CE 311S: Final Exam, Fall 2017
Solutions

Problem 1.

(a) $4 \times 5 \times 4 \times 1 = 80$

(b) $\binom{4}{2} = 6$

(c) $\binom{4}{2}\binom{5}{3}\binom{4}{1}\binom{1}{1} = 240$

(d) $2^{4+5+4+1} = 16384$

Problem 2.

(a) $H_0 : \mu = 3.5$, $H_a : \mu > 3.5$

(b) For this alternative hypothesis, we reject if $t > t_{.05,4} = 2.132$. Since $t = (\bar{x} - \mu_0)/(s/\sqrt{n})$, and since $n = 5$, $\mu_0 = 3.5$, and $s = 0.158$, we have $t > 2.132$ if $\bar{x} > 3.65$.

(c) No, because the sample mean is 3.6, which does not lie in the rejection region.

Problem 3.

(a) A is negative binomial with $m = 3$ and $p = 0.2$, so $E[A] = m/p = 15$ and $V[A] = m(1-p)/p^2 = 60$

(b) Let B be the number of offers in a given week; B is Poisson with $\lambda = 1$, so $P(B > 2) = 1 - \sum_{i=0}^{2} \frac{e^{-1}1^i}{i!} = 0.08$.

(c) C is exponential with mean and standard deviation both equal to 7 days.

(d) D is normal with mean of $E[B] = 1$ and variance of $V[B]/52 = 1/52$. $P(D > 1.1) = P(Z > 0.72) = 0.236$.

Problem 4. With the given problem data we have $S_{xx} = 3.96 \times 10^4$, $S_{xy} = -1.62 \times 10^5$, and $S_{yy} = 1.32 \times 10^6$. Furthermore $\bar{x} = 2720/60 = 45.3$ and $\bar{y} = 288$, $SSE = 6.58 \times 10^5$, and $SST = 1.32 \times 10^6$.

(a) $\beta_1 = S_{xy}/S_{xx} = -4.09$ and $\beta_0 = \bar{y} - \beta_1\bar{x} = 474$, so $y = 474 - 4.09x$.

(b) $R^2 = 1 - SSE/SST = 0.502$

(c) $\sigma = \sqrt{SSE/(n-2)} = 107$. When $x = 1$, $P(Y > 300) = P(\epsilon > -170) = P(Z > -1.59) = 0.94$.

(d) When $x = 90$, $P(Y > 300) = P(\epsilon > 194) = P(Z > 1.82) = 0.0341$.

(e) $t = \beta_1/(\sigma/\sqrt{S_x x}) = -7.64$. Since $n = 60$ we have $p \ll .0005 \approx 0$.

Problem 5. Let A, B, and C be the events corresponding to your use of each pickup line, and F the event where you receive a favorable response.

(a) $P(A \cap F) = P(F)P(A \mid F) = \frac{1}{5}\frac{1}{4} = \frac{1}{20}$

(b) $P(A) = P(A \cap F) + P(A \cap F^c) = \frac{1}{20} + \frac{4}{5}\frac{4}{10} = 0.37$

(c) $P(F \mid A) = P(A \cap F)/P(A) = 0.135$. Similarly $P(F \mid B) = 0.2$ and $P(F \mid C) = 0.384$, so line C is the most likely to receive a positive reaction (38.4% probability).

Problem 6. We have $\bar{x} = 42$, $s = 10$, and $n = 6$.

(a) The interval bounds are $\bar{x} \pm t_{0.025,5} s/\sqrt{n}$, or $(31.5, 52.5)$.

(b) The interval bounds are $\sqrt{(n-1)s^2/\chi^2_{0.05,5}}$ and $\sqrt{(n-1)s^2/\chi^2_{0.95,5}}$, or $(6.72, 20.9)$.

(c) The interval bounds are $\bar{x} \pm t_{0.025,5} s\sqrt{1 + 1/n}$, or $(14.2, 69.8)$.

(d) If the interval contains the next 5 years with 95% probability, then it contains each individual year with probability $\sqrt[5]{0.95} = 0.98979 \approx 0.99$. So the interval bounds are $\bar{x} \pm t_{0.005,5} s\sqrt{1 + 1/n}$, or $(-1.55, 85.6)$.