CE 311S: Homework 2
Due Monday, January 14

Instructions: Attempt all questions and show all the steps. Submit the homework on Canvas as a single pdf file. The file can contain scans or photographs of handwritten work *if it is legible*.

Problem 1. Consider three events with $P(A) = 0.5$, $P(B) = 0.5$, and $P(C) = 0.3$, with A and C disjoint; B and C independent; and A, B, and C exhaustive.

(a) What is the probability that none of A, B, and C occur?

(b) What is the probability that at least one of A, B, and C occur?

(c) What is the probability that exactly one of A, B, and C occur?

(d) What is the probability that exactly two of A, B, and C occur?

(e) What is the probability that A, B, and C all occur?

Problem 2. You are considering three different processes for manufacturing a batch of concrete, and conduct a series of experiments where you apply these techniques, and then test whether the batch meets the standard. Some processes were used more than others, because you wanted to do more experiments with the kinds of process you are least familiar with.

After all of your experiments, 80% of the batches of concrete ultimately met the standard, and 20% did not. Among the batches which met the minimum standard, you used Process 1 25% of the time, Process 2 50% of the time, and Process 3 25% of the time. Among the batches which did not, you used Process 1 40% of the time, Process 2 50% of the time, and Process 3 10% of the time.

(a) What percentage of *all* your experiments involved Process 1 and a concrete batch which did not meet the standard?

(b) What percentage of your experiments used Test 1?

(c) Which process will maximize the likelihood of a concrete batch which passes the standard, and what is that probability?

Problem 3. You are writing an app and want to use a "visual password" system where a user logging in is shown a total of 6 images. A password consists of 3 of these images in a particular order, and someone logging in must select the same images as the user did when creating the account, in the same order.

(a) How many passwords can you create?

(b) If ten people choose passwords independently (with all passwords equally likely), what is the probability that they all choose different passwords?

(c) Instead of ten people, now assume there are n users of your app. What is the largest value of n for which it is more likely than not for all users to have different passwords?

(d) What would increase the number of passwords more: adding a 7th image to choose from, or increasing the password length to 4?

(e) Would your answer to part (d) be different if repetition was allowed? If order didn't matter? If repetition was allowed and order didn't matter?

Problem 4. While 52-card decks are most common in the United States today, other decks are used elsewhere in the world or in history. For the purposes of this question, assume you are playing with a 32-card *Piquet pack*, with cards numbered 7, 8, 9, 10, J, Q, K, and A in each of four different suits (*trèfles*, *carreaux*, *cœurs*, and *piques*). Calculate the probability of each of the following hands, assuming that you draw five cards from a well-shuffled deck.

(a) What is the probability of a royal flush (10, J, Q, K, and A all from the same suit)?

(b) What is the probability of a flush (i.e., all five cards are of the same suit)?

(c) What is the probability of a "full house" (three cards of one number, and two cards of another)?