CE 311S: Final Exam<br>Saturday, May 10<br>7:00-10:00 PM

Name $\qquad$

## Instructions:

- SHOW ALL WORK unless instructed otherwise. You will not receive full credit for just writing down the answer.
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- If you need to make any additional assumptions, state them clearly.
- You may use two regular-sized sheets of notes and a calculator without communication abilities. No additional resources are permitted.
- The number of points associated with each part of each problem is indicated.

| Problem | Points | Possible |
| :---: | :---: | :---: |
| 1 |  | 20 |
| 2 |  | 15 |
| 3 |  | 15 |
| 4 |  | 15 |
| 5 |  | 20 |
| 6 |  | 15 |
| TOTAL |  | 100 |

Problem 1. (20 points). To deal with the food poisoning lawsuits from your failed barbecue restaurant, you end up hiring Saul Goodman and Associates. Although you were an inept manager, Saul realizes you were a good cook and introduces you to Walter "Schrödinger" Blanco and Jessica Orangewoman, who are looking for assistance with their math operation (selling answers to 311S exams). Before long you realize that probability distributions and random variables play a major role in the math trade. As you stand on the street corner at Dean Keeton \& San Jacinto, customers arrive independently, one at a time, with an average rate of 12 per hour. However, $1 \%$ of the customers are actually undercover profs who will turn you in for academic dishonesty. The number of answers each customer purchases is binomially distributed with $n=10$ and $p=0.5$.

For each of the following random variables, indicate the name of its distribution (e.g., binomial, lognormal, etc.), its mean, and its standard deviation (including units).
(a) (5) The time until the next customer arrives.
(b) (5) The number of customers who arrive in the next hour.
(c) (5) The number of real customers before an undercover prof arrives.
(d) (5) The average number of answers that the next 100 customers purchase.

Problem 2. (15 points). Unfortunately, over time the administration learns to patrol the corner of Dean Keeton \& San Jacinto, so you have to spread out your operation. There are three locations where your crew sells math: the Texas Union, Gregory Gym, and ECJ. Your arch-nemesis Frank Schraderbrau patrols these three locations with probability $20 \%, 30 \%$, and $50 \%$, respectively; furthermore, if he patrols the Union there is a $60 \%$ probability you will be caught, if he patrols Gregory there is a $30 \%$ probability you will be caught, and if he patrols ECJ there is a $10 \%$ probability you will be caught.
(a) (5) If you sell math at the Texas Union, what is the probability of being caught?
(b) (5) If you can only pick one location to sell math, which location minimizes the probability of being caught?
(c) (5) If you choose the three locations randomly (with equal probability), and are caught selling math, what is the probability you were caught at ECJ?

Problem 3. (15 points). One of your low-level sellers, Weasel, is caught at ECJ and must be defended in court by Saul. The last 6 times your sellers were caught, their legal expenses were

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8000 6400 7000 5800 3000 3400
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You feel that it is reasonable to assume these expenses are normally distributed, but you don't know the mean or variance.
(a) (10) Provide a $95 \%$ confidence interval on the average amount of legal expenses you pay for each case?
(b) (5) With $90 \%$ confidence, what is the minimum amount you will have to pay for Weasel's defense?

Problem 4. (15 points). Until this point, you have been obtaining your math from Sue de Ephedrine, a fairly reliable source; based on long experience, you know that $85 \%$ of her answers are correct. However, as you try to move into a larger market, you are considering moving to a new source who goes by the name of Math L. Meme. You obtain a sample of 100 answers from Math L. Meme and find out that $90 \%$ of them are correct. Use a hypothesis test (with $5 \%$ significance) to decide whether you should switch your supplier to Math L. Meme.
(a) (5) What are your null and alternative hypotheses?
(b) (5) What is the rejection region for this test?
(c) (5) Which supplier do you purchase your next batch of answers from?

Problem 5. (20 points). Before long, you catch the attention of a larger regional math dealer who smuggles answers alongside shipments of cat food under the guise of the successful chain Los Gatos Locos. You begin working as part of that operation and are responsible for loading the delivery trucks (which contain only cat food and exams). You know that the mean and standard deviation of the weight of the cat food in each truck are 2000 and 100 pounds, respectively; and that the mean and standard deviation of the total load in each truck are 3000 and 20 pounds. Furthermore, the correlation coefficient between the weight of the cat food and the total load in each truck is +0.5 .
(a) (5) Are the weight of the cat food and the total load in each truck independent? Why or why not?
(b) (5) What is the expected weight of the exams in each truckload?
(c) (5) What is the standard deviation of the weight of the exams in each truckload?
(d) (5) What is the correlation coefficient between the weights of the exams and weight of the cat food shipments?

Problem 6. (15 points). Unfortunately, your operation tread more heavily than it should have, and Frank Schraderbrau eventually arrests you all. Even worse for you, all your previous crimes have caught up with you and you are facing a host of charges related to selling state secrets, poisoning customers with vile-testing potato salad, and peddling math. You look at similar cases in the past, and observe the following data concerning the number of charges and the number of years served in prison:

| Number of charges | Years in prison |
| :---: | :---: |
| 7 | 23 |
| 15 | 40 |
| 4 | 21 |
| 13 | 34 |
| 20 | 25 |

(a) (5) What is the best-fit linear regression line relating the number of years served to the number of charges?
(b) (5) What is the $r^{2}$ value?
(c) (5) If you are facing a total of 10 charges, what is the probability that you spend more than 20 years in prison?

