CE 311S: Exam 2

Thursday, April 9

2:00 – 3:15 PM

Name _____

Instructions:

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no partial credit!

- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.

- If you need to make any additional assumptions, state them clearly.

- You may use one regular-sized sheet of notes. No additional resources are permitted.

- The number of points associated with each part of each problem is indicated.

Problem	Points	Possible
1		20
2		30
3		30
4		20
TOTAL		100

Problem 1. (20 points). It was a dark and stormy night. The rain fell in sheets against my window, and a streetlight flickered. It was late and the day had been slow, but I had worked this business for years and knew I'd get a client before long. Oh, I suppose I haven't introduced myself yet – my name is Walters, Sam Walters, and I'm a private eye. Chicago's where I do my work, and 1931 is no easy year to do it in. As I reminisce about the past, I realize that the number of cases I solve in any given year be modeled by a normal distribution with mean 38 and standard deviation 7.

(a) (5) What is the probability that I solve at least 52 cases this year?

(b) (5) What is the 95th percentile of the number of cases I solve?

(c) (5) What is the probability the number of cases solved is more than one standard deviation away from the mean?

(d) (5) What is the mode of the number of cases I solve?

Problem 2. (30 points). Frustrated, I rolled my chair over to the window and gazed into the street five stories below, lit only by the light of a pale October moon and a flickering streetlight. Prohibition be damned, I thought to myself as I took a swig of bourbon. Sooner or later a client had to come by. It was just that kind of day.

(a) (10) Assume that clients arrive independently, one at a time, at an average rate of once every 2 hours. What is the mean time I have to wait for a client? The standard deviation? The probability I have to wait more than 4 hours?

(b) (10) Many of my cases have to do with life insurance, with the recent popularity of double indemnity clauses, and I've noticed that the amounts tend to be gamma distributed with mean \$5,000 and standard deviation \$200. If I think ahead to the coming years, what is the mean and standard deviation of the *average* amount of the next 100 insurance cases?

(c) (10) Before long a rap at the door broke me out of my reverie. A young woman stepped in, and told me her name was Elisabeth McDaniels. She made sure I knew it was Elisabeth spelled with an 's', but I've been in the game a long time, and the only thing her name spelled to me was trouble. Her brother Frankie had recently passed away, and Elisabeth was convinced his death was anything but accidental. I don my trench coat and fedora, and step outside to wait for the streetcar, which comes every 15 minutes. Assuming I did not time my departure with the streetcar schedule, what is the mean time I need to wait? The standard deviation?

Problem 3. (30 points). I get off the streetcar in a seedy part of town, and trudge through the rain. The street was lined with the dime-a-dozen bordellos and speakeasies that seem to fill every corner of town these days, where junior members of the mob pass the nights away. Surely one of them knows what happened to Elisabeth's brother. Let X denote the number of speakeasies I visit, and Y the number of mobsters I run into. The following table shows the joint probability mass function of X and Y, except for one missing value.

			X	
		1	2	3
	0	1/16	1/8	0
	1	1/16	1/4	0
Y	2	0	1/4	1/16
	3	0	1/8	q

(a) (5) What is the missing value q?

(b) (5) What is the probability that you visit more mobsters than the number of speakeasies you visit?

(c) (5) What is the expected number of mobsters you meet?

(d) (10) What is $\rho_{X,Y}$?

(e) (5) If I pay a $2 entry fee to each speakeasy I visit, and buy a $1 drink for every mobster I run into, what is the mean and standard deviation of the amount of money I spend tonight?

Problem 4. (20 points). All of a sudden, I see a familiar face walk in and sit at the bar. It's Snake Eyes Malone, and while I've dealt with him before, he's the kind who likes to do the dealing himself, and is feeling generous if he lets you shuffle beforehand. Still, he owes me one, and before long he tells me that Elisabeth's brother was involved in the mob. He was involved in a scheme to bribe city council members in exchange for kickbacks in real estate.

Frankie would give a certain amount of cash to a councilmember, who would then pocket anywhere between $0 and $300 (uniformly distributed). The mob would then earn $300 times the square root of the remaining amount in kickbacks, and the difference is their profit. For example: if Frankie gave $10,000, the councilmember might keep $199, so the mob would earn $300 \times \sqrt{\$10000 - \$199} = \$29700$, and the profit would be $29700 - \$10000 = \$19700$. Frankie does not know in advance the amount the councilmember will keep, just that it is uniformly distributed between $0 and $300.

(a) (10) If Frankie gave a $10,000 bribe, what is the expected profit for the mob?

(b) (10) What amount should Frankie have given the councilmember to maximize the expected profit for the mob?