CE 311S: Exam 2

Tuesday, April 3

2:00 – 3:15 PM

Name ______________________________

Instructions:

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no credit!
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- If you need to make any additional assumptions, state them clearly.
- You may use one regular-sized sheet of notes; please turn in the notes with your exam. No additional resources are permitted.
- The number of points associated with each part of each problem is indicated.

Problem	Points	Possible
1		20
2		20
3		25
4		31
5		9
TOTAL		100

Problem 1. (20 points) Next fall, you decide to earn some extra cash by becoming a UT shuttle bus driver. You quickly bore of driving the same route over and over, so you start counting the number of people who ride the bus. Before long, you realize that the number of people riding the bus at any given time can be modeled by a normal distribution with mean 35 and standard deviation 7.

(a) (5) The bus can seat 49 people; what is the probability that its capacity is exceeded?

(b) (5) What is the 95th percentile of the number of riders on the bus?

(c) (5) What is the probability the number of riders is more than one standard deviation away from the mean?

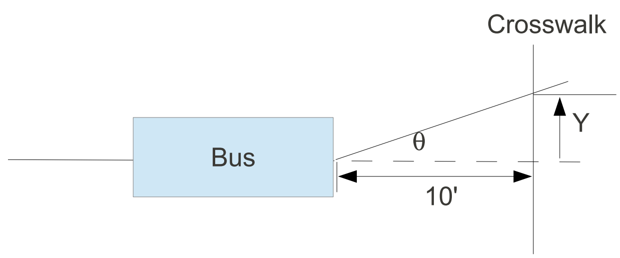
(d) (5) What is the mode of this distribution?

Problem 2. (25 points). Driving the shuttle bus doesn't pay very well, so you decide to boost your income through a shady side business — specifically, selling old CE 311S exams to students on the bus. Before the test, you have to decide how many exams to buy from your source, who charges you \$1.00 per exam. In turn, you sell exams to interested passengers for \$7.39 each. The number of passengers who are interested in buying an exam is random and can be described by an exponential distribution with mean 10.

If you buy more exams than you can sell, you're stuck with the extra and can't return them to the supplier. You also can't sell more exams than you have customers. So, if you get 15 exams from your source and 10 passengers want to buy, your profit is $7.39 \times 10 - 1.00 \times 15 = 59.9$ dollars. If you get 10 exams and 15 passengers want to buy, you can only sell 10 and your profit is $7.39 \times 10 - 1.00 \times 10 = 63.9$ dollars.

How many exams should you buy from your supplier to maximize your expected profit?

Problem 3. (25 points). While you're busy selling an exam to a passenger, the most interesting man in the world strolls in front of your bus.[1] You do what you always do in this situation — you close your eyes and swerve the bus in a random direction. In particular, the angle θ you swerve is randomly distributed between $-\pi/2$ and $+\pi/2$ radians with probability density function $(2/\pi)\cos^2\theta$. (See diagram below.) You are 10 feet away from the man when you swerve. Let the random variable Y represent the location at which the center of your bus passes the crosswalk, measured as shown in the diagram (i.e., $Y = 0$ if $\theta = 0$; $Y > 0$ if you swerve to the left, and $Y < 0$ if you swerve to the right).



(a) (5) If $Y = 0$, you strike him directly and he is injured. What is the probability of this happening?

(b) (10) If $0 < |Y| < 10$, you strike him indirectly and he is stunned (but not injured). What is the probability of this happening?

(c) (10) What is $E[Y]$?

A table of trigonometric identities and integrals is provided after the last question, in case you need them. **You must show all of your work to receive credit: do not just report the answer your calculator gives you.**

[1] He doesn't always *cruza*, but when he does, it's *enfrente del autobús*.

Problem 4. (30 points). Unfortunately, hitting pedestrians is something you do all too frequently, and your supervisor starts keeping track of all of your accidents. You think there might be some relationship between the number of consecutive shifts you drive (denoted X) and the number of pedestrians you hit that day (denoted Y... thankfully, you drive slowly enough that none are seriously injured). The following table shows the joint probability mass function of X and Y, except for one missing value.

		X		
		1	2	3
Y	0	1/16	1/8	0
	1	1/16	1/4	0
	2	0	1/4	1/16
	3	0	1/8	q

(a) (5) What is the missing value q?

(b) (5) What is the probability that you hit more pedestrians than the number of consecutive shifts you work?

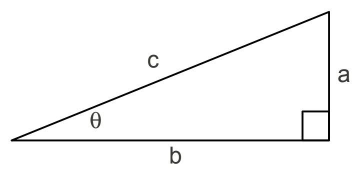
(c) (5) What is the expected number of pedestrians you hit?

(d) (15) What is $\rho_{X,Y}$?

Problem 5. (10 points, 2 each). In the end, you're fired from driving buses and spend your days answering true/false questions about probability. **You do not need to show your work, and no partial credit will be given.** Do *not* assume anything which is not given to you.

T	F	$E[X^2] \geq (E[X])^2$ for any random variable X.
T	F	f is a function which satisfies $\int_{-\infty}^{\infty} f(x)\ dx = 1$, so it must be a valid probability density function.
T	F	If F is a cumulative distribution function and $a > b$, then $F(a) > F(b)$
T	F	If X has a normal distribution, $\log X$ has a lognormal distribution.
T	F	If X and Y are independent random variables, $\mathrm{Cov}(X, Y) = 0$.

You may find some (hopefully not all) of the following identities useful as you solve Problem 3:



If $\theta = \pi/4$, then $b = a$ and $c = \sqrt{2}a$; if $\theta = \pi/6$, then $b = \sqrt{3}a$ and $c = 2a$.

$$a^2 + b^2 = c^2$$

$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{a}{b}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\frac{d}{d\theta}\sin\theta = \cos\theta$$

$$\frac{d}{d\theta}\cos\theta = -\sin\theta$$

$$\frac{d}{d\theta}\tan\theta = \frac{1}{\cos^2\theta}$$

$$\int \sin\theta\ d\theta = \cos\theta + C$$

$$\int \cos\theta\ d\theta = -\sin\theta + C$$

$$\int \tan\theta\ d\theta = -\log|\cos\theta| + C$$

$$\int \sin^2\theta\ d\theta = \frac{\theta}{2} - \frac{1}{2}\sin\theta\cos\theta + C$$

$$\int \cos^2\theta\ d\theta = \frac{\theta}{2} + \frac{1}{2}\sin\theta\cos\theta + C$$