## CE 311S: Final Exam

Friday, May 14 2:00 – 5:00 PM

## **Instructions:**

- SHOW ALL WORK unless instructed otherwise. No shown work means no partial credit!
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- $\bullet$  If you need to make any additional assumptions, state them clearly.
- You may use a calculator and two regular-sized sheets of notes. No additional resources are permitted.
- ullet The number of points associated with each part of each problem is indicated.

| Problem | Points | Possible |
|---------|--------|----------|
| 1       |        | 20       |
| 2       |        | 14       |
| 3       |        | 16       |
| 4       |        | 16       |
| 5       |        | 20       |
| 6       |        | 14       |
| TOTAL   |        | 100      |

Please copy the following statement (based on UT's honor code and the ASCE code of ethics) in your own handwriting, and sign it. For the purposes of this statement, academic dishonesty includes (but is not limited to) sharing with or receiving information from others about the exam, by any mode of communication.

"As a student of The University of Texas at Austin and as a civil engineer, I certify that I have not and will not participate in any acts of academic dishonesty related to this exam. If I witness any acts of academic dishonesty, I will report them to the instructor."

Your handwritten copy of the statement:

**Problem 1.** (20 points) You had an engineering internship offer for the summer, but your shady second cousin (also a UT CAEE student) convinces you to join their startup. The pitch is to provide online, app-based tutoring services to future CE 311S students, under the company name Knowledgeable Statisticians.

You are getting ready to make a pitch on *Shark Tank*, a reality TV show where entrepreneurs present their idea to a panel of investors, who decide whether or not to provide funding to get the company started. To prepare for your pitch, you gather data based on a trial version of your app among your friends. You record how many hours they used the app, and what their engineering GPA is, obtaining the following data:

| Friend   | Hours used | Engineering GPA |
|----------|------------|-----------------|
| Aisha    | 5          | 3.1             |
| Britney  | 8          | 2.8             |
| Chul-soo | 4          | 3.0             |
| Darnell  | 7          | 3.4             |
| Eduardo  | 9          | 3.3             |

- (a) (5) What is the best-fit linear regression line relating GPA to the hours spent on the app?
- (b) (5) What is the  $R^2$  value?
- (c) (5) Based on this data, what is the probability that someone using the app for 10 hours has a GPA greater than 3.0?
- (d) (5) Using  $\alpha = 0.05$ , perform a hypothesis test to determine whether your app has an effect on a student's GPA. Explain your conclusion in real-world terms.

**Problem 2.** (14 points) Next, you want to know whether students who use your app at all have a higher GPA, regardless of how many hours they use it. You know that the population mean engineering GPA among CAEE students is 3.0, and want to know whether the mean GPA of those using your app will be better than this.

- (a) (2) State your null and alternative hypotheses mathematically.
- (b) (4) Using the same data as the previous problem, perform a hypothesis test and determine whether you can reject the null hypothesis (with  $\alpha = 0.05$ ). What is your conclusion? Explain in real-world terms.
- (c) (4) What is the minimum sample mean GPA that would lead you to reject your null hypothesis?
- (d) (4) If the population mean GPA for students using your app is really 3.1, what is the probability of making a Type II error?

**Problem 3**. (16 points) Finally, you gather information to determine how much revenue your startup might get. You ask the same five friends what they would pay (per month) to use the Knowledgeable Statisticians app, and obtain the following five numbers:

\$0 \$30 \$20 \$40 \$35

Assuming that these five numbers correspond to a normal distribution (which CAN be negative — indicating you would have to pay *them* to use the app), calculate each of the following intervals with 95% confidence:

- (a) (4) A two-sided interval covering the willingness-to-pay of 95% of CAEE students.
- (b) (4) A two-sided interval covering the willingness-to-pay of your very first customer.
- (c) (4) A two-sided interval covering the average willingness-to-pay among all students.
- (d) (4) A two-sided interval covering the standard deviation of willingness-to-pay among all students.

**Problem 4.** (16 points) With all this data in hand, you make your pitch on *Shark Tank*. You have six main points you want to talk about in your pitch, but unfortunately you are very nervous and there is a 40% probability you will fail to describe each of the five points. You try to calm your nerves by cracking jokes about CE 311S as you go, at an average rate of two per minute. Unfortunately, the investors have not taken CE 311S and do not get your jokes — so after your third joke they interrupt and tell you to just get to the point. After this point, you will not tell any more jokes.

- (a) (4) What is the probability you properly describe at least three of your five main points?
- (b) (4) If your pitch lasts two minutes in total, what are the mean and standard deviation of the number of jokes you tell?
- (c) (4) What is the probability that your first joke comes at least 30 seconds into your pitch?
- (d) (4) What are the mean and standard deviation of the time until the investors tell you to stop joking? (For this part, you can assume that your pitch might go longer than two minutes, and that you talk long enough for them to tell you to stop.)

**Problem 5.** (20 points) Unfortunately you fail to secure funding from *Shark Tank*. Your cousin instead convinces you to invest money in two shady cryptocurrencies they are involved with, bUTcoin and CAEEthereum, as an alternative means to raise capital. The return for each cryptocurrency is lognormally distributed, with  $\mu = -1$  and  $\sigma = 1$ , and the amount of money you will end up with is your initial investment multiplied by the return. (Example: if you invest \$500 in bUTcoin and its return is 1.5, you will end up with \$750; if its return is 0.5 you will end up with \$250). The returns are correlated, with  $\rho = +0.7$ . You have a total of \$1000 to invest.

- (a) (5) What are the mean and standard deviation of the returns for each currency?
- (b) (5) If you invest \$1000 in bUTcoin, what is the mean and standard deviation of the amount of money you end up with?
- (c) (5) If you invest \$500 in bUTcoin and \$500 in CAEEthereum, what is the mean and standard deviation of the total amount of money you end up with?
- (d) (5) Based on your answers above, is it better to put all \$1000 in bUTcoin, or to split the money equally between the two?

**Problem 6**. (14 points) After your crypto adventure turns out poorly, you leave Knowledgeable Statisticians and take an engineering internship after all. When you get back to school in the fall, you and your friends talk about what you did during the summer. 45% of your friends had internships, 25% took a summer class, and 55% took at least one road trip. 60% of your friends had either an internship or a summer class; 80% of your friends had either an internship or took a road trip; 70% either took summer classes or a road trip; and 5% did all three. (Note: the "or" statements in the last sentence are inclusive and count people who did both.)

- (a) (6) How many did both a summer class and an internship?
- (b) (2) Are the events "your friend had an internship" and "your friend took a summer class" independent?
- (c) (6) How many of your friends did *none* of the three activities?