Pressure-based policies for reservation-based intersection control

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1 Introduction

Autonomous vehicles (AVs) admit new traffic behaviors, such as reservation-based intersection control [1]. Vehicles request a *reservation* from the computerized intersection manager for a specific turning movement starting at a specific time. The intersection manager simulates vehicle requests on a grid of space-time tiles representing the intersection, and accepts some subset of requests that do not conflict in any of the space-time tiles. Reservations offer more possibilities in intersection control than traffic signals because individual vehicle movements are directly controlled.

Despite the greater possibilities for control policies, optimizing reservations is a little-studied question. Reservations have been mostly studied with the first-come-first-serve (FCFS) policy. Fajardo et al. [2] found that FCFS reservations reduced delays beyond optimized signals. Reservations can mimic traffic signal phases [3], so reservations can perform as least as well as signals. However, in some situations FCFS will perform worse than signals [4]. Other studied control policies are prioritizing emergency vehicles [5] and intersection auctions [6], but those do not optimize for efficiency either.

The purpose of this paper is to study two policies with the goal of maximizing intersection efficiency:

1. **Pressure-based policy:** Tassiulas & Ephremides [7] formulated a pressure-based policy for service and routing in communication networks and proved that it maximizes throughput. Because their communications network behaves differently than a traffic network (for instance, communications networks have unbounded queues), we adapt their policy to a cell-transmission model (CTM)-based traffic network [8, 9]. CTM is necessary because communication network queue dynamics do not include congestion waves. CTM is sufficiently similar to a communications network because the congestion wave dynamics only affect cell transition flows. However, our derived policy does not require CTM to be implemented.

2. **P0 policy:** The pressure-based policy is designed for communication networks with system-assigned routes. In traffic networks, we assume that vehicles choose routes to minimize their travel time. Smith [10] proposed the P0 policy to maximize intersection throughput while accounting for user equilibrium (UE) route choice. We adapt the P0 policy for reservations in dynamic traffic assignment (DTA) by using observed travel times instead of link performance functions for the congestion pressure term.

The contributions of this paper are as follows: we formulate the traffic network using CTM as a communications network [7], and show that the corresponding pressure-based policy π^* maximizes throughput for any given route assignment. Furthermore, we adapt the P0 policy to account for UE route choice. Then, we compare both π^* and P0 on a city network. Results indicate that both improve significantly over FCFS, although π^* also outperformed P0.

2 Traffic network

This section describes the traffic network used to study the pressure-based and P0 policies. We model link flows through the cell transmission model (CTM) [8, 9], which makes this network usable in simulation-based DTA. Each cell is furthermore modeled as a FIFO queue, which admits the pressure-based policy presented in Section 3.1. Reservation-based intersection control is modeled through the conflict region integer program of Levin & Boyles [11].

2.1 Network formulation

Consider a traffic network $\mathscr{G} = (\mathscr{N}, \mathscr{A}, \mathscr{V})$ with nodes $\mathscr{N}$, links $\mathscr{A}$, and time-specific demand $\mathscr{V}$. All demand enters and exits from a centroid; let $\mathscr{Z} \subsetneq \mathscr{N}$ denote the set of centroids. We consider discrete flow, referred to as vehicles. Each vehicle v has a specific origin and destination in $\mathscr{Z}$ and chooses a path $\mathscr{p}_v$ from r to s before departing.

As with CTM,each link is divided into cells to approximately solve the hydrodynamic theory of traffic flow [12, 13] through simulation. Cells for link a have length $u_a^{\mathrm{f}} \Delta t$ where u_a^{f} is the free flow speed of link a and Δt is the simulation time step. Therefore, vehicles can traverse at most one cell per time step. Movement from cell i to cell j is permitted only when i is connected to j. Let $\mathscr{C}$ be the set of all cells, and let Γ_i^{-1} and Γ_i denote the sets of incoming and outgoing cell connectors, respectively, for cell i.

2.2 Cell flow dynamics

Let $n_i(t)$ be the number of vehicles in cell i at time t, and let $x_i(t)$ be the set of specific vehicles. Let $S_i(t)$ be the set of vehicles in cell i at time t that would leave i if there were no downstream constraints. Let $R_i(t)$ be the receiving flow — the maximum number of vehicles that can enter — of cell i at time t. Let $y_{ij}^v(t) \in \{0, 1\}$ indicate whether vehicle v moves from cell i to cell j at time t. If $y_{ij}^v(t) = 1$, v moves from i to j at t. Clearly, v cannot move from i to j at t unless v entered i at $t - 1$. Also, v will not move from i to j unless $j \in \mathscr{p}_v$. Flow between i and j is further constrained: v cannot leave i at t unless $v \in S_i(t)$. Also, the total flow into j cannot exceed $R_j(t)$.

Let $Y_n(t)$ denote the set of feasible vehicle movements across node $n \in \mathscr{N}$ at t, where the feasible region is constrained by sending flow, receiving flow, path constraints, intersection conflicts, and FIFO behavior.

Each $\mathbf{y}_{ij}(t) \in Y_n(t)$ is an *action* that may be taken for moving flow. Let $\mathbf{S}(t)$ be the vector of sending flows and $\mathbf{Y}(t)$ be the vector of feasible movements. Let $\mathscr{S}$ be the set of possible sending flows and $\mathscr{Y}$ the set of possible feasible movements. A policy $\pi = \mu_t : \mathscr{S} \to \mathscr{Y}$ defines which vehicles are moved when the sending flow is $\mathbf{S}(t)$.

3 Pressure-based policies

We present a policy π^* based on the work of Tassiulas & Ephremides [7] and adapt the P0 policy [10].

3.1 Maximum throughput policy

The policy π^* that we specify here is superstable. The policy is an algorithm executed each time step that determines intersection vehicle movements.

Stage 1

This stage determines the vehicle weights $D^v_{jk}(t)$ for vehicle v moving from cell j to cell k. Define C_j to be the set of congested cells leading up to j as follows: $j \in C_j$. Also, if $n_j(t) > Q_j(t)$, then for any $i \in \mathscr{C}$, $i \in C_j$ if $\Gamma_i \cap C_j \neq \emptyset$ and $n_i(t) > Q_i(t)$. This results in arbitrarily large pressures.

Let $p^j_i(t)$ be the proportion of vehicles in cell i that have cell j in their path. Clearly, $p^j_j(t) = 1$, and for any cell i' preceding j on the same link, $p^j_{i'}(t) = 1$ also. When queue spillback is present and i is on a different link than j, $p^j_i(t) < 1$ is possible.

Define the queue length for cell j at time t to be

$$L_j(t) = \sum_{i \in C_j} n_i(t) p^j_i(t) \tag{1}$$

L_j is the number of vehicles in the congested region C_j waiting to use cell j. Now define $D^v_{jk}(t)$ as follows:

$$D^v_{jk}(t) = (L_j(t) - L_k(t)) \min\{Q_j(t), Q_k(t)\} \tag{2}$$

Stage 2

Find a vehicle movement vector $\hat{\mathbf{y}}(t)$ satisfying the following:

$$\hat{\mathbf{y}}(t) = \arg\max_{\mathbf{y} \in \mathbf{Y}(t)} \{\mathbf{D}(t) \cdot \mathbf{y}\} \tag{3}$$

Stage 3

If $\hat{y}^v_{ij}(t) = 1$, then vehicle v is moved from cell i to cell j at time t. Otherwise, v remains in cell i. This flow is feasible because $\hat{\mathbf{y}}(t) \in \mathbf{Y}(t)$.

3.2 P0 policy

π^* is superstable for any fixed assignment, but may encourage suboptimal route choice. Smith [10] developed the P0 policy, which considers route choice in its intersection optimization. We use the following pressure calculation for the P0 policy:

$$P_i(t) = Q_i \left(\bar{\tau}_i(t) - \tau^{\mathrm{f}}_i \right) \tag{4}$$

Define $P_v(t)$ as the pressure for vehicle v, based on v's incoming link, for the P0 policy. Further, define the vector of pressures to be $\mathbf{P}(t)$ for all waiting vehicles. The objective is then to find

$$\mathbf{y}^*(t) = \arg\max_{\mathbf{y} \in \mathscr{Y}(t)} \{\mathbf{P}(t) \cdot \mathbf{y}\} \tag{5}$$

As with Stage 2 of π^*, problem (5) is also approximated through the heuristic of Levin et al. [11].

Table 1: Travel time for downtown Austin

Policy	**Total system travel time** (hours)	**Avg. travel time per vehicle** (min.)
FCFS	7577.1	7.24
π^*	6037.8	5.77 (-20.3%)
P0	6370.4	6.08 (-16.0%)

4 Results

FCFS, π^*, and P0 were tested on the downtown Austin network, which has 171 zones, 546 intersections, 1247 links, and 62836 trips. All intersections used reservations, and DTA was solved using each policy for every intersection to a cost gap of 1% of total travel time. Travel times are shown in Table 1. Both π^* and P0 improved significantly over FCFS. Surprisingly, π^* performed better than P0, even after solving DTA. This might be due to the network structure; the downtown grid has many alternative routes, and thus encouraging more optimal route choice through intersection control may be less important for reducing congestion.

References

[1] K. Dresner and P. Stone, "Multiagent traffic management: A reservation-based intersection control mechanism," in *Proceedings of the Third International Joint Conference on Autonomous Agents and Multiagent Systems-Volume 2*, pp. 530–537, IEEE Computer Society, 2004.

[2] D. Fajardo, T.-C. Au, S. Waller, P. Stone, and D. Yang, "Automated intersection control: Performance of future innovation versus current traffic signal control," *Transportation Research Record: Journal of the Transportation Research Board*, no. 2259, pp. 223–232, 2011.

[3] K. M. Dresner and P. Stone, "Sharing the road: Autonomous vehicles meet human drivers.," in *IJCAI*, vol. 7, pp. 1263–1268, 2007.

[4] M. W. Levin, S. D. Boyles, and R. Patel, "Paradoxes of reservation-based intersection controls," *In review at Transportation Research Part A: Policy and Practice*, 2016.

[5] K. Dresner and P. Stone, "Human-usable and emergency vehicle-aware control policies for autonomous intersection management," in *Fourth International Workshop on Agents in Traffic and Transportation (ATT), Hakodate, Japan*, 2006.

[6] H. Schepperle and K. Bohm, "Auction-based traffic management: towards effective concurrent utilization of road intersections," in *E-Commerce Technology and the Fifth IEEE Conference on Enterprise Computing, E-Commerce and E-Services, 2008 10th IEEE Conference on*, pp. 105–112, IEEE, 2008.

[7] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *Automatic Control, IEEE Transactions on*, vol. 37, no. 12, pp. 1936–1948, 1992.

[8] C. F. Daganzo, "The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory," *Transportation Research Part B: Methodological*, vol. 28, no. 4, pp. 269–287, 1994.

[9] C. F. Daganzo, "The cell transmission model, part ii: network traffic," *Transportation Research Part B: Methodological*, vol. 29, no. 2, pp. 79–93, 1995.

[10] M. Smith, "A local traffic control policy which automatically maximises the overall travel capacity of an urban road network," *Traffic Engineering & Control*, vol. 21, no. HS-030 129, 1980.

[11] M. W. Levin, H. Fritz, and S. D. Boyles, "Optimizing reservation-based intersection controls," *In review at IEEE: Transactions on Intelligent Transportation Systems*, 2015.

[12] M. J. Lighthill and G. B. Whitham, "On kinematic waves. ii. a theory of traffic flow on long crowded roads," in *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, vol. 229, pp. 317–345, The Royal Society, 1955.

[13] P. I. Richards, "Shock waves on the highway," *Operations research*, vol. 4, no. 1, pp. 42–51, 1956.