# CE 311S: Exam 2

Friday, April 9

9:00 – 9:50 AM

Name ______________________________________________

**Instructions:**

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no partial credit!
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- If you need to make any additional assumptions, state them clearly.
- You may use a calculator and one regular-sized sheet of notes. No additional resources are permitted. Please submit your notes along with your exam.
- The number of points associated with each part of each problem is indicated.

| Problem | Points | Possible |
|---|---|---|
| 1 | | 30 |
| 2 | | 30 |
| 3 | | 40 |
| **TOTAL** | | 100 |

**Please copy the following statement (based on UT's honor code and the ASCE code of ethics) in your own handwriting, and sign it.** For the purposes of this statement, academic dishonesty includes (but is not limited to) sharing with or receiving information from others about the exam, by any mode of communication.

"As a student of The University of Texas at Austin and as a civil engineer, I certify that I have not and will not participate in any acts of academic dishonesty related to this exam. If I witness any acts of academic dishonesty, I will report them to the instructor."

**Your handwritten copy of the statement:**

Signature ______________________________

**Problem 1.** (30 points). As you and your friends become fully vaccinated against COVID, you slowly begin engaging in activities you've avoided doing over the last year. You begin by going to karaoke, but unfortunately your friends have questionable taste in music. Their favorite songs are Bonnie Tyler's "Total Eclipse of the Heart" and Rick Astley's "Never Gonna Give You Up." Over the course of the night, let $B$ (for "Bonnie") and $R$ ("Rick") denote the number of times these two songs are performed. The following table shows the joint PMF of the frequency of singing these songs on a given night out:

| | | $B$ | | |
|---|---|---|---|---|
| | | 0 | 1 | 2 |
| $R$ | 0 | 1/8 | 0 | 0 |
| | 1 | 1/4 | 1/4 | 0 |
| | 2 | 0 | 1/4 | 1/8 |

(a) (5) Are $B$ and $R$ independent? (Explain your answer, do not just say "yes" or "no")

(b) (5) On any given night, what is the probability that "Total Eclipse of the Heart" is sung at least as often as "Never Gonna Give You Up"?

(c) (5) What are the expected values of $B$ and $R$?

(d) (15) What is $\rho_{B,R}$?

**Problem 2.** (30 points). During the pandemic, you and your classmates were also keeping track of the number of emails you receive with trite phrases such as “in these unprecedented times” or“now more than ever,” and were placing bets on who received the most of these emails.

You notice that the number of these emails each of you receives is described by a lognormal distribution with $\mu = 1$ and $\sigma = 3$.

(a) (5) What are the mean and standard deviation of the number of emails you receive?

(b) (5) What is the probability you receive more than 30 such emails?

(c) (10) Let $X_1$ and $X_2$ denote the number of emails you and your best receive. You set up a bet where you get $Y = X_1 - 2X_2$ dollars (positive means you won money, negative means you lost and have to pay). If $X_1$ and $X_2$ are independent, what are the mean and standard deviation of $Y$?

(d) (5) Instead of being independent, now assume that the number of emails you and your friend recieve is positively correlated, with $\rho_{X_1,X_2} = +0.5$. Would the mean you reported in part (c) increase, decrease, or stay the same? What about the standard deviation?

(e) (5) In total, there are 40 students in your class. Assuming the number of emails you get are independent, what is the probability that the mean number of emails received across the class is more than 40?

**Problem 3.** (40 points). Two weeks after your second vaccine, you also feel brave enough to return to Gregory Gym. However, it seems many students have the same idea. There are two cardio machines you are waiting to use (the treadmill and the stationary bike), but both are currently in use. The time that anyone spends using these two machines are exponentially distributed, with means of 20 and 30 minutes, respectively. Assume that these times are independent.

(a) (10) What is the probability that the person using the stationary bike will still be using it 60 minutes from now?

(b) (10) What is the 90th percentile for the remaining amount of time the treadmill is in use?

(c) (10) Using $X$ and $Y$ to denote the time until the treadmill and stationary bike become available, write down the joint PMF of $X$ and $Y$.

(d) (10) You decide to use whichever machine is available first. What is the probability that you end up using the treadmill?