DTA2014 Symposium: Dynamic Traffic Assignment and the Parking Search Process

Stephen D. Boyles* Shoupeng Tang† Avinash Unnikrishnan‡

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Abstract

Parking presents a major source of congestion in many urban ares. From the perspective of a driver, the parking search process is both stochastic and dynamic — the entire route to the destination cannot be predicted in advance if there is uncertainty as to where available parking spaces are located. To account for these factors, this paper develops a dynamic traffic assignment model which explicitly models these both in the traffic flow model and in the driver behavior model. The traffic flow model is built upon the cell transmission model, with added state variables to represent the number of available parking spaces on links. The driver behavior model uses the notion of policies from Markov decision processes to reflect the online decision-making process drivers face when they encounter an available parking space. A Wardrop-like equilibrium concept (defined in policies, not paths) combines the flow and behavior models.

Keywords:
Approach: within-day dynamics, CTM, discrete time modeling
Topic: road network analysis
Content: solution algorithm convergence conditions, large-scale applications

1 Introduction

Parking is a scarce good in most urban areas around the world, and the parking search process causes delays, driver frustration, increased congestion, and emissions. However, most urban planning models do not directly consider parking, perhaps because the parking search process is inherently stochastic and dynamic, and therefore difficult to model. At the same time, there is increased interest in improving parking through the use of innovative detection technologies, advanced traveler information, real-time pricing, advanced reservation, and other systems. This paper aims to support the modeling process by developing quantitative methods that can be used as a framework for evaluating such technologies.

Our model is particularly aimed at the "cruising" phenomenon when drivers are searching for parking. Comparing multiple international studies, Shoup (2011) estimated that up to 34% of congestion in urban areas can be traced to cruising behavior, and in Frankfurt, Germany up to 40% of driving time to central urban areas was spent searching for parking (Axhausen et al., 1994). By contrast, typical traffic assignment models based on equilibrium and shortest path concepts

*Assistant Professor, Civil, Architectural & Environmental Engineering, The University of Texas at Austin

†Graduate Research Assistant, Civil, Architectural & Environmental Engineering, The University of Texas at Austin

‡Assistant Professor, Civil and Environmental Engineering, West Virginia University

do not consider additional delay or stochasticity due to parking at the destination; a preliminary investigation in Tang et al. (2014) show that for short trips, this can underestimate travel times by up to 50% when parking delays are included.

As discussed in more detail in the literature review section, most methodological approaches for modeling parking delay on networks are generally based on discrete choice concepts, or the introduction of artificial parking links. In our opinion, both of these approaches have serious shortcomings: neither approach explicitly models the additional congestion caused by cruising behavior, and do not reflect the stochastic and adaptive choices drivers make as they pass multiple parking options *en route* to a destination. Simulation-based approaches have also been proposed, but these generally lack the behavioral foundations expected of modern planning models (e.g., appropriate generalizations of the equilibrium concept).

By contrast, this paper develops an equilibrium formulation accounting for stochastic and dynamic parking search by routing drivers based on *policies* rather than *paths*, using the language and framework of Markov decision processes. The mean features of this dynamic traffic assignment model are (i) incorporation of a stochastic parking model into the cell transmission model to represent traffic flow and (ii) a policy-based stochastic routing model to reflect driver behavior and adaptive choices regarding available parking spaces (for instance, if a driver sees an available space, should he or she take it or continue driving in hope of finding a more convenient space further downstream). In this way, the stochastic and dynamic nature of the parking search process can be explicitly modeled while still building on behavioral assumptions common to planning models.

These concepts are united in an equilibrium framework. While the notions of equilibrium and stochastic networks may seem mutually exclusive, this paper adopts a similar framework as the recourse equilibrium formulations of Unnikrishnan and Waller (2009) and Boyles (2009), in which the equilibrium is formulated in terms of policies (containing a set of contingent plans based on observed network conditions) rather than paths. However, in contrast to these two earlier works, the state probabilities depend on flows in addition to the state travel times, as explained in more detail below. In this way, the model developed in this paper is a generalization of these recourse equilibrium models. This model is amenable to implementation in the form of agent-based simulation, although specific details of such an implementation are not described in this paper.

The remainder of the paper is organized as follows. Section 2 presents prior research in the areas of parking search modeling and policy-based routing and equilibrium. Section 3 provides an overview of the proposed parking model, with additional details on its specific components in the following three sections: Section 4 discusses the extensions to the cell transmission model and the parking dynamics, Section 5 discusses the routing policies used by drivers, and Section 6 explains the equilibrium concept which ties them together. Finally, Section 7 concludes the paper and provides discussion of some practical considerations.

2 Literature review

Parking plays a surprisingly large role in traffic operations in dense urban areas, near universities or other demand centers, and in managing special events. Shoup (2006) reviewed parking search studies from 1927 to 2001 and found that between 8% and 74% of the traffic in congested downtowns were drivers cruising in search of parking locations. Recognizing the impact of parking searches on urban congestion, several studies have been conducted using economic, statistical, and optimization frameworks on various aspects of parking such as parking choice and pricing. Parking choice models can be classified into network assignment-based approaches, discrete choice-based approaches and simulation-based approaches.

Network assignment-based approaches model the parking choice in conjunction with the route choice. Eldin et al. (1981) used an incremental assignment approach to solve the traffic assignment problem which models the interaction between route choice, resulting vehicular flows, and parking choices. Bifulco (1993) developed a network level stochastic user equilibrium model to model parking search cost as a function of parking level occupancy as well as the route choice. Li et al. (2007a) consider mode choice between auto and transit and the simultaneous route and parking choice for automobile users using the user equilibrium framework. Lam et al. (2006) developed a variational inequality formulation for a multi-class network assignment model which considers departure time, route, parking location choice with drivers classified based on parking durations. Li et al. (2008) develop a fixed point based assignment model to study the impact of time dependent and normally distributed uncertain travel times and parking search time on network level reliability. Gallo et al. (2011) developed a stochastic user equilibrium based fixed point formulation which modeled car trip, cruising for parking, and walking to final destination in multiple network layers. Several network assignment based models were used to evaluate pricing based parking policies. Lam et al. (2002) and Li et al. (2007b) adopted a bilevel programming framework where the upper level determines the optimal tolls and parking charges and the lower level models the equilibrium assignment in response to the tolls and parking charges. d'Acierno et al. (2006) developed several optimization models to determine the parking prices taking into account the transit connectivity between origin-destination pairs with a network using a multi-modal network assignment model. However, in the these models parking is generally modeled as a deterministic phenomenon imposing a known cost to drivers, and not contributing to congestion or delay for other drivers not searching for parking.

Discrete choice models neglect the network structure and use random utility theory to understand parking choice as a function of various driver and parking location attributes. van der Goot (1982), Axhausen and Polak (1991), and Lambe (1996) use multinomial logit model to model parking location choice. Other discrete choice model forms considered include the mixed multinomial logit model (Hess and Polak, 2004b,a) and the nested logit model (Hensher and King, 2001). However, these models do not directly incorporate parking costs into the network loading and assignment.

The third category of parking choice studies has adopted an agent-based approach to model parking search. Thompson and Richardson (1998) developed an analytical model to mimic the search process where the disutility of a car park location was defined as a function of in-vehicle travel time, in-car park search time, waiting time, parking fee, parking fine, and walk time. Several other researchers have adopted an agent based simulation approach where the behavioral and parking related decision making rules were assigned to the driver (Benenson et al., 2008; Martens and Benenson, 2008; Dieussart et al., 2009; Waraich and Axhausen, 2012).

In contrast to all of the above, the model presented here is based on the concept of stochastic shortest paths with recourse, an approach pioneered for parking search in (Tang et al., 2014). Also known as the online shortest path problem (Cheung, 1998; Waller and Ziliaskopoulos, 2002; Provan, 2003), in this problem drivers progressively learn the realizations of stochastic network costs and adapt their chosen path *en route*. Although not explicitly noted in these papers, the online shortest path problem with reset takes the form of a classical Markov decision process (Bertsekas, 2005, 2007). This was adapted to parking modeling by using the recourse concept to model driver choices upon learning the state of a link (whether parking is available or not); these choices can include parking if a spot is available, or a choice of successor link to follow if no spot is available. As Provan (2003) discusses, variants of this problem can be classified as "full reset" (Waller and Ziliaskopoulos, 2002) and "no reset" (Polychronopoulos and Tsitsiklis, 1996). In the parking context, "full reset" would imply that the probability of finding parking on a link is independent of all past traversals (if

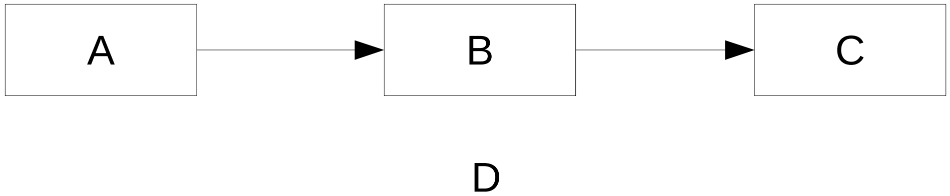

Figure 1: Small cell network to demonstrate model concepts.

no parking is found, making a U-turn gives drivers an independent chance of finding parking at the *a priori* probability), while "no reset" would imply that if no parking was found on a link, no parking space would *ever* be found in the future. Since neither of this is appropriate for parking modeling, Tang et al. (2014) developed the "asymptotic reset" concept to generalize both approaches and present an intermediate alternative.

3 Modeling overview

This section presents a general overview of the modeling components and motivating concepts using a simple example, before defining them in more general mathematical terms in the sections that follow. For illustrative purposes, the simple network in Figure 1 will be referred to throughout this section. In this network, drivers are ultimately attempting to reach the destination D. Unlike traditional transportation planning models, D is not a node in the transportation network (which represents the transportation infrastructure itself). Rather, drivers must park on a network link and then walk to D, incurring a walking cost. Each of the three links A, B, and C has a uniform travel time of 1 unit, but represents a different parking situation. Links A and B represent free on-street parking, while link C represents a paid lot with cost c. Link B is closest to the destination, while links A and C are further away and have a higher walking time $w > 1$. Assuming a uniform value of time and measuring costs in time units, we can incorporate any monetary cost into the walking time, and simply say that the walking times from A, B, and C are w, 0, and $w + c$, respectively.

Drivers clearly prefer to park on link B. Assuming that the arrival rate of vehicles is smaller than the departure rate of parked vehicles on link B, all drivers' desires can be accommodated. However, if this arrival rate increases, not all drivers will be able to park on this link. Assume that drivers have no information on the locations of available parking spaces until they traverse a link, but from experience know that the probability of finding parking on link B is p. The question is, upon seeing an available parking space on link A, will the driver choose to park there or continue in hopes of being able to park at B. Assuming that drivers wish to minimize expected travel time, they will park on link A whenever $w < 1 + (1 - p)(1 + w + c)$ (the expected additional time for continuing to link B, including driving and walking time), or equivalently $p < w/(1 + w + c)$. Likewise, whenever $p > w/(1 + w + c)$ expected travel time is minimized by continuing to link B, and then parking at link C if no space is available at B.

However, the probability p depends on these choices drivers make, and the only stable solution occurs when $p = (2 + c)/(1 + w + c)$, and the fraction of drivers choosing to park on link A is exactly the right amount for this value of p to occur. In this case, the expected travel times are equal from parking at A or continuing on to B, and drivers are indifferent between these two options. (Figure 2). For any other p value, drivers would switch their behavior at link A, with more drivers seeking to park at A if p falls below this threshold and fewer seeking to park at A if

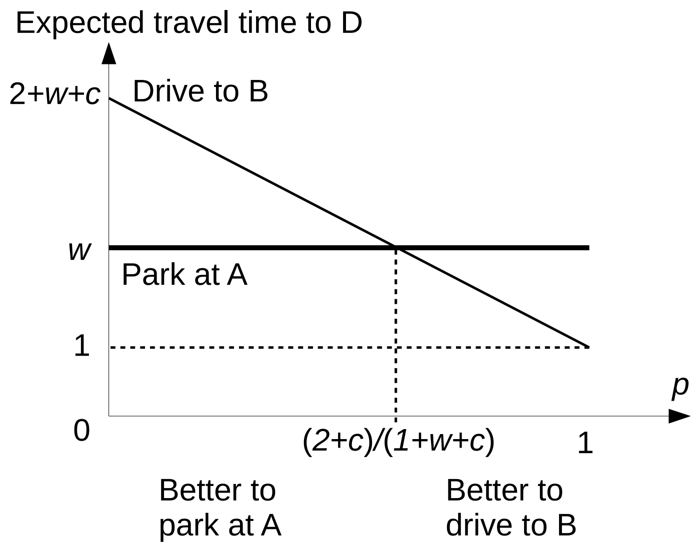

Figure 2: Optimal decisions at A as a function of parking availability at B.

p exceeds this value, and these behavioral switches would move p closer to $(2+c)/(1+w+c)$. In this way, the stable states correspond to equilibria in terms of policies, where the policy concerns the choice drivers make when an available space is found on link A.

To specify this model in a way that applies to general networks, we need to represent both (i) the traffic flow and parking dynamics, given driver policies; (ii) a policy selection model representing drivers' desire to minimize expected travel times; and (iii) the equilibrium framework uniting the first two. Each is respectively described in the following three sections.

4 Supply-side: networks and traffic flow

The flow model is built upon the cell transmission model (CTM), developed by Daganzo (1994, 1995) as a discrete solution method for the LWR hydrodynamic model (Lighthill and Whitham, 1955; Richards, 1956) based on a Godunov scheme. To implement CTM, each link must be divided into a finite number of cells whose length is the distance a vehicle travels at free-flow in one simulation tick of length Δt. Briefly recapitulating, each cell c is associated with parameters representing its capacity Q_c, the maximum number of vehicles which can fit into the cell N_c, and the ratio of backward-to-forward wave speeds δ_c, and a state variable $x_c(t)$ denoting the number of vehicles in cell c during the t-th time interval. $\tau_c(t)$ is used to indicate the time necessary for a vehicle to traverse cell c when entering during time interval $t \in \{0, 1, \ldots, T\}$.

To propagate flow, at each time interval define the sending flow $S_c(t) = \min\{x_c(t), Q_c\}$ and receiving flow $R_c(t) = \min\{\delta_c(N_c - x_c(t)), Q_c\}$. A variety of intersection models exist mapping the sending flows of incoming links and receiving flows of outgoing links to transition flows between cells. For instance, if cells c and d are in series, the number of vehicles moving between these cells during the t-th time interval is the lesser of $S_c(t)$ and $R_d(t)$. Multiple diverge and merge models have been formulated in the literature (Daganzo, 1995; Nie et al., 2008; Yperman, 2007), along with models for more general intersection types (Yperman, 2007; Tampère et al., 2011; Corthout et al., 2012). These are not discussed here for brevity, and any of these can be used as the basis for the parking model presented below, in which each cell is equipped with additional state variables used to represent the number of available parking spaces.

Consider a time-dependent network with node and arc sets N and A, respectively, and let C denote the set of cells; $\Gamma(c)$ is the set of cells immediately downstream of cell c. Let S denote the set of destinations. Unlike most network equilibrium models, the destinations S are distinct from the network nodes N, and are not directly connected to the links. Instead, for each cell c and destination s, define w_{cs} to be the walking time between cell c and destination z. Monetary and other generalized costs can easily be incorporated into this term. If parking is not permitted on cell c, by convention define $w_{cs} = \infty$ for all s. Let the parameters P_c and μ_c respectively denote the number of parking spaces associated with cell c and the mean duration vehicles park on this cell, and let the state variable $a_c(t)$ the number of available parking spaces on this cell at the start of interval t ($0 \leq a_c(t) \leq P_c$). Demand is specified by the parameters $d_{cs}(t)$, denoting the number of vehicles beginning trips at cell c during the t-th time interval, traveling towards destination s.

Let $x_c(t)$ denote the number of vehicles in cell c at time t; these vehicles are distinguished by whether they would park at this cell if a space is available, or whether they would choose to keep driving to find a parking space closer to the destination. Denote these numbers of vehicles as $x_c^P(t)$ and $x_c^{NP}(t)$, respectively, so that $x_c(t) = x_c^P(t) + x_c^{NP}(t)$. These values are determined by the policies drivers choose, as described in the next section.

After propagating flow already on the network in the t-th time interval, any vehicles originating at cell c are loaded onto that cell if space permits (otherwise, they are held back until the next time interval), along with the vehicles vacating parking spaces (denoted $d_c(t)$). Modeling parking departures as a Poisson process, the probability that any occupied space will be vacated in a simulation tick is approximately $\Delta t/\mu_c$, assuming Δt is sufficiently small. Following this, the number of parking vehicles $e_c(t)$ is calculated as the lesser of $x_c^P(t)$ and $a_c(t) + d_c(t)$; the vehicles which can park are randomly sampled from $x_c^P(t)$ and the probability of finding parking on this cell at time t is $p_c(t) = e_c(t)/[a_c(t) + d_c(t)]$.[1] Note that this ordering implies that vehicles already on the network have priority over vehicles attempting to enter the network or leave parking spaces, and that vehicles seeking to park are willing to wait to allow vehicles vacating parking spaces to do so. This process is shown in Figure 3.

5 Demand-side: parking policies and choices

Each vehicle is assigned a parking policy which determines its route and actions whenever an available parking space is found. Formally, define the state space $\mathcal{S} = C \times T \times \{P, NP\}$, whose elements $\sigma = (c, t, \rho) \in \mathcal{S}$ indicate the current cell c, time interval t, and parking status ρ (P for parking, NP for no parking) for a vehicle. The notation $c(\sigma)$, $t(\sigma)$, or $\rho(\sigma)$ is used to refer to these elements of state σ. A policy is a mapping $\pi : \mathcal{S} \to C \cup \{P\}$ associating with each state s a corresponding action to take — either a downstream cell to move towards, or the parking action P. A policy is feasible if for all σ (i) $\pi(\sigma) \in C$ implies $\pi(\sigma) \in \Gamma(c)$ and (ii) $\pi(\sigma) = P$ only if $\rho(\sigma) = 1$.

That is, each driver in a cell chooses whether or not to park on that cell if a space is available; if no space is available or if the driver chooses not to park, then the driver must choose which downstream cell to traverse next. If the driver chooses action P for a feasible policy, their trip is complete because a parking space is available. If the driver chooses a downstream cell, so $\pi(\sigma) = d \in \Gamma(c)$ when $\sigma = (c, t, \rho)$, then they enter cell d at time $t + \tau_c(t)$. With probability $p_d(t + \tau_c(t))$, the driver is next in state $(d, t + \tau_c(t), P)$, and with probability $1 - p_d(t + \tau_c(t))$ they are in state $(d, t + \tau_c(t), NP)$.

[1]This intentionally violates first-in/first-out ordering; a parking space opening just upstream of a lead vehicle may be taken by a following vehicle.

1. Initialize all cell occupancies $x_c \leftarrow 0$, $t \leftarrow 0$, and parking availability $a_c = P_c$.
2. Propagate flow at time t using cell transmission model.
3. For each cell c and each occupied parking space, generate a random real number ξ by uniformly sampling the interval $[0, 1]$; if $xi < \Delta t/mu_c$, increment a_c.
4. For each cell c identify the number of vehicles searching for parking x_c^P based on policies associated with each vehicle.
5. For each cell c randomly select $e_c = \min\{x_c^P, a_c\}$ vehicles among the x_c^P searching for parking, move them to parking spaces and $a_c \leftarrow a_c - e_c$
6. Update cell travel times and parking probabilities.
7. If $t < T$ increment t and return to step 2.

Figure 3: Cell transmission model algorithm with parking added.

Given fixed values of travel times τ and parking probabilities p, the expected travel time corresponding to a feasible policy π can be calculated as follows. Let $L_\pi(\sigma, s)$ denote the expected remaining travel time from the current state $\sigma = (c, s, \rho)$ to destination s when using policy π. These labels satisfy the recursion $L_\pi(\sigma, s) = w_{cs}$ if $\pi(\sigma) = P$ and $L_\pi(\sigma, s) = \tau_c(t) + p_d(t + \tau_c(t))L(d, t + \tau_c(t), P) + (1 - p_d(t + \tau_c(t))L(d, t + \tau_c(t), NP)$ otherwise, where $d = \pi(\sigma)$ for brevity. The first case corresponds to the parking action, while the second corresponds to driving to the next cell.

Finding an *optimal* policy π^* for a given destination s and values of p and τ is not difficult, and is accomplished by a standard label-correcting algorithm such as that in Figure 4. In our behavior model, we assume that each driver chooses a policy so as to minimize his or her expected travel time.

6 Equilibrium

The models presented in the previous two sections exhibit the mutual dependency which is typical of transportation network models: travel times and parking probabilities depend on the policies used by drivers, but the optimal policies chosen by drivers depend on the travel times and parking probabilities obtained from the flow model. As suggested in Section 3, these perspectives are harmonized by the introduction of an equilibrium principle. In particular, *an equilibrium solution associates a policy with each vehicle such that each vehicle's assigned policy is optimal for its destination, given cell travel times and parking probabilities consistent with this policy assignment.*

The model, as formulated above, assumes that vehicles are discrete; this is necessary because the parking spaces are modeled as discrete entities. Therefore, an exact equilibrium solution may

1. Initialize all labels $L(\sigma) = \infty$, create new policy π^*, set $t \leftarrow T$.
2. For each cell c:
   (a) Update $L(c, t, NP) \leftarrow \min_{d \in \Gamma(c)} \{p_d(t + \tau_c(t)) L(d, t + \tau_c(t), P) + (1 - p_d(t + \tau_c(t))) L(d, t + \tau_c(t), NP)\}$
   (b) Update $\pi^*(c, t, NP) \leftarrow \arg\min_{d \in \Gamma(c)} \{p_d(t + \tau_c(t)) L(d, t + \tau_c(t), P) + (1 - p_d(t + \tau_c(t))) L(d, t + \tau_c(t), NP)\}$
   (c) If $w_{cs} \leq L(c, t, NP)$ then update $L(c, t, P) \leftarrow w_{cs}$ and $\pi^*(c, t, P) = P$; else $L(c, t, P) \leftarrow L(c, t, NP)$ and $\pi^*(c, t, P) \leftarrow \pi^*(c, t, NP)$.
3. If $t > 0$ decrement t and return to step 2.

Figure 4: Label correcting algorithm for finding optimal policies given destination s.

not exist. Furthermore, equilibrium existence arguments generally require assumptions on the cost mapping, such as continuity, which may not be satisfied with the model presented in the previous sections, which is both discrete and stochastic — owing to the random departures from parking spaces, multiple samples may be needed to obtain reliable estimates for the parking probabilities $p\ ()$. For these reasons, we present only a heuristic which aims to produce a near-equilibrium solution with a small gap (as defined by the difference between the labels of the chosen policies and optimal policies for those travelers). This heuristic is based on the well-known method of successive averages, and is presented in Figure 5.

7 Conclusion and practical considerations

The model described in this paper represents a dynamic traffic assignment model which has been extended to include delays and congestion resulting from the parking search process. In contrast to previous network-based approaches, these delays and congestion effects are modeled explicitly, using an online shortest path approach. The resulting equilibrium state is a natural generalization of the Wardrop condition traditionally used in transportation planning. Furthermore, the algorithms presented above are easily amenable to implementation in agent-based simulation. For large-scale instances, each component algorithm can be parallelized to decrease computation time. This model can be used as is to evaluate parking policies related to pricing and duration, both for routine conditions and special events. Its general principles can also serve as the basis for more involved investigations concerning real-time parking information or dynamic pricing policies; both of these are valuable topics for future research. Other future research topics include extending the demand model to handle trip chains (a vehicle departing from one parking space may head to several other destinations, including parking at each one, before returning to the origin), or developing alternative algorithms for reaching near-equilibrium solutions.

From the standpoint of practical implementation, the additional data requirements for this model, relative to existing dynamic traffic assignment models, are the number of available parking spaces on each cell, the mean parking duration on each cell, and the walking distances between each cell and each destination. The latter data is relatively easy to estimate, given the network topology and an assumed walking speed. It may be easier to estimate the total number of available

1. Initialize travel times to free-flow and parking probabilities to 1, iteration counter $k \leftarrow 1$.
2. Identify optimal policies $\pi^*(s)$ for each destination s, and assign to all vehicles. (Figure 4)
3. Increment k
4. Simulate cell transmission model for given policies. (Figure 3)
5. *(Optional.)* Repeat previous step multiple times to generate empirical distribution of t and p based on repeated simulation.
6. Identify optimal policies $\pi^*(s)$ for each destination s. (Figure 4)
7. For each vehicle heading to destination s, switch its policy to $\pi^*(s)$ with probability $1/k$
8. Unless gap sufficiently small, return to step 3.

Figure 5: Method of successive averages equilibrium heuristic.

parking spaces on a link, and divide them evenly among the cells, or to estimate the mean parking duration with a proxy (such as a fraction of the maximum allowable time.)

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