

CE 311S: Exam 1
Thursday, March 1
2:00 – 3:15 PM

Name _____

Instructions:

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no partial credit!
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- If you need to make any additional assumptions, state them clearly.
- You may use one regular-sized sheet of notes; please turn in the notes with your exam. No additional resources are permitted.
- The number of points associated with each part of each problem is indicated.

Problem	Points	Possible
1		15
2		20
3		25
4		31
5		9
TOTAL		100

Problem 1. (15 points). Your name is Bond — Alex Bond — and you are a secret agent (a mediocre one) with a license to kill (mistakenly-awarded, perhaps by confusion with your more famous cousin James). You are currently under review because of the excessive property damage caused in the course of your last five missions (none of which were successful). In particular, the damages caused by these missions are

30 50 40 20 10

all values in millions of pounds sterling. **Show all work.**

- (a) (5) What was the mean damage you caused?
- (b) (10) What was the standard deviation of the damage you caused?

Problem 2. (20 points). Despite your past problems, you have been assigned to find out who has hijacked the *HMS Insufferable*, one of Britain's latest nuclear submarines. Before heading out on the mission, you pay a visit to P, the affably cantankerous scientist who equips you with the gadgets you need to survive. Today, P has fifteen different gadgets to show you: five types of watches, six types of pens, and four types of briefcase (exploding, of course). On any mission, you take three gadgets with you. Assuming that P picks these three gadgets randomly (with all possible assignments equally likely), answer the following questions:

- (a) (6) What is the total number of combinations of gadgets I can receive?
- (b) (7) In how many of these combinations do I have exactly two watches?
- (c) (7) What is the probability I receive exactly one watch, one pen, and one briefcase?

Problem 3. (25 points). You begin by spying on your arch-nemesis Ernst Stat-vro Blofeld and his organization, SPECTRE. You decide to pay a visit to one of his many homes. Blofeld has an odd fascination with square plots of land; every one of the 10 properties he owns is in the shape of a square. 4 of them have an area of 1 square kilometer; 3 of them have an area of 4 square kilometers; 2 of them have an area of 9 square kilometers; and 1 of them has an area of 16 square kilometers. You don't know where he is, so you pick one of the 10 properties at random (each is equally likely to be chosen).

- (a) (10) Let X be the area of the property you choose to visit. What are $E[X]$ and $V[X]$?
- (b) (10) Let Y be the length of one side of the property you choose to visit. What are $E[Y]$ and $V[Y]$?
- (c) (5) Like any decent villain, Blofeld faithfully pays the property tax on his homes. Assume that the amount of tax he pays on each property is €5000, plus an additional €2000 per square kilometer. Let Z be the amount of tax he pays on the property you choose to visit. What are $E[Z]$ and $V[Z]$?

Problem 4. (31 points). You discover that the submarine has been hijacked by Octopoisson, one of SPECTRE's operatives, and need to decide how best to proceed. You think back to your past missions. In the past, 70% of the time you've infiltrated the villain's headquarters directly, while the rest of the time you started by seducing their lover. In either case, you were captured 50% of the time. If you were captured infiltrating their headquarters, you never succeeded in completing your mission; however, if you were captured seducing their lover, they helped you escape and you succeeded 50% of the time. Regardless of how you started the mission, if you are never captured you succeed 15% of the time.

Let A be the event "you infiltrate the villain's headquarters directly," let B be the event "you are captured at any point during your mission," and let C be the event "you successfully completed the mission." **Do not just answer "yes" or "no" for (c), (d), and (e) — show all of your work.**

- (a) (15) What are $P(A)$, $P(B)$, and $P(C)$?
- (b) (5) If you were captured at some point during your mission, what is the probability that you tried to seduce the villain's lover?
- (c) (3) Are A and B independent?
- (d) (3) Are A and C independent?
- (e) (5) Are A , B , and C mutually independent?

Problem 5. (9 points, 3 each). You try to seduce Octopoisson's lover, but you fail, get captured, and handed over to the venerable businessman Boldfinger to be interrogated. He straps you to a table and points a laser directly at you! You turn to him, and defiantly ask "Do you expect me to talk?" "No, Alex Bond," Boldfinger replies. "I expect you to help me with set theory. Can you tell me which of the following equations are true?" Indicate whether each of the equations below is true or false — it's your only hope of survival! **If you say 'T', this means the equation is true *no matter what* the events A , B , and C are. You do not need to show your work, and no partial credit will be given.**

T F $(A \cup B)' = A' \cup B'$

T F $(A \cap B)' = A' \cap B'$

T F $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$