# CE 311S: Exam 1
Thursday, February 24
8:00 – 9:15 AM

Name _____

**Instructions:**

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no partial credit!

- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.

- If you need to make any additional assumptions, state them clearly.

- You may use a calculator and one regular-sized sheet of notes. No additional resources are permitted.

- The number of points associated with each part of each problem is indicated.

| Problem | Points | Possible |
|:-------:|:------:|:--------:|
| 1 | | 20 |
| 2 | | 25 |
| 3 | | 30 |
| 4 | | 25 |
| **TOTAL** | | 100 |

**Problem 1.** (20 points.) You decide that your life could use some more excitement, so you consider two possible activities: forming a band with your friends, and making shady NFTs for some pocket money. However, you also care about social approval, so you look at the 20% of your friends who are especially cool. Among these, 60% are in a band, 10% make shady NFTs, and 30% do neither. Among your friends who are not especially cool, 10% are in a band, 60% make shady NFTs, and 30% do neither.

(a) (5) What percentage of your friends are in a band?

(b) (5) Assuming you are like you friends, what is the probability that you will be especially cool if you form a band?

(c) (10) Which maximizes your probability of being especially cool: being in a band, making shady NFTs, or neither?

**Problem 2.** (25 points). Not trusting your calculations on the previous problem, you decide to form a band with your friends regardless of your answer to Problem 1. In the past you dabbled with the theremin[1] and want to make this the starring instrument in your band. You search online for used theremins, and find the following prices:

$$68 \quad 74 \quad 95 \quad 85 \quad 45 \quad 95$$

(a) (5) What is the mean amount from this sample of prices?

(b) (5) What is the median?

(c) (5) What is the mode?

(d) (5) What is the sample variance?

(e) (5) What is the sample standard deviation?

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[1]An ætherial-sounding electronic instrument controlled by waving your hands around two antennae without actually touching it; one hand controls pitch and the other volume.

**Problem 3.** (30 points). You want to predict how successful your band will be, and look back over historical data about the success of student bands at UT. You particularly pay attention to three pieces of information about each band: whether they have more than 1000 followers on social media (call this event $A$), whether they write original music (event $B$), and whether they sold enough albums to go "burnt orange" [2] (event $C$). You form the following Venn diagram showing the percentage of bands that satisfy or do not satisfy these events:
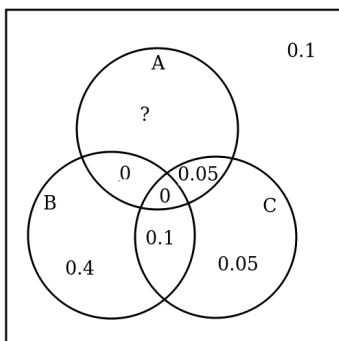
(a) (5) What is the missing value marked with a question mark?

(b) (2) Are $A$, $B$, and $C$ exhaustive? **(For all parts of this question, do not just answer yes or no — to receive any credit you must give an explanation.)**

(c) (2) Do $A$, $B$, and $C$ form a partition?

(d) (6) Are $A$ and $B$ disjoint? What about $A$ and $C$? What about $B$ and $C$?

(e) (9) Are $A$ and $B$ independent? What about $A$ and $C$? What about $B$ and $C$?

(f) (3) What is the probability that your band will go burnt orange if you have more than 1000 followers on social media?

(g) (3) What is the probability you have more than 1000 followers on social media if you go burnt orange?

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[2]This is the Longhorn equivalent of a "gold record."

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**Problem 4.** (25 points). Your band goes on tour. You randomly choose between Lubbock, Dallas, Houston, and San Antonio for the first city on your tour. After that, you have a 1/3 probability of traveling to each of the two neighboring cities, or back to Austin, as shown in the diagram below; this pattern continues until your tour ends. For example, if you are in Houston, there is a 1/3 probability of traveling to San Antonio next, a 1/3 probability of traveling to Austin next, and a 1/3 probability of traveling to Dallas next.

Your tour will continue until you return to Austin, and will end then; before that, there is no limit to how long it lasts or how many times you re-visit a city. For example, one possible tour is Austin, Lubbock, San Antonio, Lubbock, Dallas, Austin.

Lubbock    Dallas

Austin

San Antonio    Houston
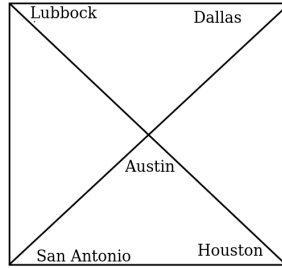
(a) (5) Let $X$ represent the number of cities you visit (excluding Austin at the start and end of the tour). Write the PMF for $X$.

(b) (5) What is the expected value of $X$? (There is more than one way to solve this problem; in one of them the series identity $1 + 2r + 3r^2 + 4r^3 + \cdots = 1/(1-r)^2$ may be useful.)

(c) (5) Traveling expenses for your band are \$50 for each leg of your trip. For example, if your tour is Austin, Lubbock, San Antonio, Lubbock, Dallas, Austin, you make five trips between cities and your expenses will be \$250. If $Y$ represents the amount of traveling expenses, write the PMF for $Y$.

(d) (5) What is the expected value of $Y$?

(e) (5) You have family in Lubbock and Houston. Let $Z$ represent the number of cities on your tour where you have family. What is $P(Z = 1)$?

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