Demand Uncertainty and Optimism in Planning Forecasts

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Transportation planning forecasts often differ widely from forecasted conditions. For instance, transit ridership is often lower than forecasted, toll revenue is lower, and construction costs and duration are higher. Many people assume that this occurs because transportation models are biased to favor selection of particular projects or alternatives for political reasons or to reflect personal biases. However, an old adage is to "never ascribe to malice that which is adequately explained by incompetence" — but rather than incompetence, this paper suggests that neglecting uncertainty in future demand may result in systematically over-optimistic forecasts, even if the modeling and prediction processes are completely unbiased.

In particular, this paper shows that even if the forecasting process is unbiased, two factors combine to produce systematically optimistic forecasts for the selected project: (1) the inability to forecast future conditions perfectly, and (2) the fact that projects are chosen to maximize one or more measures of effectiveness. The combination of this uncertainty and choice process suggest that the chosen project is likely one for which the forecast is overly optimistic. This paper studies the issue by evaluating multiple project alternatives using static traffic assignment with uncertainty in the OD matrix. Key implications for planning are: (1) introducing a large number of alternatives for comparison may in fact hinder selection of the optimal alternative, and (2) the use of multi-scenario or sampling techniques can be useful to identify overly optimistic forecasts despite imperfect knowledge of future conditions.

Consider n alternative projects under consideration, and let $\mathbf{d}$ reflect the true (unknown) future OD matrix. Let $\tilde{\boldsymbol{\epsilon}}$ be a random term representing forecasting errors in the OD matrix used for evaluation. The main result holds even when the demand modeling process is unbiased, that is, $E[\tilde{\boldsymbol{\epsilon}}] = \mathbf{0}$; but we present a slightly weaker general condition given below. Let f_i be a function mapping an OD matrix to a measure of effectiveness used to evaluate a project (say, toll revenue); the traffic assignment is embedded in this function. So, for instance, $f_1(\mathbf{d})$ is the toll revenue from alternative 1 at equilibrium, based on the true future OD matrix, and $f_3(\mathbf{d} + \tilde{\boldsymbol{\epsilon}})$ is the toll revenue from alternative 3 based on the forecasted (noisy) OD matrix. Let $\tilde{\xi}_i$ represent the error in the prediction of the measure of effectiveness for alternative i, that is $\tilde{\xi}_i = f_i(\mathbf{d} + \tilde{\boldsymbol{\epsilon}}) - f_i(\mathbf{d})$.

The selected project $i^*(\tilde{\boldsymbol{\epsilon}})$ satisfies $i^*(\tilde{\boldsymbol{\epsilon}}) \in \arg\max_i \{f_i(\mathbf{d} + \tilde{\boldsymbol{\epsilon}})\}$. Note that the selected project depends on $\tilde{\boldsymbol{\epsilon}}$, and that the realization of the noise $\tilde{\boldsymbol{\epsilon}}$ is common across all alternatives, since the same OD matrix is used to evaluate all alternatives. This is one way in which the formulation is different from standard discrete choice formulations, in which the error terms are generally independent across alternatives.

The hypothesis for this research is that $E[\tilde{\xi}_{i^*(\tilde{\boldsymbol{\epsilon}})}] \geq 0$ even if $E[\tilde{\boldsymbol{\epsilon}}] = \mathbf{0}$ (or even $E[\tilde{\xi}_i] = 0$) for all i. In particular, we show that this is true under certain general conditions, including when f

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is convex and demand is unbiased, or when f is unbiased when demand is unbiased. However, the traffic assignment mapping is complex, especially in large networks, and it is difficult to establish such properties exactly. Therefore, we explore this issue in two additional ways, deriving exact results on small transportation networks, and performing a numerical investigation of practical-sized networks. The exact result is provided below; its proof and the remaining investigations are deferred to the full paper.

Specifically, let j index an alternative for which $E[f_j(\mathbf{d}+\tilde{\boldsymbol{\epsilon}})]$ is maximal, and k an alternative for which $f_k(\mathbf{d})$ is maximal (these need not be distinct). That is, project j has maximal expected performance (that is, the one most likely to be chosen by the modeling process), and project k has maximal true performance. The following proposition provides the main theoretical result for the paper, showing that the expected deviation between the predicted and realized measure of effectiveness is nonnegative if the expected prediction error is zero or positive for each project, and providing relatively weak conditions for which this expected deviation is

Proposition 1. *If $E[\tilde{\xi}_i] \geq 0$ for each project i, then $E[\tilde{\xi}_{i^*(\tilde{\epsilon})}] \geq 0$. Furthermore, if* any *of the following criteria hold, the inequality is strict and $E[\tilde{\xi}_{i^*(\tilde{\epsilon})}] > 0$:*

1. $\Pr(f_j(\mathbf{d}+\tilde{\boldsymbol{\epsilon}}) = \max_i \{f_i(\mathbf{d}+\tilde{\boldsymbol{\epsilon}})\}) < 1$
2. $\Pr(f_k(\mathbf{d}+\tilde{\boldsymbol{\epsilon}}) = \max_i \{f_i(\mathbf{d}+\tilde{\boldsymbol{\epsilon}})\}) < 1$
3. $E[f_j(\mathbf{d}+\tilde{\boldsymbol{\epsilon}})] > E[f_k(\mathbf{d}+\tilde{\boldsymbol{\epsilon}})]$
4. $E[f_k(\mathbf{d}+\tilde{\boldsymbol{\epsilon}})] > f_k(\mathbf{d})$

Proof. To prove the first claim, we use the definitions of i^*, j, and k as follows:

$$\begin{aligned}
E[\tilde{\xi}_{i^*(\tilde{\epsilon})}] &= \int (f_{i^*(\tilde{\epsilon})}(\mathbf{d}+\tilde{\boldsymbol{\epsilon}}) - f_{i^*(\tilde{\epsilon})}(\mathbf{d}))\ d\tilde{\boldsymbol{\epsilon}} \\
&\geq \int (f_j(\mathbf{d}+\tilde{\boldsymbol{\epsilon}}) - f_{i^*(\tilde{\epsilon})}(\mathbf{d}))\ d\tilde{\boldsymbol{\epsilon}} \text{ by definition of } i^* \\
&\geq \int (f_j(\mathbf{d}+\tilde{\boldsymbol{\epsilon}}) - f_k(\mathbf{d}))\ d\tilde{\boldsymbol{\epsilon}} \text{ by definition of } k \\
&= E[f_j(\mathbf{d}+\tilde{\boldsymbol{\epsilon}})] - f_k(\mathbf{d}) \\
&\geq E[f_k(\mathbf{d}+\tilde{\boldsymbol{\epsilon}})] - f_k(\mathbf{d}) \text{ by definition of } j \\
&= E[\tilde{\xi}_k] \\
&\geq 0 \text{ by assumption}
\end{aligned}$$

The four conditions in the second claim correspond exactly to the four inequalities in the above relation, and if any hold then the inequality is strict. □

The first two conditions essentially require that the choice process be nondeterministic, that there is at least some prediction error which could affect the ranking of the top project. The third is satisfied if the projects maximizing the expected and true performance are not "tied" in expected performance, and the fourth is satisfied if the expected prediction error for the true optimal project is strictly positive. Note that if *any* of these conditions holds true, then the expected prediction error for the selected project is strictly positive.

Below we list a few specific implications and examples of functions which satisfy the condition in the first part of Proposition 1:

- Even if $E[f_i(\mathbf{d}+\tilde{\boldsymbol{\epsilon}})] = f_i(\mathbf{d})$ for all i, we may still have $E[\tilde{\xi}_{i^*(\tilde{\boldsymbol{\epsilon}})}] > 0$. (In other words, we can obtain biased results from unbiased evaluation.)
- If each f_i is convex and $E[\tilde{\boldsymbol{\epsilon}}] = \mathbf{0}$, then $E[\tilde{\xi}_{i^*(\tilde{\boldsymbol{\epsilon}})}] \geq 0$
- If f_k is strictly convex, $E[\tilde{\boldsymbol{\epsilon}}] = \mathbf{0}$, and $V[f_k(\mathbf{d}+\tilde{\boldsymbol{\epsilon}})] > 0$, then $E[\tilde{\xi}_{i^*(\tilde{\boldsymbol{\epsilon}})}] > 0$

In the full paper, this result is supplemented with numerical studies on transportation networks, where it is difficult or impossible to show that the functions f_i (embedding both the traffic assignment mapping and the measure of effectiveness) satisfy particular properties. Particular attention is paid to the implications for transportation planning, demonstrating how the level of expected error in the selected project varies with factors such as the total number of alternatives, the distribution and variance of the demand forecasting error, the particular measure of effectiveness, and also discuss strategies for mitigating this type of error.