CE 311S: Final Exam, Spring 2012
Solutions

Problem 1.

(a) A is Poisson with $\lambda = 5$, so $V[A] = \lambda = 5$ and $P(A = 0) = \exp(-5) = 0.00674$

(b) $B+1$ is negative binomial with $m = 3$ and $p = 0.05$, so $\frac{m}{p} = 60 = E[B+1]$ and $E[B] = 59$. For variance, $V[B] = V[B+1] = m(1-p)/p^2 = 1140$

(c) C is binomial with $n = 28$ and $p = 0.4$, so $E[C] = np = 11.2$

(d) D is hypergeometric with $k = 3$, $b = 4$, and $r = 11$, so $P(D = 3) = 0.00879$, $P(D = 2) = 0.145$, and $P(D = 0) = 0.362$.

Problem 2.

(a) $H_0 : \mu = 800$, $H_a : \mu < 800$

(b) Since $n < 40$ we must assume the apartment rents are normally distributed.

(c) $t = (750 - 800)/(100/\sqrt{16}) = -2$ while $t_{0.05,15} = 1.753$, so you reject the hull hypothesis and confront Pat.

(d) $\sum x_i = n\overline{x} = 12000$

Problem 3. Let C and D respectively denote the price of carnations and daffodils.

(a) $P(X \le 40) = P(C \le 40)P(D \le 40) = (1 - \exp(-40/40))^2 = 0.4$

(b) $F_X(x) = P(X \le x) = P(C \le x)P(D \le x) = (1 - \exp(-x/40))^2$

(c) $f_X(x) = \frac{d}{dx}F_X(x) = \frac{\exp(-x/40)}{20}(1 - \exp(-x/40))$

(d) $E[X] = \frac{1}{20}\left(\int_0^\infty x\exp(-x/40)\,dx + \int_0^\infty x\exp(-x/20)\,dx\right)$. Using the formulas given in the problem, this is \$100. For variance, $E[X^2] = \frac{1}{20}\left(\int_0^\infty x^2\exp(-x/40)\,dx + \int_0^\infty x^2\exp(-x/20)\,dx\right)$. Using the formulas given in the problem, this is 1800. So $V[X] = E[X^2] - (EX)^2 = 800$.

Problem 4. The first part is a prediction interval with $\bar{x} = 73$, $s = 2.82$, and $n = 2$, so the interval endpoints are $73 \pm 2.82 \times 6.314 \times \sqrt{1 + 1/2}$, or $(51, 95)$. The second part is a confidence interval with $\bar{x} = 75$, $s = 12$, and $n = 5$, so the interval endpoints are $75 \pm 12 \times 2.132/\sqrt{5}$, or $(64, 86)$.

Problem 5. Let X denote the number of Facebook posts in the next 24 hours.

(a) $H_0 : \lambda = 4$, $H_a : \lambda < 4$

(b) $\{0, 1, 2\}$ is the only rejection region reflecting the alternative hypothesis.

(c) For a Type I error, we need $P(X \leq 2)$ when X is Poisson with $\lambda = 4$. This probability is $\exp(-4)(1 + 4 + \frac{4^2}{2!}) = 0.238$.

(d) For a Type II error, we need $P(X > 2)$ when X is Poisson with $\lambda = 2$. This probability is $1 - \exp(-2)(1 + 2 + 4) = 0.0526$.

Problem 6.

(a) $E[Z] = 0$

(b) $1 = V[Z] = E[Z^2] - (E[Z])^2$, so $E[Z^2] = 1$

(c) $0 = \text{Cov}(Z, Z^2) = E[Z^3] - E[Z]E[Z^2]$, so $E[Z^3] = 0$.