

CE 311S: Final Exam

Friday, December 15

9:00 AM – 12:00 noon

Name _____

Instructions:

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no partial credit!
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- If you need to make any additional assumptions, state them clearly.
- You may use a calculator and two regular-sized sheets of notes. No additional resources are permitted.
- The number of points associated with each part of each problem is indicated.

Problem	Points	Possible
1		16
2		14
3		20
4		15
5		15
6		20
TOTAL		100

Problem 1. (16 points) 2017 had its highs and lows, and as the fall semester comes to an end you resolve to start afresh in 2018. You scribble down a number of resolutions in different categories — academics, hobbies, fitness, romance — before quickly realizing that there are far too many things you want to do. Your first order of business is to narrow down the list to something more manageable:

Academic	Hobbies	Fitness	Romance
Spring GPA over 3.5	Learn to play ukulele	Join UT mountain unicycling team	Woo Taylor
Study two hours each night	Learn to cook	Run Longhorn Run under 20:00	
Attend TA office hours	Climb Mt. Kilimanjaro	Walk 10,000 steps per day	
Do readings before class	Build a theremin	Bench press 200 pounds	
	Moonlight as a clown		

- (a) (4) In how many ways can you choose *one* resolution from each of the four categories?
- (b) (4) In how many ways can you choose *two* resolutions from the “academic” category?
- (c) (4) In how many ways can you choose two academic resolutions, three hobby resolutions, one fitness resolution, and one romantic resolution?
- (d) (4) If you can pick any number of resolutions from each category (even zero), in how many ways can you choose your resolutions for 2018?

Problem 2. (14 points) You first resolve that your spring 2018 GPA will be over 3.5. This number has particular significance to you because this happens to be the average GPA of your five closest friends — after obsessively tracking their grades for years, you are very sure that their GPAs are normally distributed with a mean of 3.5. This fall, your friends have tried to improve their grades by studying with a metal pasta strainer on their heads, to better focus their brain waves.¹ You haven't joined them, because that's just weird. However, at the end of the semester, you notice their fall GPAs:

3.4 3.7 3.8 3.6 3.5

Thankfully, you know how to conduct a hypothesis test, so you can rigorously decide whether or not to adopt their strategy.

- (a) (4) What are your null and alternative hypotheses?
- (b) (5) If you conduct the test with a 5% significance, what is your rejection region for this test?
- (c) (5) Do you wear a pasta strainer on your head next semester? (Explain your answer, don't just say yes/no.)

¹Or perhaps they simply converted to Pastafarianism. You're not sure.

Problem 3. (20 points) You next turn to your hobby resolutions, and hope that they go more smoothly than the academic ones.

- (a) (5) You quickly discover that cooking is harder than it looks on the Food Network, and that there is a 20% chance that the fire department will be called to your apartment each time you try to cook. After the third time this happens, your roommates will stage an intervention and force you to stop cooking. Let A be the number of times you try to cook in 2018. What are $E[A]$ and $V[A]$?
- (b) (5) You have better luck with your clown gig, and before you know it you start receiving offers to perform at various parties, festivals, and *soirées* around Austin. Assume that you receive an average of one offer per week, and that offers come independent of each other. What is the probability that you get more than two offers in the last week of 2018?
- (c) (5) Let C be the time between successive offers for clown gigs, *measured in days*. What are the mean and standard deviation of C ?
- (d) (5) At the end of 2018, you will calculate D , the *average* number of gigs per week you received over the entire year. What is the probability that D is more than 1.1?

Problem 4. (15 points) To prepare for your fitness resolutions, you first gather data on how crowded Gregory Gym will be at different points in time during the semester. Gathering data from a friend who works for RecSports, you build a table of usage statistics from the fall semester, looking at 60 randomly selected days. Let x_i denote the day of the semester the measurement was taken (1, 2, etc., up to the number of days in the semester), and let y_i be the number of students who went to Gregory on that day. You find that $\sum x_i = 2720$, $\sum y_i = 17.3 \times 10^3$, $\sum x_i^2 = 163 \times 10^3$, $\sum y_i^2 = 6.31 \times 10^6$, and $\sum x_i y_i = 622 \times 10^3$.

- (a) (4) What is the best-fit line for predicting the number of students at Gregory each day of the semester?
- (b) (4) What is the R^2 value for your line?
- (c) (2) What is the probability that more than 300 students will use Gregory on the 1st day of the semester?
- (d) (2) What is the probability that more than 300 students will use Gregory on the 90th day of the semester?
- (e) (3) What is the P value for your line? Can you claim a significant relationship between these two variables with 5% significance?

Problem 5. (15 points) In pursuit of your resolution to woo lovely Taylor, you start practicing pickup lines on random people studying in the EERC. Hoping that your common love of engineering will spark a lasting relationship, you have perfected your delivery of three pickup lines:

- A You must be $\sqrt{-1}$, because you can't be real.
- B I wish I were $\sin^2 \theta$ and you were $\cos^2 \theta$, so that together we'd be one.
- C My love for you is like π , irrational and never ending.

You keep meticulous records of your practicing, and find that 20% of people responded favorably to your romantic overtures, while 80% responded unfavorably. Of those who responded favorably, you used line A 25% of the time, line B 50% of the time, and line C 25% of the time. Of those who responded unfavorably, you used line A 40% of the time, line B 50% of the time, and line C 10% of the time.

- (a) (5) For any given attempt, what is the probability that you both used line A and received a favorable response?
- (b) (5) During your practicing, what percent of the time did you use line A?
- (c) (5) Which pickup line will maximize the probability of a positive reaction from Taylor, and what is that probability?

Problem 6. (20 points) You think back over the last 6 years, and write down the percentage of your new years resolutions that you were successfully able to keep. Within this sample, the mean is 42% and the standard deviation is 10%. Assuming that the percentage of kept resolutions is normally distributed, answer the following questions.

- (a) (5) Form an interval containing the mean percentage of successful resolutions, with 95% confidence.
- (b) (5) Form an interval containing the standard deviation, with 90% confidence
- (c) (5) Form an interval containing the percentage of successful resolutions for 2018, with 95% confidence.
- (d) (5) Form an interval containing the percentages of successful resolutions for *the next five years*, with 95% confidence.