DTA2012 Symposium: A Continuous DUE Algorithm Using the Link Transmission Model

Nezamuddin Stephen D. Boyles

Abstract

This paper describes a continuous-flow, continuous-time model for which a dynamic Wardrop equilibrium provably exists. This formulation is general, but is particularly designed to include the link and node models of Yperman's Link Transmission Model as a special case. Rather than using path flows to describe route choice, travelers are aggregated by destination and node-specific routing parameters are used to reduce the number of control variables needed. Furthermore, this formulation allows efficient solution methods from static traffic assignment, such as Linear User Cost Equilibrium (LUCE), to be applied in a fairly straightforward manner. Demonstrations on a small network verify the effectiveness of this dynamic LUCE algorithm in our model, showing favorable performance comapred to the method of successive averages.

1 Introduction

Despite several decades of active research and growing acceptance among practitioners, dynamic traffic assignment (DTA) models have yet to reach their full potential in transforming traffic assignment in the field. While explicitly representing the time-dependent nature of travel demand and congested behavior can model important phenomena more accurately, including traffic control and queue spillbacks, doing so introduces a host of additional modeling difficulties not present in static models. Broadly speaking, two major difficulties concern the nature of the traffic flow model employed, and the nature of dynamic equilibrium itself. These are briefly explained below; readers seeking additional detail can consult Peeta and Ziliaskopoulos (2001) for a more thorough review of DTA approaches.

These difficulties can be highlighted by comparing approaches which have been developed to date. The first class of DTA models was based on analytic representation of link delay using "exit functions" (Merchant and Nemhauser, 1978a,b). While such functions lend themselves to convergence analysis, properly enforcing first-in first-out (FIFO) discipline results in nonconvex constraints (Carey, 1992) which form an obstacle to large-scale application. Another class of DTA formulations attempts to simplify a traffic-flow model so that it can scale tractably to large-scale systems; such efforts often approximate the hydrodynamic model of Lighthill and Whitham (1955) by introducing a piecewise-linear flow-density diagram, and applying a space-time discretization to form the cell-transmission model (Daganzo, 1994, 1995), or methods based on cumulative counts (Newell, 1993a,b,c), as in the link transmission model (LTM) (Yperman, 2007). These issues are compounded when questions of traffic control arise, including signals, roundabouts, or stop-controlled junctions. In all cases, a tension exists between accuracy and tractability, and the lack of consensus among researchers likely hinders adoption of DTA in practice.

The second, more subtle difficulty is central to the definition of dynamic equilibrium. Many practical DTA packages employ a mesoscopic simulator to represent traffic flow, in which individual vehicles are propagated through the network. From a mathematical standpoint, these models are discontinuous in the representation of vehicles, and often discontinuous in the representation of traffic control as well (in the case of signals, for instance). In such cases, it is not difficult to produce examples where *no* dynamic equilibrium exists. Even when continuous models are employed, the equilibrium solution need not be unique (see, for instance Nie, 2010). These may seem to be technical issues more than practical ones, but are in fact fundamental to the application and interpretation of DTA models in practice — if multiple equilibria exist, or no equilibria, then what should planners design for?

This paper describes a continuous-vehicle, continuous-time dynamic equilibrium formulation which addresses these issues. The corresponding traffic flow model is relatively general; for concreteness the demonstrations in this paper use the link transmission model (LTM) developed by Yperman (2007) and further developed by Gentile (2010) and others. The LTM is amenable to representing traffic control, building on established results from traffic operations, such as the Highway Capacity Manual. In particular, continuity can be provided by representing *average* delay due to traffic control, rather than explicitly simulating gap acceptance and signal control. This reduces computational delay and, as Yperman (2007) argues, travelers presumably base route choice on average control delay anyway, so the loss of "accuracy" is negligible. Continuous vehicle flows are addressed by using node routing parameters, avoiding path enumeration and aggregating travelers by destination.

Section 2 presents the model and associated notation, and Section 3 defines and proves existence of at least one dynamic Wardrop equilibrium. Section 4 give two potential solution methods, the well-known method of successive averages, and a method resembling the LUCE algorithm from static traffic assignment, utilizing derivative information from the flow model. These methods are demonstrated on a small network in Section 5 before conclusions are given in Section 6

2 Notation and Model

Let the network $G = (N, A, Z)$ have node set N, link set A, and zone set Z. Let M denote the set of turning movements $\{(i, j, k) : (i, j) \in A, (j, k) \in A\}$. Not all potential turning movements need exist in the set M; only those permissible for use by travelers need be included. Turning movements are used to represent control delay and capacity restrictions at intersections; links are used to represent delay and flow restrictions which do not occur at intersections. Let $\mathcal{T} = [0, \overline{T}]$ denote the temporal modeling period. For each origin r and destination s, $d^{rs}(t)$ denotes the rate of demand for travel between these zones at time t.

The fundamental state variables are given by the flow at the upstream and downstream ends of each link ($q^{\uparrow}_{ij}(t)$ and $q^{\downarrow}_{ij}(t)$, respectively) and turning movement ($q^{\uparrow}_{ijk}(t)$ and $q^{\downarrow}_{ijk}(t)$). These are supplemented by auxiliary state variables $\mathbf{x_{ij}}(t)$ and $\mathbf{x_{ijk}}(t)$ denoting exogenous parameters relevant to the traffic-flow model, such as free-flow times and capacities. The history of the system can be conveniently represented using cumulative vehicle counts of the form $N(t) = \int_0^t q(t')dt'$ defined identically for the upstream and downstream ends of links and turning movements; the *full history* $N[t_i, t_j)$ represents the graph of the mapping N on the half-open interval $[t_i, t_j) \subset \mathcal{T}$, that is, $N[t_i, t_j) = \{(t, n) : t \in [t_i, t_j), n = N(t)\}$.

Rather than representing route choice with an explicit path enumeration, travelers are aggregated by destination[1] and their route choices represented by *routing parameters* $\phi^s_{ijk}(t)$ representing the proportion of travelers who arrive at node j from link (i, j) at time t destined for zone s who choose to leave by link (j, k). This dramatically reduces the number of control variables required to represent all feasible route choices, without eliminating any potential equilibria. This equivalence is discussed in more detail following the presentation of the remainder of the flow model.

The flow variables, cumulative counts, and full histories are disaggregated by destination; thus the *full state* of a link is given by $S_{ij}(t) = \{\mathbf{x_{ij}}(t), \times_{s \in Z}(N^{\uparrow s}_{ij}[0, t), N^{\downarrow s}_{ij}[0, t))\}$. Likewise, the full state of a node j is $S_j(t) = \{\times_{(i,j,k) \in M}(\mathbf{x_{ijk}}(t), \times_{s \in Z}(N^{\uparrow s}_{ijk}[0, t), N^{\downarrow s}_{ijk}[0, t)))\}$ and the system evolution is governed by

[1]Unlike in static traffic assignment, origin- and destination-based aggregations are not identical in dynamic assignment, and only a destination-based aggregation is sensical.

$$q_{ijk}^{\uparrow s}(t) = \phi_{ijk}^{s}(t) q_{ij}^{\downarrow s} \qquad \forall (i,j,k) \in M \tag{1}$$

$$q_{ij}^{\uparrow s}(t) = \sum_{(h,i,j)\in M} q_{ijk}^{\downarrow s} \qquad \forall (i,j) \in A \tag{2}$$

$$q_{ijk}^{\downarrow s}(t) = f_{jk}^{\uparrow}(S_j(t), S_{jk}(t)) \qquad \forall (i,j,k) \in M \tag{3}$$

$$q_{ij}^{\downarrow s}(t) = f_{ij}^{\downarrow}(S_j(t), S_{ij}(t)) \qquad \forall (i,j) \in A \tag{4}$$

where $f^{\uparrow}$ and $f^{\downarrow}$ represent the flow propagation functions at the upstream and downstream ends of the links; the upstream functions include the role of the upstream intersection.

These four equations can be explained as follows; the reader may find it useful to consult Figure 1 for a graphical depiction. Equation (1) splits flow arriving at a node according to the proportions $\boldsymbol{\phi}$; note the disaggregation by destination. Conversely, equation (2) gathers flow leaving a node onto a particular arc from all corresponding turn movements. The two equations (3) and (4) are the most interesting in the sense that these can distinguish one flow model from another. Equation (3) defines the rate at which vehicles exit turning movement (i, j, k), which can depend on the nature of the traffic control corresponding to this movement, the state of the node (for instance, the volume on conflicting movements), and the state of the downstream link (for instance, if the entire link is congested and no flow may enter). Equation (4) defines the rate at which vehicles exit a link to be split onto turning movements, as a function of the link parameters (such as capacity) and the state of the downstream node j.

Origins and destinations can be handled in this framework without loss of generality by introducing additional turning movements. Trips originating at origin r begin at the downstream end of an artificial arc with r as its head whose downstream flows $q_{ir}^{\downarrow s}$ are given by the trip table, essentially forming a boundary condition. Likewise, upon arrival at a destination, trips are all routed onto an artificial turning movement with infinite capacity.

The link transmission model developed by Yperman (2007) can be seen as a special case of the framework (1)–(4). The correspondence is as follows: for each link, the parameters $\mathbf{x_{ij}}(t)$ contain the link's free-flow time T_{ij}^0, length L_{ij}, jam density, K_{ij}, and capacity Q_{ij}, and the destination-specific upstream/downstream cumulative counts imply the aggregate upstream/downstream counts needed to propagate flow using Newell's method. The node states vary according to the type of node (merge, diverge, etc.) and include information specific to each turning movement (priority parameters for merge, movement capacities, and so forth). The

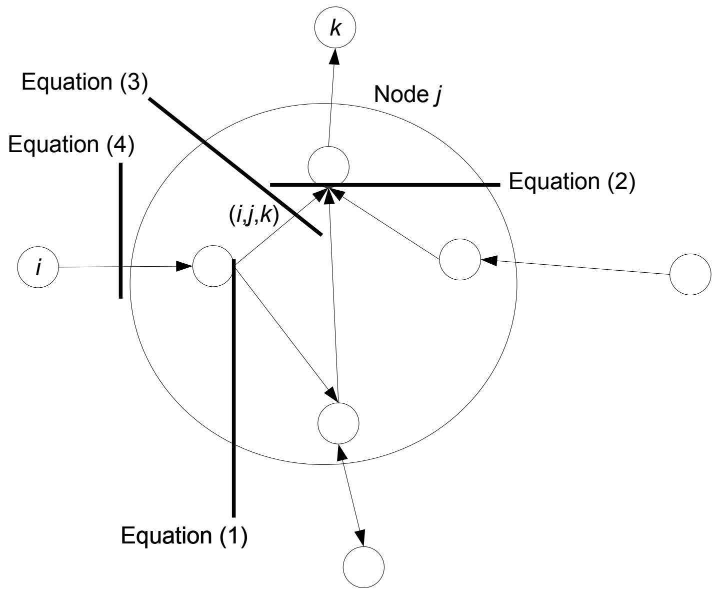

Figure 1: Distinction between links and turning movements, and location where each equation acts. Note that not all turning movements are permissible.

link and node flow models in the LTM correspond to the functions $f^{\uparrow}$ and $f^{\downarrow}$ in equations (3) and (4), respectively.

With the flow model specified, we can show that the routing parameter formulation is equivalent to a path-based formulation, in the following sense: given any particular set of time-dependent path departures, there exists a vector $\boldsymbol{\phi}$ such that the node and link states $S_j(t)$ and $S_{ij}(t)$ are identical for all $i \in N$, $(i,j) \in A$, and $t \in \mathcal{T}$. Let $h_{\pi}^{rs}(t)$ denote the number of travelers departing origin r for destination s via path π at time t. Given these path flows, $q_{ijk}^{\uparrow s}(t)$ is uniquely determined for all s, i, j, k, and t from the model (2)–(4) (where (1) is not needed because the path flows determine route choices). Thus, we can define $\phi_{ijk}^{s}(t) \equiv q_{ijk}^{\uparrow s}(t)/q_{ij}^{\downarrow s}$ whenever $q_{ij}^{\downarrow s} > 0$, and arbitrarily otherwise; this clearly produces equation (1), which can be adopted instead of specifying path flows.

3 Equilibrium Formulation

Together with boundary conditions representing the time-dependent OD matrix, the above system of equations defines a generic dynamic flow model. Define the leaving time of the n-th vehicle on link (i,j) to be $\tau_{ij}^{\downarrow}(n) = \inf\{N_{ij}^{\downarrow} = n\}$ and the travel time to be $T_{ij}(t) = \max\{\tau_{ij}^{\downarrow}(N_{ij}^{\uparrow}(t)) - t, T_{ij}^{0}(t)\}$. The travel time of a turning movement is defined similarly. The travel time to destination s from turning movement (i,j,k) is denoted as

$$C^s_{ijk}(t) = T_{ijk}(t)+T_{jk}(t+T_{ijk}(t))+\sum_{(j,k,\ell)\in M}\phi^s_{jk\ell}(t+T_{ijk}(t)+T_{jk}(t+T_{ijk}(t)))C^s_{jk\ell}(T_{ijk}(t)+T_{jk}(t+T_{ijk}(t)))$$

together with the boundary condition $C^s_{jks} \equiv 0$.

With this in mind, the definition of Wardrop's principle in the context of our model can be given as follows:

Definition 1. Wardrop's principle. *The routing parameters ϕ satisfy Wardrop's equilibrium principle if*

$$\phi^s_{ijk}(t) > 0 \Rightarrow C^s_{ijk}(t) = \min_{(i,j,k')\in M}\{C^s_{ijk'}(t)\}$$

for all s, i, j, k, and t, where the functions $\mathbf{C}$ are determined by $\boldsymbol{\phi}$ as described above.

Existence of equilibrium can be shown under regularity assumptions on the travel time functions $C^s_{ijk}(t)$ which are satisfied if the flow propagation functions f are continuous. This is shown in the following proposition, which defines a correspondence $\mathrm{Br}(\boldsymbol{\phi})$ returning the set of routing parameters placing positive weight only on minimum-cost turning movements, assuming that the travel times are determined by *fixed* routing parameters $\boldsymbol{\phi}$. Every fixed point of this correspondence (that is, every $\boldsymbol{\phi^*}$ such that $\boldsymbol{\phi^*} \in \mathrm{Br}(\boldsymbol{\phi^*})$) is a Wardrop equilibrium. The remainder of this section establishes the existence of at least one fixed point to this mapping by establishing properties of the feasible sets of routing parameters and the correspondence Br and appealing to Kakutani's Theorem (Kakutani, 1941).

Define the function norm $||f|| = \sup_{t\in\mathcal{T}} f(t)$, so a sequence of functions f^k converges to a function f^* if this sequence converges uniformly over $\mathcal{T}$. Let Φ denote the set of feasible routing parameters $\Phi = \{\phi^s_{ijk}(t) : \sum_{(j,k)\in A}\phi^s_{ijk}(t) = 1, \phi^s_{ijk}(t) \geq 0\}$ and $\mathbf{T}(\boldsymbol{\phi})$ the vector of travel time functions given routing parameters $\boldsymbol{\phi} \in \Phi$ under the above definitions for a flow pattern satisfying the model (1)–(4). Define the best response correspondence $\mathrm{Br} : \Phi \to 2^\Phi$ to be

$$\mathrm{Br}(\mathbf{T}(\boldsymbol{\phi})) = \times_{s\in Z,(i,j,k)\in M}\{\phi^s_{ijk}(t) \in \Phi : \phi^s_{ijk}(t) > 0 \Rightarrow C^s_{ijk}(t) = \min_{(i,j,k')\in M}\{C^s_{ijk'}(t)\}\} \quad (5)$$

Φ is clearly nonempty, compact, and convex.

For any $\boldsymbol{\phi}$, the set $\mathrm{Br}(\boldsymbol{\phi})$ is nonempty because the set of turning movements is finite. Furthermore, this set is convex: let $\boldsymbol{\phi_1}$ and $\boldsymbol{\phi_2}$ be arbitrary elements of $\mathrm{Br}(\boldsymbol{\phi})$, and consider $\lambda \in [0, 1]$. For any strictly positive component of $\lambda\boldsymbol{\phi_1} + (1-\lambda)\boldsymbol{\phi_2}$, we must have the corresponding component of either $\boldsymbol{\phi_1}$ or $\boldsymbol{\phi_2}$ to be strictly positive, and therefore representing a minimum-cost turning movement; thus $\lambda\boldsymbol{\phi_1} + (1-\lambda)\boldsymbol{\phi_2} \in \mathrm{Br}(\boldsymbol{\phi})$ as well.

Finally, the graph $\{\boldsymbol{\phi}, \mathrm{Br}(\boldsymbol{\phi})\}$ is closed: consider any sequence $(\boldsymbol{\phi_n}, \mathrm{Br}(\boldsymbol{\phi_n})$ converging to the point $(\hat{\boldsymbol{\phi}}, \hat{y})$. Since Φ is closed, $\hat{\boldsymbol{\phi}}$ is a feasible vector of routing parameters. Consider any s, i, j, k, and t such that $\hat{\phi}^s_{ijk}(t) > 0$. There exists $N^s_{ijk}(t)$ such that $n > N^s_{ijk}(t) \Rightarrow (\phi_n)^s_{ijk}(t) > 0$. For all such n and for all $(i,j,k') \in M$, we have $C^s_{ijk}(t) - C^s_{ijk'}(t) \leq 0$ from the definition of the sequence. This holds at the limit if the mapping $\mathbf{C}$ is continuous, proving that the graph is closed. If the functions $f^{\uparrow}$ and $f^{\downarrow}$ in equations (3) and (4) are continuous in their arguments, and if the node and movement parameters $\mathbf{x_{ij}}$ and $\mathbf{x_{ijk}}$ are continuous, $\boldsymbol{\tau}$ is continuous in $\boldsymbol{\phi}$, in turn implying continuity of $\mathbf{T}$ and $\mathbf{C}$. Thus, having established all of the necessary conditions, Kakutani's Theorem asserts the existence of a fixed point of Br, which corresponds to a dynamic Wardrop equilibrium.

4 Solution Methods

In practice, the continuous model described above is solved by choosing a suitably fine time discretization and solving equations (1)–(4) in increasing order of time, interpolating between time intervals as necessary. The remaining question is how to find equilibrium routing parameters. Given the generality of the flow model presented in the previous section, we do not aim to provide a provably convergent algorithm for all instances (a notion which in any case necessitate a careful definition given the potential nonuniqueness of equilibria); rather, the more modest aim in this paper is to present two heuristic strategies. Both strategies are tested in the following section in a small network, and appear to converge towards equilibrium.

The first method is the well-known method of successive averages (MSA), which can be stated as follows:

1. Initialize $\boldsymbol{\phi}$ by choosing some vector such that $\phi^s_{ijk}(t) = 1$ and $\phi^s_{ijk}(t) \geq 0$ for all $(i,j,k) \in M$.

2. Load flow by solving (1)–(4) in increasing order of time.

3. Calculate travel times $\mathbf{T}$ and times to destinations $\mathbf{C}$.

4. Choose a "best response" $\boldsymbol{\phi}^* \in \mathrm{Br}(\mathbf{T}(\boldsymbol{\phi}))$.

5. Update routing parameters: $\boldsymbol{\phi} \leftarrow \lambda\boldsymbol{\phi} + (1-\lambda)\boldsymbol{\phi}^*$ for some $\lambda \in [0,1]$.

6. Check for convergence; if not converged, return to step 2.

A few notes on this procedure:

- One natural choice for initializing the algorithm in Step 1 is to choose $\boldsymbol{\phi}$ to place unit weight on the shortest path trees rooted at each destination, using free-flow travel times. Alternately, a "warm-start" can be applied if one has an equilibrium or approximate equilibrium on a similar network. See Nezamuddin (2011) for further discussion on warm-starting DTA models, and for empirical comparison of several options.

- Step 3 essentially finds a *local* best response; there is no shortest path algorithm which is run over the entire network.

- In step 5, λ is often taken to be the reciprocal of the iteration number, but other choices are possible as well. (Bar-Gera and Boyce, 2006)

- In step 6, one potential convergence measure is the average excess cost, defined in the following section.

The second method attempts to use derivative information from the mapping $\mathbf{C}(\mathbf{T}(\boldsymbol{\phi}))$ to improve the rate of convergence. Such methods have been found effective in other DTA algorithms, such as B-Dynamic (Ramadurai and Ukkusuri, 2011). In particular, for now assume that $T'_{ij}(t) \equiv \partial T_{ij}(t)/\partial q_{ij}(t)$ exists for all (i,j), s, and t. Then we can define a derivative of the travel time from the upstream end of a turning movement to the destination using the recursion

$$D^s_{ijk}(t) = T'_{ijk}(t)+T'_{jk}(t+T_{ijk}(t))+\sum_{(j,k,\ell)\in M}(\phi^s_{jk\ell})^2(t+T_{ijk}(t)+T_{jk}(t+T_{ijk}(t)))D^s_{jk\ell}(T_{ijk}(t)+T_{jk}(t+T_{ijk}(t))) \tag{6}$$

and the boundary condition $D^s_{jks} \equiv 0$.

Then, we can solve for an approximate local equilibrium using the linear approximations $C^s_{ijk}(\phi^{s*}_{ijk}) \approx C^s_{ijk} + (\phi^{s*}_{ijk} - \phi^s_{ijk})D^s_{ijk}$ in a matter analogous to the LUCE algorithm Gentile (2009) developed for static traffic assignment, such that $\phi^{s*}_{ijk} > 0 \Rightarrow C^s_{ijk}(\phi^{s*}_{ijk}) = \min_{ijk'} C^s_{ijk'}(\phi^{s*}_{ijk'})$. This can be done extremely quickly, and the actual routing parameters updated using an averaging rule such as that in MSA. The algorithm can be presented as

1. Initialize $\boldsymbol{\phi}$ by choosing some vector such that $\phi^s_{ijk}(t) = 1$ and $\phi^s_{ijk}(t) \geq 0$ for all $(i,j,k) \in M$.

2. Load flow by solving (1)–(4) in increasing order of time.

3. Calculate travel times $\mathbf{T}$, times to destinations $\mathbf{C}$, and derivatives $\mathbf{D}$.

4. Choose $\boldsymbol{\phi}^*$ by solving "local" equilibrium problems using linear approximations based on $\mathbf{C}$ and $\mathbf{D}$.

5. Update routing parameters: $\boldsymbol{\phi} \leftarrow \lambda\boldsymbol{\phi} + (1-\lambda)\boldsymbol{\phi}^*$ for some $\lambda \in [0,1]$.

6. Check for convergence; if not converged, return to step 2.

Because of the derivative information incorporated, the weighting factor λ can be higher than that in MSA. In the experiment reported in the following section, the algorithm starts with $\lambda = 1$, halving λ whenever a gap measure increases between successive iterations, and leaving it unchanged otherwise.

The model defined by (1)–(4) may not be everywhere differentiable; however, when f_{ijk} and f_{ij} are given by the link transmission model, the mapping $\mathbf{C}(\mathbf{T}(\boldsymbol{\phi}))$ is piecewise differentiable with $dT_{ij}/dq^{\uparrow}_{ij} = 1/Q_{ij}$ if there is a downstream bottleneck and 0 otherwise. In the demonstration reported in the following section, one of these two values was chosen arbitrarily whenever a derivative was required at the intersection of these pieces; determining whether this is generally a wise strategy is a worthwhile topic for future research.

5 Demonstration

Both of the algorithms described in the previous section are tested on the familiar Braess network (Figure 2) containing four nodes and five links. Units are chosen so that each link is given unit length and free-flow time, a capacity of 50 vehicles per unit time, and a jam density of 200 vehicles per link. Travel demand is 100 vehicles per unit time for $t \in [0, 11]$ and zero thereafter. The initial solution is generated by splitting flows evenly at every diverge, that is, $\phi_{\cdot 12}(t) = \phi_{\cdot 13}(t) = \phi_{123}(t) = \phi_{124}(t) = 1/2$.

Figures 3 and 4 plot the progress of the two algorithms according to two metrics; numerical values can be found in Table 1. The average excess cost, used in Figure 3, is the difference between the total experienced travel time, and the theoretical total travel time obtained if all travelers experienced the shortest possible travel times between their origin and destination among all available paths, scaled by the total travel demand. Mathematically, let the shortest path travel time from the upstream end of turning movement (i,j,k) to destination s be given by

$$U^s_{ijk}(t) = T_{ijk}(t) + T_{jk}(t + T_{ijk}(t)) + \min_{(j,k,\ell)\in M} (t + T_{ijk}(t) + T_{jk}(t + T_{ijk}(t)))U^s_{jk\ell}(T_{ijk}(t) + T_{jk}(t + T_{ijk}(t))) \quad (7)$$

and the boundary condition $U^s_{js\cdot} \equiv 0$.

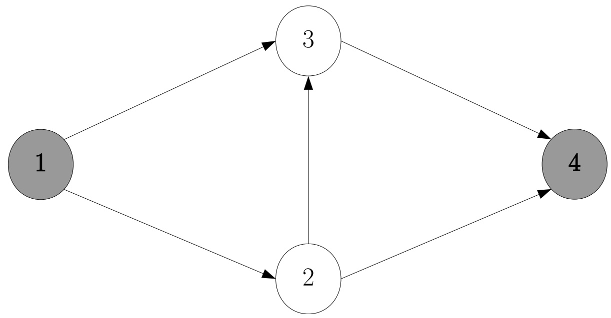

Figure 2: Network used in demonstration.

Table 1: Average excess cost and total system travel time for two solution methods.

	MSA		LUCE	
Iteration	AEC	TSTT	AEC	TSTT
1	1.75	4625.0	1.75	4625.0
2	2.054	4504.1	4.739	7462.5
3	0.432	2841.7	2.685	5178.2
4	0.883	3194.8	1.297	3693.5
5	0.277	2605.0	0.462	2848.7
6	0.549	2819.9	0.389	2632.4
7	0.174	2462.4	0.288	2533.0
8	0.422	2676.8	0.108	2319.0
9	0.143	2412.9	0.031	2234.3
10	0.3	2540.2	0.011	2212.5
11	0.124	2382.1	0.004	2205.8
12	0.255	2488.3	0.001	2201.6

Then the average excess cost is given by

$$AEC = \frac{\sum_{(r,s)\in Z^2} \sum_{(r,j)\in A} \int_0^{\overline{T}} (T^s_{\cdot rj}(t) - U^s_{\cdot rj})dt}{\sum_{(r,s)\in Z^2} \int_0^{\overline{T}} d^{rs}(t)dt} \tag{8}$$

while the total system travel time is simply

$$TSTT = \sum_{(r,s)\in Z^2} \sum_{(r,j)\in A} \int_0^{\overline{T}} T^s_{\cdot rj} dt \tag{9}$$

While the latter is not strictly a gap measure (since Wardrop equilibria do not generally correspond to system-optimal solutions) it nevertheless indicates the relative quality of the solutions. By either measure, the LUCE-based method reaches faster precision after the initial iterations.

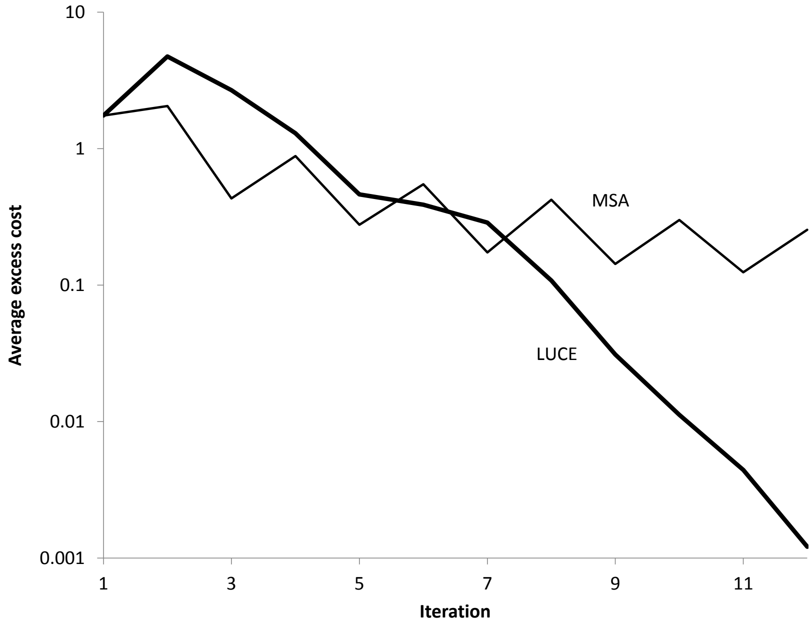

Figure 3: Average excess cost for two solution methods.

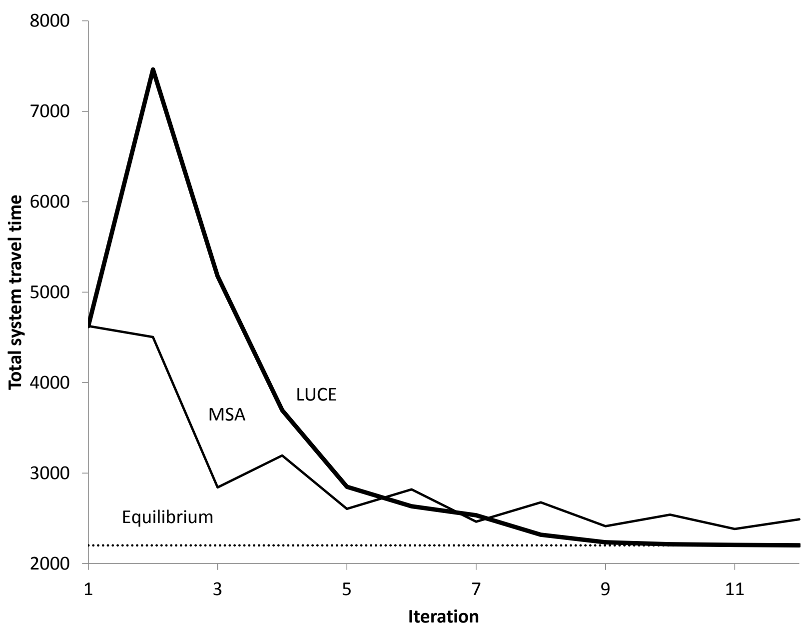

Figure 4: Total system travel time for two solution methods.

6 Conclusions

This paper described a generic continuous-flow DTA model for which (1) existence of dynamic Wardrop equilibrium is guaranteed; (2) generic solution methods can be easily developed and implemented; and (3) a variety of node models can be introduced to represent average traffic control delay. This model was demonstrated on a small network using dynamic versions of MSA and LUCE. In particular, the LUCE algorithm was easily adapted to our dynamic model, without reducing either its efficacy or its efficiency.

Future research should explore solution algorithms in further depth, exploring issues of convergence, practical performance across a broader suite of test networks, and sensitivity analyses to various parameters. By presenting a generic model, our hope is that a variety of link and node models can be compared and calibrated using this model as a common framework.

References

Bar-Gera, H. and D. Boyce (2006). Solving a non-convex combined travel forecasting model by the method of successive averages with constant step sizes. *Transportation Research Part B 40*, 351–367.

Carey, M. (1992). Nonconvexity of the dynamic traffic assignment problem. *Transportation Research Part B 26*(2), 127–133.

Daganzo, C. F. (1994). The cell transmission model: a dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B 28*(4), 269–287.

Daganzo, C. F. (1995). The cell transmission model, part II: network traffic. *Transportation Research Part B 29*(2), 79–93.

Gentile, G. (2009). Linear User Cost Equilibrium: a new algorithm for traffic assignment. Working paper.

Gentile, G. (2010). *The general link transmission model for dynamic network loading and a comparison with the DUE algorithm*. Edward Elgar Publishing.

Kakutani, S. (1941). A generalization of Brouwers fixed point theorem. *Duke Mathematical Journal 8*(3), 457–459.

Lighthill, M. J. and G. B. Whitham (1955). On kinematic waves II. A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society A 229*, 314–345.

Merchant, D. and G. Nemhauser (1978a). A model and an algorithm for the dynamic traffic assignment problems. *Transportation Science 12*, 183–199.

Merchant, D. and G. Nemhauser (1978b). Optimality conditions for a dynamic traffic assignment model. *Transportation Science 12*, 200–207.

Newell, G. F. (1993a). A simplified theory of kinematic waves in highway traffic, part I: General theory. *Transportation Research Part B 27*, 281–287.

Newell, G. F. (1993b). A simplified theory of kinematic waves in highway traffic, part II: Queueing at freeway bottlenecks. *Transportation Research Part B 27*, 289–303.

Newell, G. F. (1993c). A simplified theory of kinematic waves in highway traffic, part III: Multi-destination flows. *Transportation Research Part B 27*, 305–313.

Nezamuddin (2011). *Improving the Efficiency of Dynamic Traffic Assignment through Computational Methods Based on Combinatorial Algorithm.* Ph. D. thesis, The University of Texas at Austin.

Nie, Y. M. (2010). Equilibrium analysis of macroscopic traffic oscillations. *Transportation Research Part B 44*, 62–72.

Peeta, S. and A. K. Ziliaskopoulos (2001). Foundations of dynamic traffic assignment: the past, the present, and the future. *Networks and Spatial Economics 1*, 233–265.

Ramadurai, G. and S. V. Ukkusuri (2011). B-dynamic: An efficient algorithm for dynamic user equilibrium assignment in activity-travel networks. *Computer-Aided Civil and Infrastructure Engineering 26*(4), 254–269.

Yperman, I. (2007). *The Link Transmission Model for Dynamic Newtork Loading.* Ph. D. thesis, Katholieke Universiteit Leuven, Belgium.