CE 311S: Exam 2

Tuesday, April 2

2:00 – 3:15 PM

Name ____

Instructions:

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no partial credit!
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- If you need to make any additional assumptions, state them clearly.
- You may use one regular-sized sheet of notes; please turn in the notes with your exam. No additional resources are permitted.
- The number of points associated with each part of each problem is indicated.

Problem	Points	Possible
1		20
2		20
3		30
4		20
5		10
TOTAL		100

Problem 1. (20 points). You are a hugely devoted fan of Canadian teen idol Justin Bieber, and are a self-proclaimed "belieber." Your most prized possession is a bootleg recording of *Beauty and a Beat* from the Believe Tour, and you listen to this song incessantly, especially when working on homework for 311S. You put your engineering skills to good use by programming your MP3 player so that the time between successive playbacks of *Beauty and a Beat* is a random variable T which has the probability density function $f(t) = (1/10)\exp(-t/10)$ when $t \geq 0$ and 0 otherwise, assuming t is measured in minutes.

(a) (5) What is the mean time between playbacks of *Beauty and a Beat*?

(b) (5) What is the median of T?

(c) (5) What is the mode of T?

(d) (5) Assuming you spend 6 hours on the homework assignment, let B denote the number of times *Beauty and a Beat* starts to play while working on the homework. What distribution does B have, and what is $E[B]$?

Problem 2. (20 points). Needing an extra burst of energy to solve a difficult problem, you load up Lady Gaga's *The Edge of Glory*. You are such a devoted Lady Gaga fan (a "Little Monster") that you spent the latter half of January road-tripping around the country to attend her Born This Way Ball performances. Over time, you notice that the volume of the crowd noise, when measured in bels[1], can be described by a normal distribution with a mean of 10 and a standard deviation of 0.5. Ignoring the irony of using the "normal" distribution to describe a Gaga performance, answer the following questions:

(a) (5) Let V represent the volume level at a Lady Gaga concert in bels. Write the pdf for V.

(b) (5) What is the the probability that the volume is more than 11 bels?

(c) (5) The relationship between the crowd size C and the volume level V is $C = e^V$. What is the probability that the crowd size is more than 35,000?

(d) (5) What are the mean and standard deviation crowd size?

[1] A tenth of a decibel, naturally

Problem 3. (30 points) Before finishing the homework, you get online to see if there have been any developments in the Nicki Minaj/Lil' Kim feud, which started after the debut of Nicki Minaj's *Pink Friday* when Lil' Kim was quoted as saying "If you are going to steal my swag, you gonna have to pay." As a huge Minaj fan[2], you find such threats unconvincing. In fact, you have conducted extensive research, measuring the number of artists who stole Kim's swag each year (denoted by the random variable X) and the number who had to pay (Y). Based on this data, you have constructed the following table containing the joint pmf of X and Y in any given year.

		X		
		0	1	2
	0	1/8	1/4	0
Y	1	0	1/4	1/4
	2	0	0	1/8

(a) (5) Are X and Y independent? (Explain your answer, do not just say "yes" or "no")

(b) (5) In any given year, what is the probability that everybody who steals Lil Kim's swag has to pay?

(c) (5) What are the expected values of X and Y?

(d) (15) What is $\rho_{X,Y}$?

[2] A "Barbie"

Problem 4. (20 points). For some reason, your roommate doesn't share your taste in music, so you try to get as much studying done as possible before they come home. However, you have an exam tomorrow and can't study without music, so you continue to blast your playlist through the apartment. The current time is 6:00 PM. Assume that the time at which your roommate comes home is uniformly distributed between 6:00 and 6:30 PM. Once they come home, they will try to get you to turn the music off for a certain period of time before giving up and leaving; the length of that period of time is uniformly distributed between 0 and 30 minutes. The length of this latter period is independent of the time they come home. Let W be the number of minutes between now and when your roommate gives up and leaves, and let $F_W(w)$ be the CDF of W, and $f_W(w)$ the PDF.

(a) (5) What are the earliest and latest times at which your roommate will give up and leave?

(b) (5) What is $E[W]$?

(c) (5) What are $F_W(w)$ and $f_W(w)$ when w is between 0 and 30 minutes?

(d) (5) What are $F_W(w)$ and $f_W(w)$ when w is greater than 30 minutes?

Problem 5. (10 points, 2 each). True/False. **You do not need to show your work, and no partial credit will be given.** Do *not* assume anything which is not given to you.

T	F	If $\rho_{X,Y} = 0$, then X and Y are independent.
T	F	If $\rho_{X,Y} < 0$, then $\mathrm{Cov}(X, Y) < 0$.
T	F	The gamma distribution is a special case of the exponential distribution.
T	F	If X is a continuous random variable, it can only have one mode.
T	F	The cumulative distribution function of any random variable is nondecreasing.