# CE 311S: Exam 2
## Tuesday, April 3
## 8:00 – 9:15 AM

Name _____

**Instructions:**

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no partial credit!

- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.

- If you need to make any additional assumptions, state them clearly.

- You may use one regular-sized sheet of notes; please turn in the notes with your exam. No additional resources are permitted.

- The number of points associated with each part of each problem is indicated.

| Problem | Points | Possible |
|---------|--------|----------|
| 1       |        | 30       |
| 2       |        | 20       |
| 3       |        | 20       |
| 4       |        | 30       |
| **TOTAL** |      | 100      |

**Problem 1.** (30 points). It was a dark and stormy night. The rain fell in sheets against my window, and a streetlight flickered. It was late and the day had been slow, but I had worked this business for years and knew I'd get a client before long. Oh, I suppose I haven't introduced myself yet — my name is Walters, Sam Walters, and I'm a private eye. Chicago's where I do my work, and 1931 is no easy year to do it in. I've managed to keep busy, though, and think back over the last eight months. Here's a table showing different possibilities of how many cases I've had each month, and how many I've been able to solve; the numbers show how many of the last eight months fell into each category.

| Number of months | | Number of cases | | |
|---|---|---|---|---|
| | | 0 | 1 | 2 |
| | 0 | 1 | 1 | 1 |
| Cases solved | 1 | 0 | 1 | 3 |
| | 2 | 0 | 0 | 1 |

(a) (5) Assuming that this table is representative of the true probabilities, what is the mean number of cases I solve each month?

(b) (5) Are the number of cases I get each month, and the number I solve each month, independent? Explain your answer, do not just say yes or no.

(c) (5) What is the expected number of *unsolved* cases each month?

(d) (15) What is the correlation coefficient between the number of cases I get each month, and the number I solve each month?

**Problem 2.** (20 points). Before long, a rap at the door broke me out of my reverie. A young woman stepped in, and told me her name was Elisabeth McDaniels. She made sure I knew it was Elisabeth spelled with an 's', but I've been in the game a long time, and the only thing her name spelled to me was trouble. It was an insurance case, and her brother had recently passed away. His wife was set to receive a massive insurance payout, double indemnity and all, but Elisabeth was convinced her brother's death was anything but accidental.

I start by reviewing the distribution of life insurance payoffs, which I'll call $X$. In my experience, life insurance payoffs range between \$1000 and \$2000 (hey, it's 1931), and the probability density function for a life insurance payoff is proportional to $(x - 1000)(2000 - x)$ between these values, and zero elsewhere.

(a) (5) Write the complete pdf for $X$, including the proportionality constant.

(b) (5) What is the expected value of $X$?

(c) (5) What is the variance of $X$?

(d) (5) What is the expected value of the *absolute difference* between $X$ and its mean value? (The absolute difference between two numbers $x$ and $y$ is $|x - y|$.)

**Problem 3.** (20 points). Elisabeth handed me a folder, showing a photo of her sister-in-law meeting with a shadowy figure in a dark alley, and left. I was stunned, and set to work. My mind was reeling so fast you'd think it was a movie projector, but the only pictures moving here were in the dossiers I was shuffling through, trying to see who might be behind this. Not making much progress, I don my trench coat and fedora, and step out into the rain. A good walk always clears my head.

As I walk, I think about whether my cases will clear enough dough to pay the rent on my office. In an average month my cases bring in $500 (with a standard deviation of $120), and my expenses are $400 (with a standard deviation of $50); but Elisabeth's case is shaping up to be anything but average.

(a) (5) What are the mean and standard deviation of my profit in a given month? Assume that the correlation coefficient between revenue and expenses is 0.5.

(b) (5) Now assume that I don't know the correlation coefficient. What would be the highest and lowest possible values for the standard deviation of my monthly profit?

(c) (10) I also need to give Uncle Sam his due, so for each dollar I earn I only keep ninety cents after taxes. What are the mean and standard deviation of my profit after accounting for taxes? Assume that the correlation coefficient between revenue and expenses is 0.5.

**Problem 4.** (30 points). Before long, I reach the local gin joint, a dime-a-dozen speakeasy like those filling every corner of this town. A familiar face walks by. It's Snake Eyes Malone, and while I've dealt with him before, he's the kind who likes to do the dealing himself, and is feeling generous if he lets you shuffle beforehand. Still, if anyone knows what happened to Elisabeth's brother, it's him. I ply him with a few drinks, reveal some dirt I have on him, and he points me to Muggsy Nelson. Clues start falling into place like a jigsaw puzzle, but knowing my luck this one's been cut crooked and is missing a few pieces.

In fact, the clues are falling into place like a Poisson distribution, with an average of 3 clues per day. I will solve Elisabeth's case once I find seven clues – it's my lucky number, I always solve a case when I get the seventh clue. For each of the following quantities, give the name of the continuous distribution; its probability density function; its mean; and its standard deviation.

(a) (10) The time *between* successive clues, in days.

(b) (10) The time it takes me to solve Elisabeth's case, in days.

(c) (10) The *average* of the times it takes me to solve the next 49 cases, in days.