CE 311S: Final Exam

Saturday, May 12

2:00 – 5:00 PM

Name ____

Instructions:

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no credit!
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- If you need to make any additional assumptions, state them clearly.
- You may use two regular-sized sheet of notes; please turn in the notes with your exam. No additional resources are permitted.
- The number of points associated with each part of each problem is indicated.

Problem	Points	Possible
1		20
2		20
3		15
4		20
5		15
6		15
TOTAL		105

Problem 1. (20 points) You are sitting in Dr. Boyles' lecture class and, as entertaining and useful as his lectures are, sometimes your mind can't help but wander to other topics. However, you feel guilty, so you decide to apply your knowledge of discrete distributions whenever you get distracted. **You may make aditional (reasonable) assumptions that you need for solving these problems, as long as you state what they are.**

(a) (5) You are bored so you start counting the number of times Dr. Boyles says "easy easy" in lectures. After several weeks, you notice that, on average, he says "easy easy" 5 times per class. Let A represent the number of times he says "easy easy" in a given lecture. What distribution best describes A? What is the variance? What is the probability that he doesn't say "easy easy" at all in a lecture?

(b) (5) Next, you start counting how often there are typos in the slides. You find that 5% of the slides have typos in them. Let B be the number of slides you see before you see the third one with a typo (so if the third typo occurs on the 25th slide, $B = 24$). What is the expected value of B? the variance?

(c) (5) By sitting in the back row, you find that you can work on the crossword in the Daily Texan while making it look like you're just taking notes and looking thoughtful. After extensive practice you find you can solve 40% of the crosswords (assume the difficulty of each crossword is independent of any other). You attempt one puzzle each lecture, so let C be the number of puzzles you successfully solve over the semester (28 lectures). What distribution does C have? What is the expected number of puzzles you solve during the semester?

(d) (5) When all else fails, you use your laptop to play Scrabble$^{\text{TM}}$, a game where you try to make words out of letters drawn from a bag. You aren't doing very well, and the game is almost over. You have the four letters AGIN and you are randomly drawing three more letters from the bag. Suddenly, you realize that if you draw three O's, you can make the word OOGONIA which will give you enough points to win. There are fifteen letters left in the bag, four of which are O's. What is the probability that all three letters you draw are O's? the probability that you only draw two O's? that you don't draw any O's at all?

Bonus: (1 extra point). What does OOGONIA mean?

Problem 2. (20 points). One day you tire of the pedestrian distractions from Problem 1, and your mind takes a turn for the romantic — perhaps it's the spring air. As you leaf through the classified ads in the Daily Texan looking for apartments, you dream wistfully of Pat, the man/woman you've (unsuccessfully) tried to woo all semester. Pat thinks the average rent of apartments in Austin has gone up to over \$800, but you disagree and think they're much cheaper. Having more engineering skills than social skills, you decide the best course of action is to prove Pat wrong by performing a hypothesis test on the mean rent. Since you are madly in love and terrified of rejection, you use a significance level of 0.05 and will confront Pat with your results only if you are very confident that the average apartment rent is less than \$800.

(a) (4) Mathematically state your null and alternative hypotheses.

(b) (1) There are sixteen apartment ads in the Daily Texan, with a mean rent of \$750 and a standard deviation of \$100. What additional assumption do you need to perform the hypothesis test?

(c) (10) Perform the hypothesis test under this assumption. Do you confront Pat?

(d) (5) What is the sum of the rents for each of the sixteen apartemnts advertised?

Problem 3. (15 points). After dreaming more of Pat and scribbling a sonnet in the margin of your notes, you decide to buy Pat a gift of flowers — either carnations or daffodils, depending on the prices. Since you are trying to impress Pat *you will buy whichever ones are more expensive.* Assume that the price of carnations and daffodils are given by independent exponential distributions, both having a mean of $40 per dozen. As a budget-conscious student, you are curious about the distribution of X, the price that you will actually pay (i.e., the price of the more expensive flowers).

The first step is to write the cumulative distribution functon $F(x)$, which is the probability that you pay no more than x dollars per dozen. ($F(x) = P(X \leq x)$). Since you buy the more expensive one, if you pay less than x dollars, that means that both carnations and daffodils cost less than x dollars.

(a) (3) What is the probability that you pay less than $40 for a dozen flowers?

(b) (2) What is the cumulative distribution function (cdf) of X?

(c) (5) What is the probability density function (pdf) of X?

(d) (5) What are the mean and variance of X? Hint: $\displaystyle\int_0^\infty xe^{-ax}\, dx = \frac{1}{a^2}$ and $\displaystyle\int_0^\infty x^2e^{-ax}\, dx = \frac{2}{a^3}$

Problem 4. (20 points). Before class is over, you wonder about how you are going to do on the final. Your scores on the first two exams were 71 and 75. Assuming that your scores on both of these exams and the final are independent, normally distributed random variables with the same mean and variance, find an interval that will contain your score on the final with 90% confidence.

Also, you weren't paying attention when Dr. Boyles told you what the average was on the last test. However, you have a random sample of five friends who were willing to tell you their scores:

56 72 77 84 86

Find an interval that contains the true mean with 90% confidence.

Problem 5. (15 points). After chatting with your friends, the lecture still isn't over, so you log on to Facebook to see what Pat is up to. However, Pat hasn't made many wall posts lately, and in a flash of terror you suspect that s/he may be blocking some posts from you — perhaps your love is unrequited after all. You decide to perform a hypothesis test by counting the number of wall posts Pat makes in the next 24 hours. Over the semester, you have carefully determined that Pat makes an average of 4 wall posts in any given 24-hour period, and that the number of posts follows a Poisson distribution. As a hopeless romantic, you will not conclude Pat is blocking you unless you are very sure you are seeing fewer than 4 posts per day.

(a) (2) Mathematically state your null and alternative hypotheses.

(b) (3) Which of the following rejection regions is the most appropriate: $\{0, 1, 2\}$, $\{0, 1, 7, 8, \ldots\}$, $\{3, 4, 5\}$, or $\{6, 7, \ldots\}$?

(c) (5) For the rejection region you chose, what is the probability of a Type I error?

(d) (5) It turns out that Pat really is blocking some posts from you, so you can only see an average of 2 wall posts per day. For the rejection region you chose, what is the probability of a Type II error?

Problem 6. (15 points). Let Z be a continuous random variable with a standard normal distribution. Recall that Z and Z^2 are uncorrelated, but dependent.

(a) (5) What is $E[Z]$?

(b) (5) What is $E[Z^2]$?

(c) (5) What is $E[Z^3]$? (Hint: $Z^3 = Z \cdot Z^2$)