CE 311S: Final Exam, Spring 2014

Solutions

Problem 1.

(a) Exponential with $\lambda = 12$/hr, so the mean and standard deviation are $1/\lambda = 5$ minutes.

(b) Poisson with mean 12 and standard deviation $\sqrt{12}$.

(c) Let C be the quantity in the problem. Then $C + 1$ is negative binomial with $m = 1$ and $p = 0.01$, so $E[C] = m/p - 1 = 99$, $V[C] = m(1-p)/p^2 = 9900$ and the standard deviation is 99.4.

(d) The number each customer purchases is binomial with $n = 10$ and $p = 0.5$, and thus has a mean of $np = 5$ and a variance of $np(1-p) = 2.5$. By the Central Limit Theorem, the *average* of the next 100 customers is normally distributed with mean 5 and variance 0.025, so a standard deviation of 0.16.

Problem 2. Let UNB, GRE, and ECJ be the events "Schraderbrau patrols the Union," "Schraderbrau patrols Gregory," and "Schraderbrau patrols ECJ." Further let C be the event "you are caught."

(a) $P(UNB \cap C) = P(UNB)P(C \mid UNB) = 0.2 \times 0.6 = 0.12$

(b) Repeating part (a) gives a 9% probability of being caught if you sell math at Gregory, and a 5% probability of being caught if you sell math at ECJ. ECJ is the best choice.

(c) Define new events as follows: $\overline{UNB}$, $\overline{GRE}$, and $\overline{ECJ}$ are the events "you sell math at the Union," "you sell math at Gregory", and "you sell math at ECJ," respectively. Then

$$P(\overline{ECJ} \mid C) = \frac{P(C \mid \overline{ECJ})P(\overline{ECJ})}{P(C \mid \overline{UNB})P(\overline{UNB}) + P(C \mid \overline{GRE})P(\overline{GRE}) + P(C \mid \overline{ECJ})P(\overline{ECJ})}$$
$$= \frac{0.05 \times \frac{1}{3}}{0.12 \times \frac{1}{3} + 0.09 \times \frac{1}{3} + 0.05 \times \frac{1}{3}} = \frac{5}{26}$$

Problem 3. With this data set $\bar{x} = 5600$ and $s = 2000$.

(a) The interval endpoints are $\bar{x} \pm st_{0.025,5}/\sqrt{n}$ or $(3500, 7700)$.

(b) This is a one-sided prediction interval with lower endpoint $\bar{x} - st_{0.1,5}\sqrt{1 + 1/n}$ or 2410

Problem 4.

(a) $H_0 : p = 0.85$, $H_a : p > 0.85$

(b) For this alternative hypothesis, you reject the null hypothesis if $z > z_{0.05} = 1.64$; and for a test on proportions $z = (p' - p_0)/\sqrt{p_0(1-p_0)/n}$, so substituting $p_0 = 0.85$, $n = 100$, we find $z > 1.64$ if $p' > 0.909$

(c) We cannot reject the null hypothesis, so we purchase the next batch from Sue de Ephedrine.

Problem 5. Let C be the weight of cat food in a truck, X the weight of exams, and T the total weight (so $T = C + X$)

(a) No, because the correlation coefficient is nonzero.

(b) $E[X] = E[T - C] = E[T] - E[C] = 3000 - 2000 = 1000$ pounds.

(c) $V[X] = V[T - C] = V[T] + V[C] - 2\rho_{T,C}\sigma_T\sigma_C = 20^2 + 100^2 - 2(0.5)(100)(20) = 8400$, so $\sigma_X = 91.7$ pounds.

(d) $V[T] = V[C + X] = V[C] + V[X] + 2\rho_{C,X}\sigma_C\sigma_X$. We know everything in this equation except for $\rho_{C,X}$, so we can solve for it: $\rho_{C,X} = -0.98$.

Problem 6. Let x denote the number of charges and y the number of years in prison.

(a) The best-fit line is $y = 21.4 + 0.612x$

(b) $R^2 = 0.233$

(c) $SSE = 200$, so $\sigma = 8.17$ and $P(Y > 20) = P(Z > -0.918) = 0.82$.