w.p. > 0$ at time t. The EOA labels can now be computed using a label correcting algorithm for one-to-all TDSP with waiting allowed.

3.2 Earliest All-to-Destination TDSP (EAD)

The EAD problem is a straight forward extension of the EOA problem. The time dependent arc costs $\zeta_{ij}(t)$ defined earlier are used to compute labels $\gamma_n(t)$, which represent the earliest we can reach the destination $w.p. > 0$, given that we depart from node n at time t. We assume $\forall \; t \geq T,\; \gamma_n(t) = \gamma_n(T-1)$, which is reflective of a steady state (i.e. arc costs are no longer time dependent) after time T. This assumption is not restrictive as long as T is sufficiently large, in which case the arc costs equal the walking travel times. The EAD problem can thus be formulated and solved as an all-to-one TDSP.

3.3 Latest Origin-to-all TDSP (LOA)

The LOA labels can be computed using a recursive application of a TDSP algorithm by varying the time dependent arc costs. We first assume that the time dependent arc costs are same as the shortest walking arc travel times and solve a TDSP to obtain the earliest time by which we can reach a node in the network. Using these labels, we find the first stop at which a bus in the network can be boarded $wp1$. The time dependent arc costs are then updated assuming that the bus takes the longest possible time to traverse links on trips beyond the stop at which it could have been boarded $wp1$. The TDSP labels are computed again and this process is repeated until the labels do not change. An elaborate discussion of solution techniques for these problems can be found in Rambha *(13)*.

3.4 Preprocessing procedure

The optimal strategy of the ATR problem is not necessarily influenced by all buses in the network. For instance, while traveling between an OD pair, buses that ply on routes far away from the OD pair may not have an impact on the optimal policy. The preprocessing steps described in this section aims at identifying the buses and individual states that could affect travel between an OD pair using the concept of risk aversion and the algorithms described earlier and significantly reduces the size of the state space. In the first step of the preprocessing procedure, we compute λ_D which is used to define the *cutoff* as explained earlier and then solve for the EOA and EAD labels.

In the final stage we define indicator variables, eliminate states and create absorbing states. The ideas used in the process of elimination are primarily motivated by the following questions:

1. *Suppose the destination of a traveler is to the south of the origin node. Should he/she consider the states of a bus heading in the opposite direction (i.e. NB)? If yes, do all the individual states of the bus play a role in finding the optimal strategy?*
2. *Suppose a traveler missed a bus. Can we exclude it from the set of buses to be considered while populating the system states?*

3.5 Elimination based on EAD labels

Consider the first question. A traveler heading south might reach the destination faster by transferring to another bus after boarding the NB bus, thus making a case for its inclusion in constructing the system state space. In other words, a traveler may travel away from the destination geographically, in the process of minimizing the travel time. Let us now address the second part of the question. We might be able to limit the individual states of a bus by discarding the ones from which we cannot make it to the destination before the *cutoff*. This is accomplished using the EAD labels by defining an indicator variable δ_{ead} as shown below.

$$\delta_{ead}(s_b) = \begin{cases} 1 & \text{if } \gamma_{\tilde{n}(s_b)}(\tilde{t}(s_b)) > \lambda_D \\ 0 & \text{otherwise} \end{cases}$$

Let us study the elimination process using the example shown in figure 3a. Assume that the *cutoff* obtained by solving the LOA algorithm is 40. Clearly states beyond node 4 can be discarded. Let the set of individual states of a bus be represented as an acyclic network as shown in figure 3b. A node in the network denotes an individual state and arcs are used to connect adjacent states. Let all states that can be reached from an individual state and all states from which a particular state can be reached be referred to as *descendant* and *ancestor* states respectively. For the ease of illustration a few states have been replicated (indicated by the dashed connectors). All states with $\delta_{ead} = 1$ are marked in gray. Note that if for a particular state $\delta_{ead} = 1$, we cannot reach the destination within the *cutoff* with positive probability from all descendant states. The elimination of individual states can be divided into the following two phases.

Phase I:

From figure 3b, observe that if a bus at state $(1, 2)$ takes 16 minutes to travel to node 2, we can be certain that it does not influence the optimal strategy. Thus if the travel time between nodes 1 and 2 is 16, we assume the bus exits the network. This is achieved by creating an absorbing/sink state $(0, -2)$ which indicates that the bus is back in the garage. The phase I elimination procedure can be described as follows. In the acyclic network of individual states, delete arcs that connect marked states (shown by the dotted lines in figure 3b) and then delete all marked states. Arcs orphaned after the removal of marked states are then connected to the garage state $(0, -2)$. If multiple arcs are created between an individual state and the sink state, replace them with a single arc between the two states. Finally, connect all unmarked states (excluding the garage states) with outdegree 0 to the state $(0, -2)$. Buses from these states are assumed to incur a cost of 0 to reach the sink. The resulting network of individual states is shown in figure 3d.

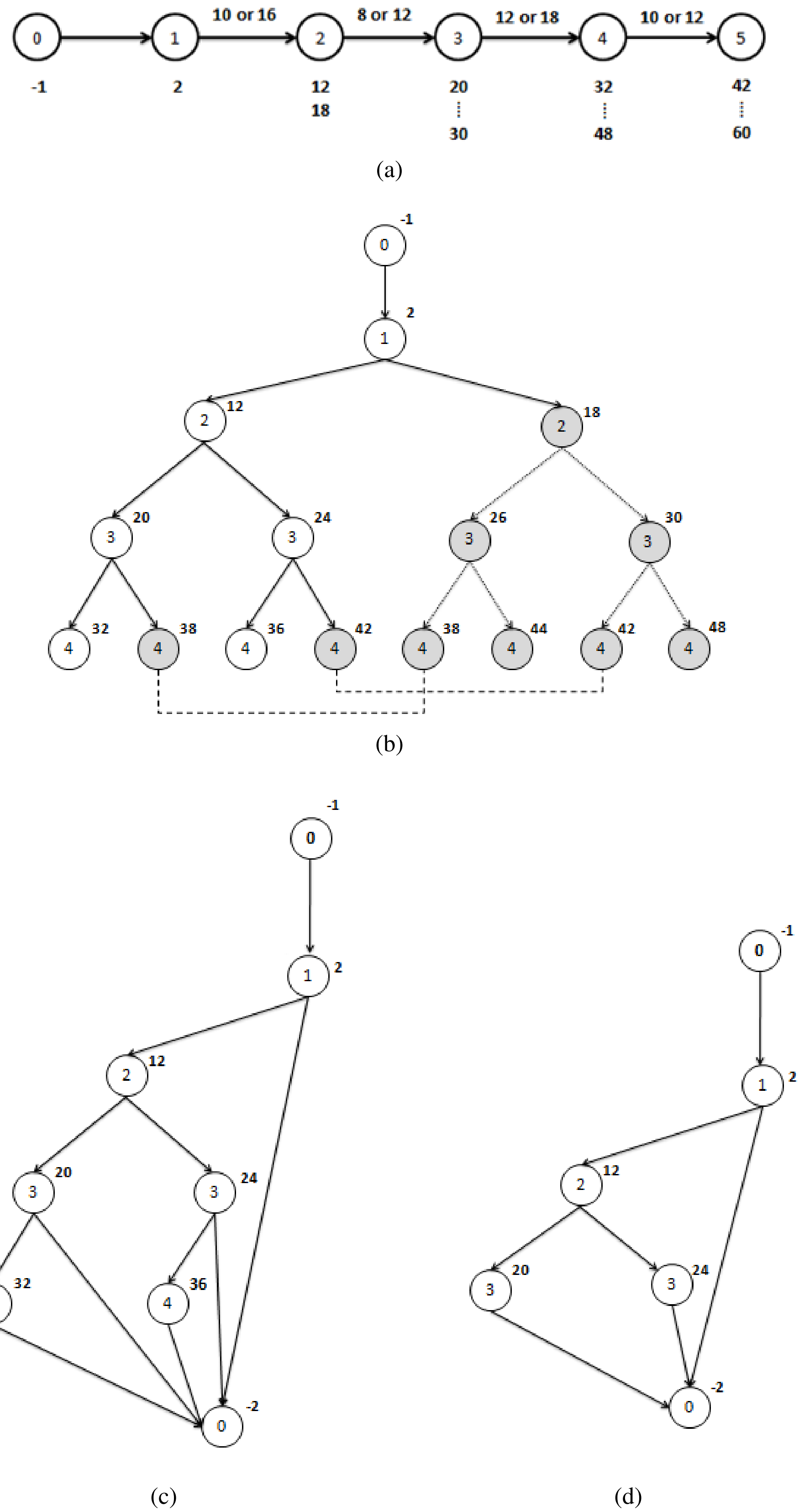


FIGURE 3: Preprocessing of individual states

Phase II:
Suppose a traveler is about to board the bus at node 3 at $t = 20$. Although the destination can be reached within the *cutoff* with positive probability, the risk aversion condition may be violated if the bus takes 18 minutes to travel on its next arc. Hence, the state $(4, 32)$ can be eliminated and the bus may be assumed to proceed immediately to the garage from state $(3, 20)$. The state $(4, 36)$ can also be eliminated using a similar argument. Assuming the state with the highest order is denoted by y_{max} we proceed as follows. If y_{max} is 1 the bus is completely ignored. Else, pick individual states with order y_{max}. If at least one of the outgoing arcs from each predecessor state of the state with order y_{max} is connected to the garage state $(0, -2)$, we delete the state being examined and the orphaned arcs. If no state is eliminated we terminate, else y_{max} is recalculated and the procedure is repeated.

Effect of elimination the labels and delta values:
The elimination of individual states of each bus was carried out independently without regard to its impact on the labels. In phase I, if a particular state is removed, we know that by boarding the bus at that state we cannot reach the destination within the *cutoff*. If this state was used in finding the optimal EAD label of another individual state (of the same or different bus), δ_{ead} of the later state would still be 1. Thus, eliminating the former state might increase the EAD labels but the delta values remain unaffected. Now consider phase II of elimination using the EAD labels. Since we may eliminate unmarked states, the delta values are likely to change in addition to the EAD labels. Depending on the new EAD labels, states which were originally unmarked (i.e. $\delta_{ead} = 0$) can get marked. Thus we may iterate between the calculation of the EAD labels and the elimination phases until no states are excluded.

3.6 Elimination based on EOA labels

Let us now try to address the second question. If a traveler misses a bus it might still be possible to catch the missed bus at a later stop using some faster service. On this line of thought, we can employ the EOA labels to find the earliest possible arrival time at every node. If the latest time at which the bus arrives at a node is lesser than the EOA label, then the states of the bus at that node do not have any bearing on the optimal policy. This can be mathematically translated by defining a new indicator variable δ_{eoa} as follows:

$$\delta_{eoa}(s_b) = \begin{cases} 1 & \text{if } \tilde{t}(s_b) < \lambda_{\tilde{n}(s_b)} \\ 0 & \text{otherwise} \end{cases}$$

Consider the network shown in figure 3a to illustrate the use of the δ_{eoa} indicator variable. Let the vector [15 25 35 50] represent the earliest we can reach nodes 1, 2, 3 and 4 respectively(note that the states at node 5 were discarded). The δ_{eoa} values for all states are set to 1 and hence we can disregard the bus while finding the optimal policy. Now consider another scenario in which the EOA labels are [15 25 30 42]. Clearly, we cannot arrive before the last possible states of the bus at nodes 1 and 2, but since the bus can be reached at node 3 $w.p. > 0$, it can be boarded at any other node along the remainder of its journey with positive probability. However, the states at node 1 and 2 cannot be eliminated as it leads to a loss of information (i.e. knowledge of the actual travel time on links $(1, 2)$ and $(2, 3)$ sheds more light on the state of the bus at subsequent nodes).

In such situations, we can completely ignore the bus if for each individual state, either δ_{eoa} or δ_{ead} is 1. Intuitively, this implies that we cannot catch the bus at some individual states, and in cases in which we can board the bus, the destination cannot be reached within the *cutoff*.

3.7 Light Cones

A light cone is a flash of light from an event (E) that travels in spacetime. It comprises of two cones, a past and a future light cone. While, the past light cone represents events/points in spacetime from which a flash of light can be observed at E, the future light cone includes all points that can be reached by a light pulse from E. Light cones serve as a perfect tool to understand causality, i.e. only the events which occur in the past light cone can possibly affect event E and event E can possibly influence only the events in the future light cone.

Finding buses/individual states that do not affect the optimal routing policy and eliminating them resembles the idea behind light cones. For illustrative purposes, we assume that the shape of a diagram based on how fast a traveler reaches other nodes in the network is a cone. Let us now draw a parallel between light cones and the elimination procedure using the example in figure 4. Suppose that the network comprises of a single bus and a traveler starts at the origin at $t = 0$. The cone at the origin can be obtained using the EOA labels. Consider three individual states of a bus (1,2 and 3) as shown in the figure. Let the cones at these states represent the earliest possible time taken to travel through the network. Individual state 3 lies outside the cone at the origin and hence cannot be reached. This corresponds to $\delta_{eoa}(3) = 1$.

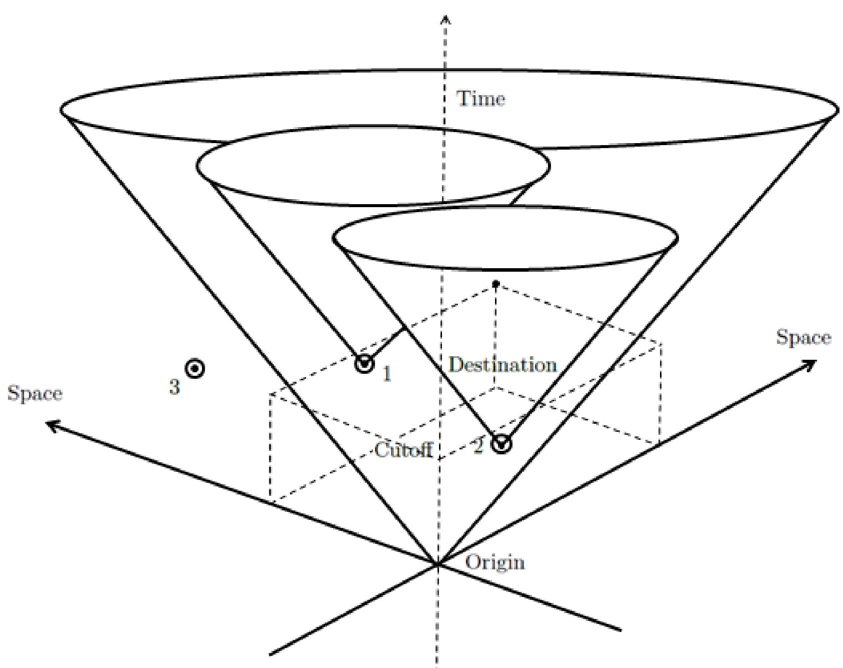


FIGURE 4: Light cones and indicator variables

On the other hand, individual states 1 and 2 can be reached from the origin as they lie within the cone (i.e. $\delta_{eoa}(1) = \delta_{eoa}(2) = 0$). While the point represented by the destination and the *cutoff* lies inside the cone at state 2, it falls outside the cone at state 1. This is equivalent to the values of δ_{ead} being 1 and 0 for individual states 1 and 2 respectively.

4 COMPONENTS OF DYNAMIC PROGRAM

In this section, we define briefly define the elements required to model the ATR problem as an MDP. The components of the dynamic program includes the state space, decision/action space, transition and value functions.

4.1 State space

As defined earlier, the state space(S) is a subset of $\mathbb{S}$ where $\mathbb{S} = S_{b_1} \times S_{b_2} \times S_{b_3} ... \times S_{b_{|B|}} \times N \times T$. Note that the preprocessing methods reduce the sizes of the sets B and S_b and thus results in a smaller system state space. We may further redefine the set T to be $\{t_O, t_O + 1, ..., \lambda_D\}$.

Let us motivate the idea behind the construction of the system state space using an example. Consider the individual state network (see figure 3d) obtained as a result of eliminating states using EAD labels. Notice that if a traveler is in the network at some node at say $t = 15$, the bus cannot be at states $(1, 2)$, $(3, 20)$ and $(3, 24)$. It cannot be at the latter two states as they represents points in future. A bus at state $(1, 2)$ would have advanced to one of its successor states by $t = 12$. Hence the bus can only be present at either $(2, 12)$ or $(0, -2)$. In general, for every $t \in T$, we mark the individual states at which a bus in the network is likely to be at and take the cartesian product of the set of all marked states of each bus and the set N. This procedure is then repeated for all $t \in T$ to obtain the entire the system state space S.

4.2 Decision space

The decision space consists of the set of available actions at a particular state. Let us denote the decision at a particular state by $x(s)$ and the set of all available actions by $X(s)$. At each state, recognize that a traveler can either walk, wait or board a bus when it arrives. Thus, we define $X(s)$ as $X_w(s) \cup X_p(s) \cup X_r(s)$, where $X_w(s)$, $X_p(s)$ and $X_r(s)$ comprise of the walking, waiting (loops of cost 1) and transit arcs available at state s. While the cost of choosing to walk/wait is known deterministically, the actual travel time incurred by choosing a transit arc is random and is defined by the distributions $\Omega_{a_{\beta_b}}$. In general, let $\tilde{\xi}_{x(s)}$ be a random cost variable associated with a decision $x(s)$. Further, let $\xi_{x(s)}$ and $\Xi_{x(s)}$ denote a generic realization and the support of $\tilde{\xi}_{x(s)}$ respectively.

4.3 Transition functions

Given the decision made at a state, the transition functions for an MDP are used to describe the evolution of the system. More precisely, it gives the probability of ending up in a state s', assuming that a decision maker incurs a cost of $\xi_{x(s)}$ by choosing $x(s)$ at a state s. We denote the transition functions by $\mathbb{P}\big[s'|(s, x(s), \xi_{x(s)})\big]$, where $s, s' \in S, x(s) \in X(s)$ and $\xi_{x(s)} \in \Xi_{x(s)}$.

The fact that the uncertainty in travel time of the buses in the ATR problem is assumed to be independent of each other (see assumption 7) can be exploited in calculating the probabilities of future states of each bus separately, i.e. $\mathbb{P}\big[s'_b|(s_b, x(s), \xi_{x(s)})\big]$ where $s_b, s'_b \in S_b$. Further, owing to its repeated use it would be wise to calculate these probabilities and store it as a lookup table before solving the MDP.

4.4 Value functions

The value function of a state (denoted by $V(s)$) is the expected time taken to the destination from that particular state. The value function for all states in which n is the destination node is set to 0 and the value of all other states is set to ∞.

4.5 Optimality Criteria

Using the definitions and notation described so far, the the Bellman's equation of optimality can be expressed as follows:

$$V(s) = \max_{x(s) \in X(s)} \left[\mathbb{E}_{\Xi_{x(s)}} \left[\tilde{\xi}_{x(s)} + \sum_{s' \in S} \mathbb{P}\big[s' | (s, x(s), \tilde{\xi}_{x(s)})\big] V(s') \right] \right]$$

The value functions may be estimated using the value iteration method, but the size of the state space after the preprocessing stage might still be very large. In such cases it could be worthwhile to explore approximate dynamic programming methods in which value functions may be approximated using sampling methods or aggregation of arcs and pmfs. A more detailed discussion of the elements of the dynamic program can be found in Rambha *(13)*.

5 COMPUTATIONAL EXPERIMENTS

In this section, an analysis of the state space reduction on a small instance of the Austin transit network is presented. Eight routes (route 3,5,7 and 10 in NB and SB directions) were chosen, and information available on the Capital Metropolitan Transportation Authority's website (http://www.capmetro.org/) was used for this study. All trips that begin between $6:00$ AM and $12:00$ PM were included in the model. A total of 48 buses and 78 stops were considered, and the shortest walking arcs between each pair of nodes was obtained using the Google Distance Matrix API. Trips on routes were manually assigned to buses (as this information was not available), using which itineraries for each bus were developed. In some cases, buses were assigned to trips across several routes (for example buses on route 10 also served route 3). A current state vector and pmfs on transit arcs were randomly generated (the size of the support of pmfs on each link was restricted to two) and were used to construct the individual states of each bus in the network.

The graph in figure 5 shows the shows the logarithm of the cardinality of the cartesian product of the individual state space (which is proportional to the size of the actual state space) at the end of each step in the preprocessing procedure for two scenarios. For the first OD pair, it was found that the destination could be reached within 48 minutes $wp1$. The second OD pair corresponds to a longer trip in which the *cutoff* was found to be 216 minutes. While the magnitude of reduction in state space is significant using the phase I, further reduction due to phase II appears to be marginal. In the case of the second OD pair, since the cutoff was higher, relatively lesser states were eliminated using the risk aversion condition and the elimination methods compared to elimination of states for OD pair 1. As mentioned earlier it might be possible to further reduce the state space by recursive application of the Phase I and Phase II methods. We did not observe any reduction in the state space using the EOA labels.

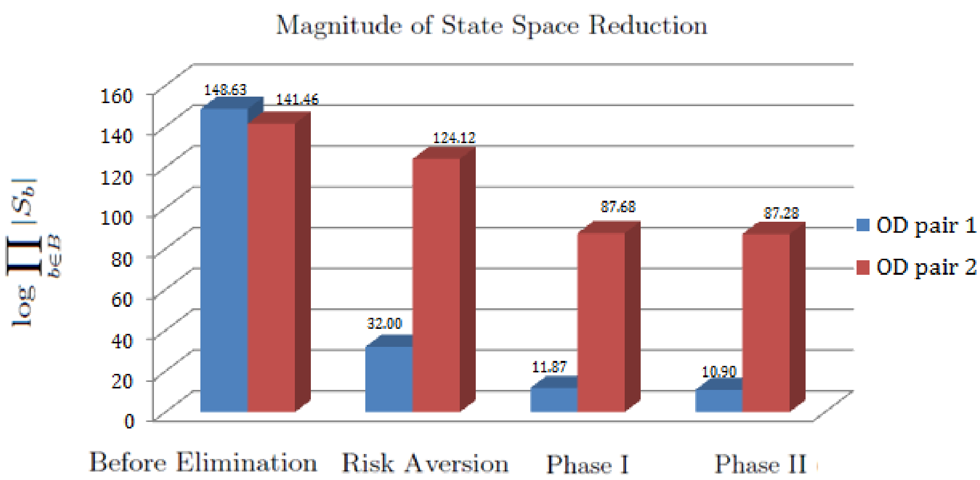


FIGURE 5: A plot of the individual state space reduction

The results in general are influenced by several factors such as the OD pair, density of transit network and the pmfs, which in turn affect the *cutoff* and the EAD labels. The numerical results presented here serve as a rough reflection of the abilities of these approaches and it is difficult to expect similar results if these methods were replicated on a different network. For instance, one might assume that having a denser network of buses makes it easier to reach the destination from individual states within the *cutoff* and hence, lesser number of states may be eliminated. But the LOA algorithm in a dense transit network might yield a smaller *cutoff* in the first place.

6 CONCLUSIONS

The ATR problem addresses the effects of congestion and its impacts on route choice and transfers using adaptive strategies in a stochastic time dependent transit network. Some of the major contributions of this paper includes the ability of using a vast amount of real time information that can potentially result in lower expected travel times compared to an a priori strategy. In addition, state space reduction methods that may be applied independently on each bus in the network, were developed along the lines of causality and the concept of light cones. Also, the bus based approach and assumptions made lets us model the strategies and the slack in schedules between trips in a much broader and realistic manner. However, the proposed model is limited by the assumptions made, some of which are restrictive in nature. For instance, the travel time distributions on links are assumed to be independent of each other; but buses on routes that share links are likely to be affected in a similar manner in the presence of congestion. Another assumption that limits the scope of the ATR problem has to do with a travelers reluctance in transferring multiple times before reaching the destination.

The study of the ATR problem presents some useful pointers for future research on this topic. While the preprocessing methods look promising, an exact estimation of the optimal strategy may be hindered by the size of the state space. In such cases, one would have to resort to approximation techniques, in which case it is necessary to investigate the quality of solutions obtained using these methods. It is also of prime interest to determine the travel time savings in using these methods in comparison with an a priori strategy or other simpler trip planning approaches. As noted earlier in the case of the benefits of the state space reduction, the savings in travel time are also dependent on the OD pair, pmfs and the density of the transit network, etc. For instance, in the example in figure 1, the travel time savings is 3, which is a 40 percent decrease over the expected travel time of the optimal a priori strategy. Instead, if the travel time on the link $(4, 5)$ is either 1 or 2, the optimal a priori strategy and the adaptive solution are the same. Hence, it is very difficult to judge the benefits of using this model unless extensive testing on transit networks with accurate probability distributions and information of buses itineraries is carried out. Although the ATR problem remains to be explored in greater detail, this study develops a sound theoretical framework and novel state space reduction techniques that contribute significantly to the study of adaptive routing in transit networks.

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