CE 311S: Final Exam, Spring 2018
Solutions

Problem 1. For this problem we have $\bar{x} = 97.8$ and $s = 14.7$.

(a) This is a prediction interval with endpoints $\bar{x} \pm st_{0.975,5}\sqrt{1 + 1/n}$, or $(57.0, 139)$.

(b) This is a confidence interval on the mean with endpoints $\bar{x} \pm st_{0.975,5}/\sqrt{n}$, or $(82.4, 113)$.

(c) This is a confidence interval on the standard deviation with endpoints $\left(\sqrt{(n-1)s^2/\chi^2_{0.025,5}}, \sqrt{(n-1)s^2/\chi^2_{0.975,5}}\right) =$ $(9.17, 36.0)$

(d) This is a tolerance interval with endpoints $\bar{x} \pm 5.775s$, or $(13.0, 183)$

Problem 2.

(a) Poisson with mean and variance 3

(b) Exponential with mean 1/3 and variance 1/9

(c) Normal with mean 3 and variance 3/100

(d) Negative binomial with $m = 3$ and $p = 1 - \exp(-3)(1 + 3/1 + 3^2/2! + 3^3/3! + 3^4/4! + 3^5/5!) = 0.0839$, so mean and variance are 35.7 and 390, respectively.

Problem 3. For parts (a) and (b) of this problem, $p(x)$ is obtained by dividing the survey results by 100.

(a) $E[X] = \sum xp(x) = 1.2$

(b) $E[X^2] = \sum x^2 p(x) = 3.2$, so the standard deviation is $\sqrt{E[X^2] - [EX]^2} = 1.33$.

(c) In this part of the problem, the pmf changes to condition on $X > 0$; the resulting pmf is 1/2, 1/6, 1/6, and 1/6 for x values of 1, 2, 3, and 4, respectively. Repeating with this pmf gives $E[X \mid X > 0] = 2$ and $\sigma[X \mid X > 0] = 1.15$.

Problem 4. From the previous problem we have $p = 0.3$.

(a) $H_0 : p = 0.25$, $H_a : p > 0.25$

(b) Expanding the business when not many students are frequent users; not expanding the business when there are many frequent users.

(c) $z = (p' - p_0)/\sqrt{p_0(1 - p_0)/n} = 1.15$. For this alternative hypothesis we reject H_0 iff $z > z_\alpha = 1.645$, so you do not expand the business.

(d) From the previous equation $z > 1.645$ iff $p > 0.321$.

Problem 5. In this problem we have $\sum x = 472$, $\sum y = 284$, $\sum x^2 = 45096$, $\sum y^2 = 17930$, and $\sum xy = 26191$. Thus $S_{xx} = 539$, $S_{yy} = 1799$, $S_{xy} = -619$, $\bar{x} = 94.4$, and $\bar{y} = 56.8$. Finally $SSE = 1089$ and $SST = 1799$, so $\sigma = 19.0$.

(a) $\beta_1 = S_{xy}/S_{xx} = -1.15$ and $\beta_0 = \bar{y} - \beta_1\bar{x} = 165.1$, so $y = 165.1 - 1.15x$ is the best-fit line.

(b) $R^2 = 1 - SSE/SST = 0.394$.

(c) $P(Y = 165.1 - 1.15x + \epsilon > 60) = P(\epsilon > -15.061) = P(Z > -1.05) = 0.705$.

Problem 6. Let R be revenue, S scooter costs, and F the cost of fines. Profit is given by $\Pi = R - S - F$.

(a) $E[\Pi] = E[R] - E[S] - E[F] = -200$

(b) $\sigma[\Pi] = \sqrt{V[R] + V[S] + V[F]} = 768$.

(c) $E[\Pi]$ is the same as before. But now

$$\sigma[\Pi] = \sqrt{V[R] + V[S] + V[F] - 2\rho_{R,S}\sigma_R\sigma_S - 2\rho_{R,F}\sigma_R\sigma_F + 2\rho_{S,F}\sigma_S\sigma_F} = 812$$