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A Stochastic Delay Prediction Model for Real-Time Incident Management

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Abstract

Incident management policies always involve making real-time decisions under uncertain and rapidly changing conditions. To support this process, a number of computer models have been developed to predict incident severity, although few of these can truly accommodate the uncertainty that exists in this process. In this paper, we demonstrate that failing to account for this consistently and systematically underestimates the impact of incidents, potentially to a large enough degree that faulty incident management decisions are made (simulation results indicate underestimation on the order of 20-50%). As a result, we develop a new delay prediction model that explicitly provides predictions in the context of uncertain incident duration, which eliminates this source of error. Analytical incident delay formulae are extended to account for uncertain incident duration, and simulation with Monte Carlo sampling is undertaken to study scenarios which are too complicated for exact analysis. These indicate that different demand profiles also play a large role in determining the impact of an incident, and should also be taken into account in any delay prediction model applied in practice.

1 Introduction and Motivation

The adverse effects of traffic congestion are widely known, and much effort has been invested in studying its causes in order to alleviate this problem in urban areas. To support this process, many previous efforts focused on recurring congestion patterns corresponding to known bottlenecks or problem areas. However, it has long been recognized that an equal, if not greater, amount of congestion is due to non-recurring events [13]; in particular, incidents can result in large delays that contribute significantly to the total congestion experienced by commuters.

Accordingly, effective incident management techniques have the potential to substantially reduce congestion. Some of these techniques, such as the use of variable message signs (VMS) and highway advisory radio (HAR) to inform travelers of incidents, can be used in response to nearly all incidents, regardless of severity. However, other techniques, such as the temporary lifting of high-occupancy vehicle (HOV) restrictions on lanes to increase capacity, or the closing of upstream freeway onramps, involve more operational overhead and are only appropriate for incidents which are expected to have a severe impact on the system. Alternately, some incident response strategies, such as the use of dedicated incident response vehicles to

Table 1: Small example to demonstrate importance of modeling uncertainty.

	Incident A	Incident B
Duration	5 or 25	17
Expected duration	15	17
Delay using expected duration	225	289
True expected delay	325	289

reduce clearance times, involve limited resources and require real-time dispatching decisions to be used most efficiently.

Thus, effective use of incident management techniques requires the ability to predict the severity of an incident in real time, allowing one to determine the best response. One frequently-used measure of incident severity is the total delay caused to travelers, because this captures the impact on all users of the system. Thus, the primary intent of this paper is to form an improved model for predicting incident delay. However, there are other possible measures of severity, such as the clearance time, length of queues caused by the incident, or the degree of personal injury and property damage. As a result, this model should only be viewed as one component of an integrated incident management policy.

There are models that can predict incident delay when provided the duration of closure. Although described more fully in the next section, this suggests a natural incident management framework, depicted in Figure 1. Incident management personnel learn characteristics of an incident, such as the number of vehicles involved or the weather conditions, through a variety of means. This information is then used to estimate incident duration, which in turn is used to predict delay caused by this incident, using models described in Section 2. Finally, this is used to enact the best response to this incident, with this process repeated as more information is learned and the prediction is refined.

However, incident duration can never be predicted with absolute certainty, and failure to account for this can lead to inefficient incident response. Particularly, using only the expected incident duration as input to the delay model will always give a wrong prediction and systematically underestimate expected incident delay. In essence, it is impossible to obtain an accurate prediction without considering the full range of possible incident durations.

The following sections give mathematical proof of this fact, but here we present a small motivating example which, while admittedly simplistic, illustrates this principle. A traffic management center learns of two incidents, A and B (see Table 1), but there is only one available incident response vehicle, so they decide to dispatch it to the incident which will cause the greater delay.

Based on the information they have about these incidents, they predict that Incident A will either last for 5 minutes, or for 25 minutes (with equal probability); thus the incident duration model reports an expected duration of 15 minutes. Incident B, on the other hand, will last exactly 17 minutes with absolute certainty. Typically, total incident delay is proportional to the square of incident duration. So, if the expected duration is used, one predicts a delay of 225 and 289 units for

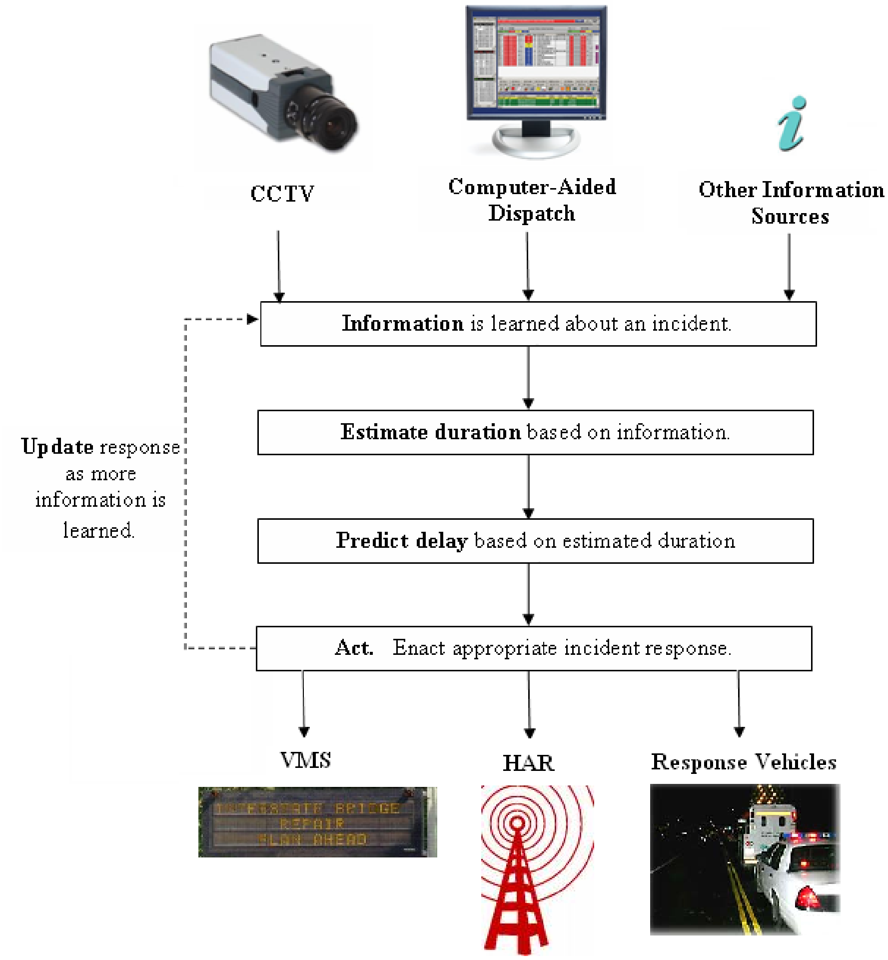

Figure 1: Incident delay prediction and response framework.

Incidents A and B, respectively; thus, the vehicle is sent to assist with Incident B.

Now, instead of using the average duration, let's calculate the delay for both scenarios. If Incident A only lasts 5 minutes, it will create a delay of 25 units, but if it lasts 25 minutes, it will create a delay of 625 units. Since these are equally likely, the true expected delay due to Incident A is the average of these, or 325 units. Since there is no uncertainty with Incident B, it will still have a delay of 289 units. Thus, when estimating delay in this manner, we conclude that the vehicle should be dispatched to Incident A instead!

This inconsistency does not occur because of the simplicity of this example; rather, in the next section, we show that this is due to fundamental principles of probability. Thus, it is incorrect to use the average duration to predict delay, and one must consider the underlying probability distribution. Surprisingly, to the authors' knowledge, this phenomenon has not been described in the incident management literature, implying that all current incident delay prediction models systematically underestimate true delay. Thus, this paper attempts to remedy this situation by proposing a *stochastic* incident delay prediction model.

In the next section, we describe past work on incident duration and delay prediction. In Section 3 we define the stochastic delay prediction problem more precisely, and derive exact formulae to account for incident duration probability distributions that have been encountered in the literature. Section 4 describes numerical results attained from multiple simulation experiments using Monte Carlo sampling to investigate this phenomenon in greater depth, for scenarios which are too complicated for exact analysis. Finally, Section 5 summarizes the key findings and concludes the paper.

2 Background

A variety of models have been proposed in the literature to predict delay caused by an incident: Wirasinghe [20] and Morales [14] develop analytical formulae based on the classical shockwave theory of Lighthill and Whitham [12]; given a predicted incident duration, one can estimate the resulting delay. Al-Deek et al. [1] propose an online prediction model using loop detectors to calculate delay *a posteriori*; while practical and useful in the case of multiple incidents (a property few incident delay prediction models share), this method as presented is not suited for real-time prediction, but rather post-incident analysis. However, Garib et al. [6] combine this work with a linear regression model that can predict delay while an incident is in progress.

While linear regression models have the advantage of simplicity, they carry several major disadvantages. In the context of incident management, the most serious is that they require an input value for each independent variable, at least nominally. While techniques exist to interpolate missing data from other observations (see, for instance, Bhat [2]), this reduces accuracy and complicates the prediction processs. Since information about incidents is often incomplete and obtained sequentially (rather than all at once), this suggests another method is more appropriate. Generally speaking, for delay prediction, models based on traffic flow theory are more transferable and flexible, since they are founded on more fundamental principles.

These delay models require the incident duration to be predicted beforehand, and provided as input. Many models exist to estimate incident duration: linear regression models are common, using characteristics such as incident type, weather condition, and number of vehicles and lanes involved as independent variables; Khattak et al. [9], Garib et al. [6], and Ozbay and Kachroo [16] use variations of this approach. Jones et al. [11] apply a Poisson regression on similar variables to incidents in the Seattle area. Giuliano [7] aggregates incidents into broad categories and estimates models for each category. Nam and Mannering [15] use hazard-based models which provide information not only on the total incident duration, but also on the probability that an incident, known to have already existed for a certain period of time, will be cleared in the next small time interval. Ozbay and Kachroo [16] construct decision trees which do not require knowledge of all observable incident characteristics. Smith and Smith [18] also suggest the use of nonparametric regression, where incident duration is estimated based on similar incidents in the past.

However, as demonstrated in the example in the previous section, using a single, expected value of incident duration always underestimates delay in the presence of uncertainty. This effect can be traced to an inequality of Jensen [10] regarding convex functions: if f is convex, and X is a random variable, we must have $E[f(X)] \geq f(E[X])$. Specifically, let f represent incident delay, and X represent the uncertain incident duration. Since f is proportional to the square of X, $f(X)$ is strictly convex, and thus the expected incident delay must be greater than delay which would result from an incident of exected duration. This result applies no matter what the distribution of incident duration is.

Therefore, we prefer incident duration models that give information about the entire distribution of possible incident durations to those which only report single values. Models of the former sort can be found in Golob [8] and Sullivan [19] in the form of lognormal distributions fitted to incident data, in Boyles et al. [4] as a naïve Bayesian classifier, and in Ozbay and Kachroo [17] with a Bayesian network. Bayesian models are particularly well suited to incident duration prediction under uncertainty, as they are robust to outliers and perform well under incomplete information. In the next section, we develop analytical formulae for total incident delay based on both the Bayesian models and lognormal regression.

3 Analytical Stochastic Delay Formulae

The analysis in this section is based on Wirasinghe's [20] delay formula. The other commonly-used formula is that of Morales [14] which is based on the same principles, but involves considerably more parameters. Thus, the approach demonstrated here is equally applicable to this formula as well, although a parallel derivation is omitted for brevity. In particular, Wirasinghe's formula for total delay due to a stationary incident is

$$D_u = \frac{1}{2}\tau^2 \frac{(q_1 - q_4)(q_3 - q_4)}{q_3 - q_1}$$

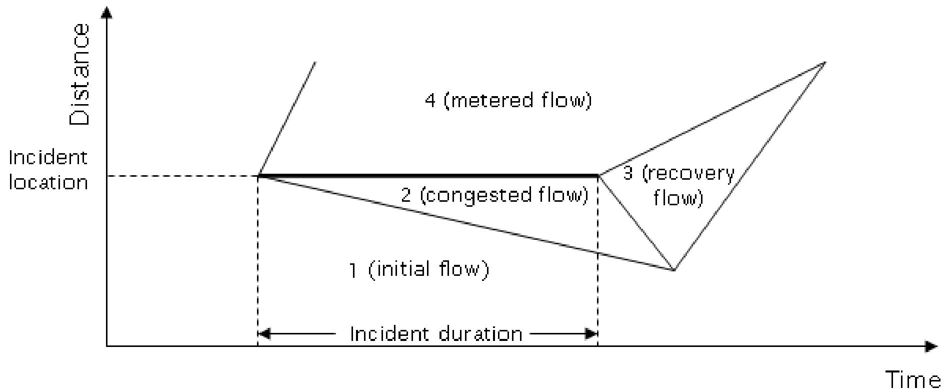

Figure 2: Idealized shockwave diagram for a stationary incident.

Table 2: Example output from a Bayesian incident duration model.

Incident duration range	(0,15)	(15, 25)	(25,35)	(35, 50)	(50, 75)
Probability	0.05	0.13	0.37	0.34	0.11

where D_u is total delay, τ is the incident duration, and q_1, q_3, and q_4 are the initial, recovery, and metered vehicle flow rates (see Figure 2 for identification of these regions on a shockwave diagram). Real-time values of q_1, q_3, and q_4 can be estimated from vehicle detectors in the area.

Assuming some probability distribution $f(\tau)$ for incident duration, we can express the expected delay as

$$E[D_u] = \frac{(q_1 - q_4)(q_3 - q_4)}{2(q_3 - q_1)} E[\tau^2] = \frac{(q_1 - q_4)(q_3 - q_4)}{2(q_3 - q_1)} \int_0^\infty \tau^2 f(\tau)\, d\tau$$

which, depending on f, can be evaluated analytically or numerically. Therefore, the remainder of this section considers different probability distributions for incident duration which have been proposed in the literature, and derives corresponding formulae. In particular, we consider probability distributions implied by Bayesian classification and the lognormal distribution, both of which are repeatedly seen in the incident duration prediction literature.

Bayesian methods classify incidents into categories based on incident duration; for instance, reporting the probability that an incident will last between 0 and 15 minutes, between 15 and 25 minutes, between 25 and 35 minutes, and so on (see Table 2 for an example). To calculate expected delay, we need to specify the distribution more precisely. The simplest approach is to assume a uniform distribution within each category, as in Figure 3, such that the area under the distribution in each category is equal to the probability that the incident lasts that duration.

Mathematically, let there be n categories of incident duration where each cate-

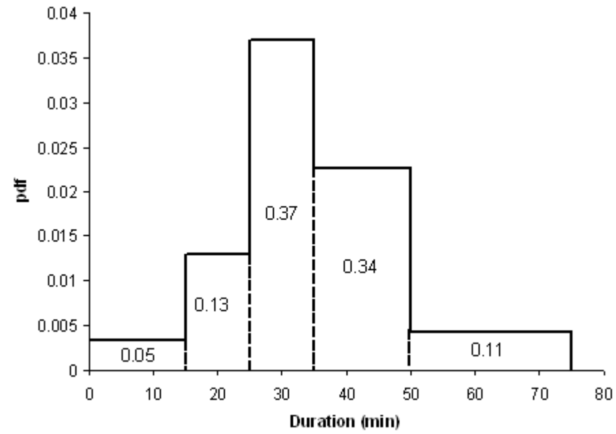

Figure 3: Probability density function corresponding to Bayesian example.

gory i contains incidents lasting longer than ξ_{i-1} but no longer than ξ_i units of time; the probability that an incident's duration is of category i is p_i. Thus these categories form the intervals $(\xi_0, \xi_1], (\xi_1, \xi_2], ..., (\xi_{n-1}, \xi_n]$ and the probability density function can be written

$$f(\tau) = \begin{cases} 0 & \tau \leq \xi_0 \\ \dfrac{p_i}{\xi_i - \xi_{i-1}} & \xi_{i-1} < \tau \leq \xi_i \\ 0 & \tau > \xi_n \end{cases}$$

Then

$$\begin{aligned} E[\tau^2] &= \int_0^\infty \tau^2 f(\tau)\, d\tau \\ &= \sum_{i=1}^n \int_{\xi_{i-1}}^{\xi_i} \frac{p_i \tau^2}{\xi_i - \xi_{i-1}}\, d\tau \\ &= \sum_{i=1}^n \frac{p_i}{3} \frac{\xi_i^3 - \xi_{i-1}^3}{\xi_i - \xi_{i-1}} \\ &= \sum_{i=1}^n \frac{p_i}{3} \left(\xi_i^2 + \xi_i \xi_{i-1} + \xi_{i-1}^2\right) \end{aligned}$$

and

$$E[D_u] = \frac{(q_1 - q_4)(q_3 - q_4)}{2(q_3 - q_1)} \sum_{i=1}^n \frac{p_i}{3} \left(\xi_i^2 + \xi_i \xi_{i-1} + \xi_{i-1}^2\right)$$

Alternately, in some cases, the n-th interval may be unbounded; that is, it contains all incidents which have length greater than ξ_{n-1}, and has the form (ξ_{n-1}, ∞).

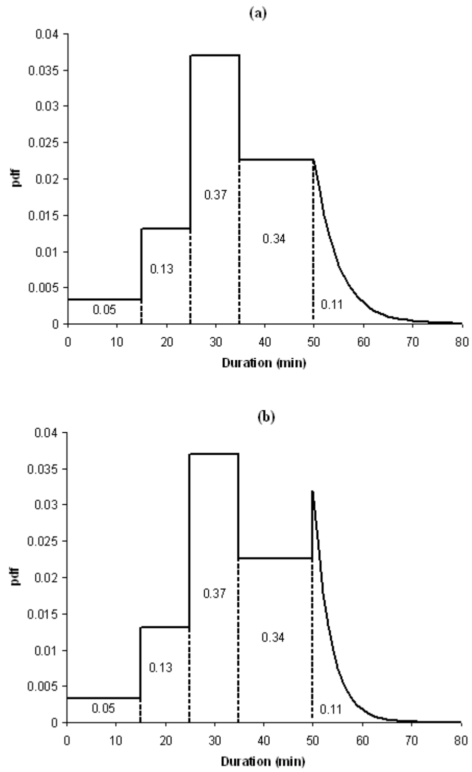

Figure 4: Choices of λ resulting in (a) continuous and (b) discontinuous densities.

In this case, a uniform distribution cannot be used for this last category. A suitable substitute is the exponential distribution, because (1) it has unbounded support, (2) the probability of longer and longer incidents approaches zero asymptotically, (3) long duration incidents may be characterized by a relatively constant hazard rate, and (4) it is tractable and amenable to analysis.

We must choose an appropriate rate parameter λ for the exponential distribution; here we choose this such that the density function is continuous at ξ_{n-1} to avoid a sudden jump at this point (see Figure 4). More precisely, let $\overline{\tau} = \max\{\tau - \xi_{n-1}, 0\}$ represent the additional duration past ξ_{n-1}, so $\overline{\tau}$ has an exponential distribution, given $\tau > \xi_{n-1}$. To preserve continuity, we must choose $\lambda = f(\xi_{n-1}) = \dfrac{p_{n-1}}{p_n\left(\xi_{n-1} - \xi_{n-2}\right)}$. Thus

$$E[\tau^2] = \sum_{i=1}^{n} \frac{p_i}{3}\left(\xi_i^2 + \xi_i\xi_{i-1} + \xi_{i-1}^2\right) + p_n E[\tau^2|\tau > \xi_{n-1}]$$

To evaluate the latter term, realize that

$$\begin{aligned} E[\tau^2|\tau > \xi_{n-1}] &= E\left[(\overline{\tau} + \xi_{n-1})^2|\tau > \xi_{n-1}\right] \\ &= E\left[\overline{\tau^2} + 2\overline{\tau}\xi_{n-1} + \xi_{n-1}^2|\tau > \xi_{n-1}\right] \\ &= \frac{2}{\lambda^2} + \frac{2\xi_{n-1}}{\lambda} + \xi_{n-1}^2 \end{aligned}$$

using linearity of expected value, and well-known formulas for the first two moments of the exponential distribution. Substituting for λ and back into the previous equation, we obtain the final formula

$$\begin{aligned} E[\tau^2] = \sum_{i=1}^{n-1} \frac{p_i}{3}\left(\xi_i^2 + \xi_i\xi_{i-1} + \xi_{i-1}^2\right) \\ + 2p_n\left(\frac{p_n^2(\xi_{n-1} - \xi_{n-2})^2}{p_{n-1}^2} + \frac{p_n\xi_{n-1}(\xi_{n-1} - \xi_{n-2})}{p_{n-1}} + \frac{\xi_{n-1}^2}{2}\right) \end{aligned}$$

Another common assumption is a lognormal distribution fitted using regression. Recall that the lognormal distribution is characterized by two parameters μ and σ^2, and its density function is

$$f(\tau) = \frac{1}{\sqrt{2\pi\sigma^2}\tau} e^{(\log\tau - \mu)^2/2\sigma^2}$$

for $\tau \geq 0$. Thus

$$\begin{aligned} E[D_u] &= E\left[\frac{1}{2}\tau^2\frac{(q_1 - q_4)(q_3 - q_4)}{q_3 - q_1}\right] = \frac{(q_1 - q_4)(q_3 - q_4)}{2(q_3 - q_1)} E\left[\tau^2\right] \\ &= \frac{(q_1 - q_4)(q_3 - q_4)}{2(q_3 - q_1)}\left\{Var[\tau] + (E[\tau])^2\right\} \\ &= \frac{(q_1 - q_4)(q_3 - q_4)}{2(q_3 - q_1)}\left\{e^{2\mu+\sigma^2}(e^{\sigma^2} - 1) + e^{2\mu+\sigma^2}\right\} \end{aligned}$$

using well-known values for the mean and variance of the lognormal distribution. Simplifying, we obtain

$$E[D_u] = \frac{(q_1 - q_4)(q_3 - q_4)}{2(q_3 - q_1)} e^{2(\mu+\sigma^2)}$$

4 Simulation Results

While the previous section presents analytical extensions of traditional delay formulas to account for uncertain incident duration, these formulas are not always applicable in practice without further refinement, due to several assumptions made in the derivation. In particular, having a uniform initial region in a space-time

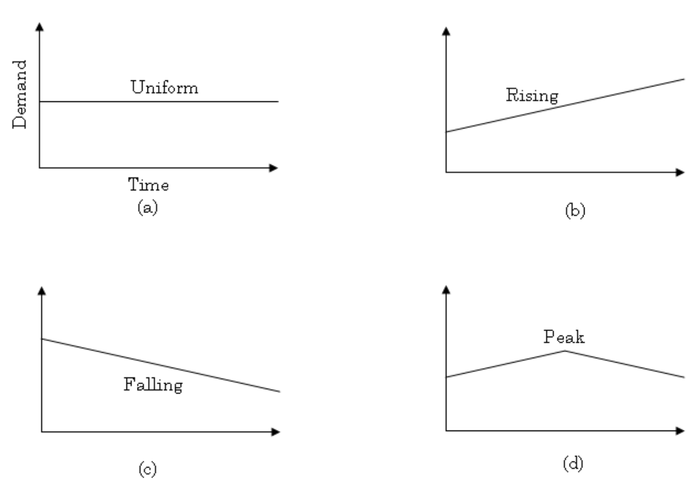

Figure 5: Four demand profiles used for simulation.

diagram (region 1 in Figure 2) requires the assumption of constant vehicle demand. While this may be applicable for incidents of short duration, or those that occur during off-peak periods, this assumption becomes problematic when considering extended incidents during periods of heavy congestion – which is precisely when these models are most useful.

Thus, we employ a simulation approach to both validate the theoretical results for the case of uniform demand, and investigate the impacts of varying demand. These simulations are based on the cell transmission model developed by Daganzo [5], a mesoscopic simulator which is consistent with basic shockwave theory, yet well suited to efficient computer implementation. We consider the impact of an incident on an isolated three-lane freeway segment, with initial capacity if 6600 vehicles per hour. For the duration of the incident, freeway capacity is reduced to 3000 vehicles per hour, perhaps representing complete blockage of one lane and additional capacity reduction due to the presence of emergency vehicles and merging at the incident site. The duration of the incident is obtained from a truncated lognormal distribution with parameters $\mu = 3$ and $\sigma = 1.6$, with a maximum incident duration of approximately 50 minutes. This choice of distribution results in an average incident length of approximately 15 minutes, with a standard deviation of 9 minutes.

Four different demand profiles are considered, four of which are shown in Figure 5. Figure 5(a) shows a uniform profile, where demand is constant over the analysis period. As mentioned above, this is the assumption in the Wirasinghe and Morales formulae. Figure 5(b) indicates "rising" demand where the number of travelers is increasing (as at the beginning of a peak period), while Figure 5(c) indicates "falling" demand, as would occur at the end of a peak period. Figure 5(d) shows a "peaking" profile where demand first rises, then falls. Regardless of the profile, the average demand during the incident is 5000 vehicles per hour, indicating heavy

Table 3: Total delay results from Monte Carlo simulation (all values in vehicle-hours).

Profile	Average delay	Standard deviation	Skewness
Uniform	179.60	247.93	1.69
Rising	122.30	137.07	1.04
Falling	343.58	524.22	1.95
Peak	206.67	281.69	1.65

(but uncongested) flow prior to the incident, but a congested state exceeding the freeway capacity during the incident.

For each of these demand profiles, we simulate 1000 incidents, obtaining their duration from Monte Carlo sampling applied to the truncated lognormal distribution described above; these are attained by applying the Box-Muller transform [3] to generate samples from a normal distribution, then exponentiating these to generate lognormal samples. To determine the delay caused by an incident, the travel time corresponding to each demand profile without any incidents was determined through simulation; this was subtracted from the total travel time when incidents occured, this giving the additional delay due to the incident. Table 3 summarizes these resuls.

Clearly, the average delay differs substantially according to how demand is changing while the incident occurs. Compared to the uniform profile, delay was lower when demand was rising during the incident, and higher when demand was falling. This makes intuitive sense: when demand is falling, more vehicles encounter the incident while it is in progress, and thus must face a long queue. When demand is rising, the majority of vehicles encounter the incident after it has cleared, and encounter the queue in a recovery state, rather than a lengthening state. All profiles exhibit positive skewness, indicating asymmetry: when an incident is worse than average, its effects are much worse than average; when an incident is better than average, its effects are only slightly less than average.

Since the motivation for this work is delay underestimation that occurs when using just the average value of incident duration, we quantify the benefit of accounting for uncertainty by comparing the prediction yielded by this model to what is predicted by simulating a single incident of average duration. These results are shown in Table 4, and show that, depending on the profile, only using a single average value underestimates true average delay by 20-50%, certainly a significant amount. Using the average value with a falling demand profile results in particularly bad underestimation.

Also, as shown in the previous section, for the case of uniform demand, incident delay is directly proportional to the square of incident duration. For other demand profiles, the relationship will not be exact, but the ratio of delay to squared duration still gives an indication of the rate at which incident delay increases with delation. Table 5 shows the average and standard deviations of this ratio for the different demand profiles. The zero value of standard deviation for the uniform profile indicates that the proportionality relationship is exact; for the other profiles,

Table 4: The degree of underestimation from ignoring stochasticity.

Profile	True average delay	Estimated delay	Underestimation
Uniform	179.60	106.22	41%
Rising	122.30	95.49	22%
Falling	343.58	159.04	54%
Peak	206.67	119.63	42%

Table 5: Proportionality results from simulation.

Profile	Average ratio	Standard deviation
Uniform	0.50	0.00
Rising	0.45	0.08
Falling	0.75	0.18
Peak	0.56	0.04

the larger the standard deviation, the more the delay-duration relationship differed from quadratic. Combining this result with the skewness results from Table 3, it is evident that even when accounting for uncertainty, the uniform demand assumption may introduce additional underprediction in delay estimation, depending on the actual demand profile.

5 Conclusion

Although a variety of incident management strategies exist to reduce the impact of these events on a roadway network, limited resources and implementation requirements require decisions to be made about which strategies to apply to which incidents. Thus, it is useful to be able to predict both the duration and the delay caused by a given incident. In particular, models to do both can be used in sequence, with the former providing input for the latter, which can in turn be used to guide the incident management process.

In this paper, we considered the impact of uncertainty in predicting incident duration, and show that failing to properly account for this will inevitably result in underestimating the delays caused by incidents, possibly up to a factor of two. Since limited knowledge and uncertainty are highly characteristic of incident management, and since, to the authors' knowledge, no previous incident delay prediction model explicitly incorporates these factors, filling this gap is vital for optimally managing incidents and maximizing system performance.

In addition to providing the stochastic counterparts to well-known analytic delay prediction formulas, we conduct a series of computer simulation experiments to a variety of changing demand profiles, applying Monte Carlo sampling to draw conclusions about the impact of uncertainty for scenarios which are too complicated for analytic formulas. As a result, it appears that accounting for how demand is changing plays a major role in delay prediction; incidents at the beginning of the peak period will have a much smaller impact than incidents occuring just as demand begins to decrease. While no computer model or formula is sophisticated enough to replace the judgment and experience of a trained human operator, we nevertheless believe that such models can supplement human knowledge and experience, leading to improved incident management and, in the end, to improved freeway performance.

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