

## CE 311S: Exam 2

Tuesday, April 1

2:00 – 3:15 PM

Name \_\_\_\_\_

### Instructions:

- **SHOW ALL WORK** unless instructed otherwise. You will not receive full credit for just writing down the answer.
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- If you need to make any additional assumptions, state them clearly.
- You may use one regular-sized sheet of notes and a calculator without communication abilities. No additional resources are permitted.
- The number of points associated with each part of each problem is indicated.

Problem	Points	Possible
1		30
2		25
3		25
4		20
<b>TOTAL</b>		100

**Problem 1.** (30 points). Thankfully, the secret document you released at the end of the last exam did not get you killed. You go deep underground, content that you will never be caught, until you realize that one of the documents you have is Salt Lick BBQ's secret recipe. You return to Austin under a false identity and set up a competing barbecue restaurant (Pepper Rick's BBQ) downtown.

You begin by conducting market research for one week, to see what the demand is for brisket and gluten-free organic free-range vegan kabobs (a new menu item you introduced to appeal to the Austin crowd). Based on your research, you construct the following table, which shows the joint probability of selling a certain amount of brisket (measured in hundreds of pounds) and a certain number of kabobs (again measured in hundreds).

		Kabobs (in hundreds)		
		0	1	2
Brisket (hundreds of pounds)	0	2/18	3/18	4/18
	1	1/18	2/18	3/18
	2	0	1/18	2/18

- (a) (5) What is the probability you sell more kabobs than pounds of brisket?
- (b) (5) How many pounds of brisket do you expect to sell on a typical day?
- (c) (5) How many kabobs do you expect to sell on a typical day?
- (d) (15) What is the correlation between the number of kabobs sold and pounds of brisket sold?

**Problem 2.** (25 points). Based on these results you start ordering enormous numbers of vegan kabobs. Unfortunately you chose SXSW week to do your market research, so the influx of hipsters ironically threw off your estimates and business is not going so well. To make matters even worse, you have been ordering your potato salad from your shady brother-in-law's discount company.

The potato salad is delivered in cubical containers which are supposedly 2 ft on a side, but due to shoddy manufacturing the actual side length  $X$  varies substantially from shipment to shipment. Furthermore, customers often become ill from the potato salad; let  $Y$  denote the proportion of customers who fall ill on a given day.  $X$  and  $Y$  are jointly distributed with pdf  $f_{X,Y}(x,y) = \frac{1}{4}(3 - x - 2y + 2xy)$  for  $1 \leq x \leq 3$  and  $0 \leq y \leq 1$ , and 0 otherwise.

- (a) (10) What is expected side length of the potato salad containers?
- (b) (5) What is the expected volume of the potato salad containers?
- (c) (10) Are  $X$  and  $Y$  independent? (Explain your answer, do not just say yes or no.)

**Problem 3.** (25 points). As more and more people fall ill from eating at your restaurant, the lawsuits start coming in. To distract yourself from all of the court proceedings, you think back fondly to your 311S days and notice that the amount you have to pay for each lawsuit seems to be lognormally distributed with  $\mu = 5$  and  $\sigma = 2$ .

- (a) (5) What is the mean amount you have to pay for each lawsuit?
- (b) (5) What is the standard deviation?
- (c) (10) What is the probability that you have to pay more than \$50,000 for a single lawsuit?
- (d) (5) What is the 95th percentile?

**Problem 4.** (20 points). After losing a fortune on lawsuits, you decide that representing yourself in court may not be the best of options, so you hire a lawyer. Unfortunately, by this point things are going so poorly that you have to choose whoever is cheapest. You have to decide whether to hire a criminal lawyer (Dewey, Cheatham & Howe, LLC) or a “criminal lawyer” (Saul Goodman & Associates). Assume that the cost of legal services from Dewey, Cheatham & Howe and Saul Goodman have independent exponential distributions with mean \$60,000 and \$30,000, respectively. You will pick the law firm with the lower cost; let the random variable  $C$  be the cost you actually pay.

- (a) (5) What is the probability that you pay more than \$40,000 for legal services? (Hint: If you pay more than \$40,000, then **both** law firms have a higher cost than \$40,000).
- (b) (5) What is the cdf for  $C$ ?
- (c) (10) What are the mean and standard deviation of  $C$ ?