

CE 311S: Exam 2

Friday, April 12

9:00 – 9:50 AM

Name _____

Instructions:

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no partial credit!
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- If you need to make any additional assumptions, state them clearly.
- You may use a calculator and one regular-sized sheet of notes. No additional resources are permitted.
- The number of points associated with each part of each problem is indicated.

Problem	Points	Possible
1		30
2		35
3		35
TOTAL		100

Problem 1. (30 points). After finding out that Priyadarshan won an intramural racquetball tournament last semester¹ you decide that you want to beat him at all costs. You relentlessly pester your friend Rishabh to train you, even though he is busy with his studies. On occasion he reluctantly agrees to give you a lesson; the time between lessons is well-approximated by an exponential distribution with a *median* of 5 days. Assume that each lesson occurs independently of every other.

- (a) (10) What are the mean and variance of the amount of time until your first lesson? What is the probability that you have to wait more than 5 days?
- (b) (10) After five lessons, you finally feel ready to challenge Priyadarshan. What is the mean and standard deviation of the time until you have completed five lessons?
- (c) (10) During your practice games, you find that the number of points you score in each game follows a normal distribution with a mean of 8 and a standard deviation of 3. What is the probability that you score more than 11 points in a single game? Over the next 36 games, what is the probability that you score an *average* of more than 9 points per game?

¹This is true.

Problem 2. (35 points). Unfortunately, your racquetball skills grow only slowly under Rishabh's tutelage. You grow impatient and begin playing every practice match as if your life depended on it — diving for balls, pushing past your limits, and if all else fails closing your eyes and swinging your racquet wildly. Perhaps not unsurprisingly, this results in a number of minor injuries, but perhaps it is paying off in terms of wins. The following table shows the joint probability mass function of A (the number of games you win in a three-game match) and B (the number of minor injuries you suffer during the match).

		Games won A			
		0	1	2	3
Minor injuries B	0	4/40	3/40	2/40	1/40
	1	8/40	6/40	4/40	2/40
	2	4/40	3/40	2/40	1/40

- (a) (10) Write the marginal PMFs for A and B .
- (b) (10) What are the mean and standard deviation of A and B ?
- (c) (10) What is ρ_{AB} ?
- (d) (5) You make a bet with Rishabh where you will earn \$10 for every game you win, but have to pay \$5 for every minor injury you suffer. What are the mean and standard deviation of the amount you will earn from this bet? (Positive means you make money; negative means you lose money.)

Problem 3. (35 points). You continue playing racquetball even as minor injuries accumulate. After a certain amount of time X , measured in months, you will suffer a major injury; assume that X is described by the PDF $3e^{-3x}$ for $x \geq 0$, and 0 otherwise. Also, the time until you beat Priyadarshan Y can be described by the PDF $2e^{-2y}$ for $y \geq 0$, and 0 otherwise; Y is also measured in months. Assume that these occur independently.

- (a) (5) Write the joint PDF for X and Y .
- (b) (10) What is the probability that you both beat Priyadarshan *and* suffer a major injury during the next month?
- (c) (10) What is the probability that you beat Priyadarshan anytime before you suffer your first major injury?
- (d) (10) Now assume that you quit racquetball once you either suffer a major injury or beat Priyadarshan (whichever happens first). On average, how many months will pass until you stop playing racquetball?