CE 3500: Homework 1
Solutions

If the answer to any of the following question is "it depends," say what it depends on, and explain how your answer might differ for the different possibilities.

Problem 1*. You have collected data (shown on the next page) from travel surveys about the number of recreational trips a household takes in a month:*

1. *Perform a linear regression on the number of recreational trips made.*
2. *On average, how many trips would you expect a household of four members to make in a given month, assuming two of them are children under 12, one of them is a worker, and their annual income is $56,000?*
3. *What is the effect on the number of recreational trips if the household size is increased by one?*
4. *Your supervisors are curious about the role household size and composition plays in recreational trip-making. However, they haven't taken any courses in statistics or engineering (that's why they hired you as a specialist!). Answer their question* without using any technical or mathematical language.

1. The regression equation on the number of monthly recreational productions P_r is
$$P_r = 0.52 + 0.047n_T + 1.00n_C - 0.47n_W + (1.04 \times 10^{-5})I$$
where n_T, n_C, n_W, and I are the total number of household members, number of children under 12, number of workers, and annual income, respectively.
2. Substituting $n_T = 4$, $n_C = 2$, $n_W = 1$, and $I = 56000$ into the regression equation gives 1.56 recreational trips per month, on average.
3. **It depends.** If the new household member is a child under 12, the household will make an average of 1.00 + 0.05 = 1.05 more recreational trips per month. If the new household member is a worker, the household will make an average of 0.42 fewer recreational trips per month (0.47 - 0.047). If the new household member neither works, nor is a child under 12, the average monthly recreational trips would increase by 0.05.
4. The primary factors influencing recreational tripmaking are the number of young children (under 12) and the number of employed household members. The more young children, the more recreational trips each month; the more workers, the fewer trips, although the effect is not as strong as with chidlren. The total size of the household has only a small effect, what really matters is how many of them work, and how many are children.

Problem 2*. You are given the following (unbalanced) production and attractions data for the three zones in your city, as well as these friction factors:*

Zone	A_w *(Raw)*	A_s *(Raw)*	P_w	P_s
1	*48,000*	*33,000*	*63,000*	*33,000*
2	*53,000*	*50,000*	*92,000*	*12,000*
3	*72,000*	*19,000*	*56,000*	*62,000*

Φ	*1*	*2*	*3*
1	*1*	*1/3*	*1/4*
2	*1/3*	*1*	*1/2*
3	*1/4*	*1/2*	*1*

Create the (balanced) OD matrix for the morning peak period.

The total number of work productions and attractions must be equal. We have more confidence in predicting trip productions, so we multiply the number of attractions by 211/173 (so the total number of attractions is 211,000, the same as the total number of productions). This gives $\mathbf{A_w} = \begin{bmatrix} 59,000 & 65,000 & 88,000 \end{bmatrix}$. Now we create an OD matrix for the morning peak period, by following the procedure on page 11 of the planning notes.

Step 1: During the morning peak, trips are starting at home and ending at work. So $\mathbf{S} = \begin{bmatrix} 48,000 & 53,000 & 72,000 \end{bmatrix}$ and $\mathbf{E} = \begin{bmatrix} 59,000 & 65,000 & 88,000 \end{bmatrix}$.

Step 2: The initial adjustment factors are $\boldsymbol{\mu} = \mathbf{1}$.

Step 3: We use the gravity model equation to calculate the initial entries in the table, giving:

$$
\begin{aligned}
d_{11} &= \frac{\mu_1 S_1 E_1 \phi_{11}}{\mu_1 E_1 \phi_{11} + \mu_2 E_2 \phi_{12} + \mu_3 E_3 \phi_{13}} = \frac{1 \times 63,000 \times 59,000 \times 1}{1 \times 59,000 \times 1 + 1 \times 65,000 \times 1/3 + 1 \times 88,000 \times 1/4} = 36,000 \\
d_{12} &= \frac{\mu_2 S_1 E_2 \phi_{12}}{\mu_1 E_1 \phi_{11} + \mu_2 E_2 \phi_{12} + \mu_3 E_3 \phi_{13}} = \frac{1 \times 63,000 \times 65,000 \times 1/3}{1 \times 59,000 \times 1 + 1 \times 65,000 \times 1/3 + 1 \times 88,000 \times 1/4} = 13,000 \\
d_{13} &= \frac{\mu_3 S_1 E_3 \phi_{13}}{\mu_1 E_1 \phi_{11} + \mu_2 E_2 \phi_{12} + \mu_3 E_3 \phi_{13}} = \frac{1 \times 63,000 \times 88,000 \times 1/4}{1 \times 59,000 \times 1 + 1 \times 65,000 \times 1/3 + 1 \times 88,000 \times 1/4} = 14,400 \\
d_{21} &= \frac{\mu_1 S_2 E_1 \phi_{21}}{\mu_1 E_1 \phi_{21} + \mu_2 E_2 \phi_{22} + \mu_3 E_3 \phi_{23}} = \frac{1 \times 92,000 \times 59,000 \times 1/3}{1 \times 59,000 \times 1/3 + 1 \times 65,000 \times 1 + 1 \times 88,000 \times 1/2} = 14,000 \\
d_{22} &= \frac{\mu_2 S_2 E_2 \phi_{22}}{\mu_1 E_1 \phi_{21} + \mu_2 E_2 \phi_{22} + \mu_3 E_3 \phi_{23}} = \frac{1 \times 92,000 \times 65,000 \times 1}{1 \times 59,000 \times 1/3 + 1 \times 65,000 \times 1 + 1 \times 88,000 \times 1/2} = 46,000 \\
d_{23} &= \frac{\mu_3 S_2 E_3 \phi_{23}}{\mu_1 E_1 \phi_{21} + \mu_2 E_2 \phi_{22} + \mu_3 E_3 \phi_{23}} = \frac{1 \times 92,000 \times 88,000 \times 1/2}{1 \times 59,000 \times 1/3 + 1 \times 65,000 \times 1 + 1 \times 88,000 \times 1/2} = 32,000 \\
d_{31} &= \frac{\mu_1 S_3 E_1 \phi_{31}}{\mu_1 E_1 \phi_{31} + \mu_2 E_2 \phi_{32} + \mu_3 E_3 \phi_{33}} = \frac{1 \times 56,000 \times 59,000 \times 1/4}{1 \times 59,000 \times 1/4 + 1 \times 65,000 \times 1/2 + 1 \times 88,000 \times 1} = 6100 \\
d_{32} &= \frac{\mu_2 S_3 E_2 \phi_{32}}{\mu_1 E_1 \phi_{31} + \mu_2 E_2 \phi_{32} + \mu_3 E_3 \phi_{33}} = \frac{1 \times 56,000 \times 65,000 \times 1/2}{1 \times 59,000 \times 1/4 + 1 \times 65,000 \times 1/2 + 1 \times 88,000 \times 1} = 13,000 \\
d_{33} &= \frac{\mu_3 S_3 E_3 \phi_{33}}{\mu_1 E_1 \phi_{31} + \mu_2 E_2 \phi_{32} + \mu_3 E_3 \phi_{33}} = \frac{1 \times 56,000 \times 88,000 \times 1}{1 \times 59,000 \times 1/4 + 1 \times 65,000 \times 1/2 + 1 \times 88,000 \times 1} = 36,000
\end{aligned}
$$

giving the table

d	1	2	3
1	36,000	13,000	14,000
2	14,000	46,000	32,000
3	6100	13,000	36,000

Step 4: The table is not balanced (e.g., 73,000 trips are destined for zone 2, as compared to the 65,000 that should be heading there), so we create adjustment factors: $\mu_1 = 59,000/(36,000 + 14,000 + 6100) = 1.04$, $\mu_2 = 65,000/(13,000 + 46,000 + 13,000) = 0.88$, $\mu_3 = 88,000/(14,000 + 32,000 + 36,000) = 1.08$. Then we repeat Step 3 to generate a new table.

Step 3: Repeating the gravity model calculations with the new μ values:

$$d_{11} = \frac{\mu_1 S_1 E_1 \phi_{11}}{\mu_1 E_1 \phi_{11} + \mu_2 E_2 \phi_{12} + \mu_3 E_3 \phi_{13}} = \frac{1.04 \times 63,000 \times 59,000 \times 1}{1.04 \times 59,000 \times 1 + 0.88 \times 65,000 \times 1/3 + 1.08 \times 88,000 \times 1/4} = 37,000$$

$$d_{12} = \frac{\mu_2 S_1 E_2 \phi_{12}}{\mu_1 E_1 \phi_{11} + \mu_2 E_2 \phi_{12} + \mu_3 E_3 \phi_{13}} = \frac{0.88 \times 63,000 \times 65,000 \times 1/3}{1.04 \times 59,000 \times 1 + 0.88 \times 65,000 \times 1/3 + 1.08 \times 88,000 \times 1/4} = 12,000$$

$$d_{13} = \frac{\mu_3 S_1 E_3 \phi_{13}}{\mu_1 E_1 \phi_{11} + \mu_2 E_2 \phi_{12} + \mu_3 E_3 \phi_{13}} = \frac{1.08 \times 63,000 \times 88,000 \times 1/4}{1.04 \times 59,000 \times 1 + 0.88 \times 65,000 \times 1/3 + 1.08 \times 88,000 \times 1/4} = 14,400$$

$$d_{21} = \frac{\mu_1 S_2 E_1 \phi_{21}}{\mu_1 E_1 \phi_{21} + \mu_2 E_2 \phi_{22} + \mu_3 E_3 \phi_{23}} = \frac{1.04 \times 92,000 \times 59,000 \times 1/3}{1.04 \times 59,000 \times 1/3 + 0.88 \times 65,000 \times 1 + 1.08 \times 88,000 \times 1/2} = 15,000$$

$$d_{22} = \frac{\mu_2 S_2 E_2 \phi_{22}}{\mu_1 E_1 \phi_{21} + \mu_2 E_2 \phi_{22} + \mu_3 E_3 \phi_{23}} = \frac{0.88 \times 92,000 \times 65,000 \times 1}{1.04 \times 59,000 \times 1/3 + 0.88 \times 65,000 \times 1 + 1.08 \times 88,000 \times 1/2} = 42,000$$

$$d_{23} = \frac{\mu_3 S_2 E_3 \phi_{23}}{\mu_1 E_1 \phi_{21} + \mu_2 E_2 \phi_{22} + \mu_3 E_3 \phi_{23}} = \frac{1.08 \times 92,000 \times 88,000 \times 1/2}{1.04 \times 59,000 \times 1/3 + 0.88 \times 65,000 \times 1 + 1.08 \times 88,000 \times 1/2} = 35,000$$

$$d_{31} = \frac{\mu_1 S_3 E_1 \phi_{31}}{\mu_1 E_1 \phi_{31} + \mu_2 E_2 \phi_{32} + \mu_3 E_3 \phi_{33}} = \frac{1.04 \times 56,000 \times 59,000 \times 1/4}{1.04 \times 59,000 \times 1/4 + 0.88 \times 65,000 \times 1/2 + 1.08 \times 88,000 \times 1} = 6200$$

$$d_{32} = \frac{\mu_2 S_3 E_2 \phi_{32}}{\mu_1 E_1 \phi_{31} + \mu_2 E_2 \phi_{32} + \mu_3 E_3 \phi_{33}} = \frac{0.88 \times 56,000 \times 65,000 \times 1/2}{1.04 \times 59,000 \times 1/4 + 0.88 \times 65,000 \times 1/2 + 1.08 \times 88,000 \times 1} = 12,000$$

$$d_{33} = \frac{\mu_3 S_3 E_3 \phi_{33}}{\mu_1 E_1 \phi_{31} + \mu_2 E_2 \phi_{32} + \mu_3 E_3 \phi_{33}} = \frac{1.08 \times 56,000 \times 88,000 \times 1}{1.04 \times 59,000 \times 1/4 + 0.8 \times 65,000 \times 1/2 + 1.08 \times 88,000 \times 1} = 38,000$$

giving the table

d	1	2	3
1	37,000	12,000	14,000
2	15,000	42,000	35,000
3	6200	12,000	38,000

Step 4: At this point, the predicted and assigned trips agree to within 1%, so we stop.

Alternate solution method using a spreadsheet.

	A	B	C	D	E	F	G	H
1	36143.43	13302.79	13553.78	63000		1.00	0.33	0.25
2	14019.05	46438.1	31542.86	92000		0.33	1.00	0.50
3	6081.448	13429.86	36488.69	56000		0.25	0.50	1.00
4	58543.35	64641.62	87815.03					
5								
6								
7								
8	1	1	1					
9								
10	58543.35	21547.21	21953.76					
11	19514.45	64641.62	43907.51					
12	14635.84	32320.81	87815.03					

Figure 1: Example spreadsheet for trip distribution.

Note that the gravity model formula can be rewritten as

$$d_{ij} = S_i \frac{\mu_j E_j \phi_{ij}}{\sum_j \mu_j E_j \phi_{ij}}$$

which suggests that the terms $\mu_j E_j \phi_{ij}$ (the *trip distribution factors*) can be tabulated separately to write the above formulas compactly. So if $T_{ij} = \mu_j E_j \phi_{ij}$ is the trip distribution factor, then $d_{ij} = S_i T_{ij} / \sum_j T_{ij}$.

If your OD matrix is contained in the range A1:C3, the zone productions in the range D1:D3, the zone attractions in the range A4:C4, the friction factors in F1:H3, the adjustment factors $\boldsymbol{\mu}$ in A8:C8, and the trip distribution factors in A10:C12 (see Figure 1), enter the formula `=A$8*A$4*F1` in A10 (to give the trip distribution factor $\mu_1 E_1 \phi_{11}$ between zone 1 and itself), and fill it to the right and down. (Note the use of the dollar signs before the row indicator for the adjustment factors and zone attractions, since these are all found in rows 8 and 4, respectively, while the friction factors are found in multiple rows.)

So, using the compact form of the gravity model equation, enter the formula `=$D1*A10/SUM($A10:$C10)` in cell A1 to give the first value of the OD matrix and fill to the right and down. (Again, the dollar signs: all of the zone productions are found in column D, and the sum of all of the trip distribution factors is always between columns A and C, while the individual factors are found in different rows and columns).

Now we update the adjustment factors in A8:C8. Set the three factors equal to `A4/SUM(A1:A3)`, `B4/SUM(B1:B3)`, and `C4/SUM(C1:C3)`. (If you enter the formula directly in these cells, you will get a circular reference warning. Either use a calculator, or make the new adjustment factors in a different cell and "paste values" into A8:C8). This will automatically update the trip distribution factors and OD matrix, which is now balanced to within 1%.

Problem 3*. Think about your own household, and how you travel to Denver. Assume that CDOT wants to improve US-287 so it is a divided, multilane facility (like the upgrade WYDOT recently made near Tie Siding). Since funding is scarce, they choose to levy a $10 toll for each vehicle entering Colorado on US-287 (this way, the users of the road pay, and they don't have to raise taxes across the state). Which of the following travel decisions would change for your household? (1) the total number of trips made to Denver;*

	A	B	C	D	E	F	G	H
1	37055.28	11575.4	14369.32	63000		1.00	0.33	0.25
2	14988.3	42138.68	34873.02	92000		0.33	1.00	0.50
3	6168.229	11561.06	38270.71	56000		0.25	0.50	1.00
4	58543.35	64641.62	87815.03					
5								
6								
7								
8	1.040883	0.883435	1.076358					
9								
10	60936.79	19035.56	23630.1					
11	20312.26	57106.68	47260.21					
12	15234.2	28553.34	94520.42					

Figure 2: Final spreadsheet for trip distribution.

(2) the mode of transportation you use; (3) the route you take to get to Denver. For each of these, say why or why not.

Multiple answers are possible. For me, I probably wouldn't change the total number of trips I make or the mode (the $10 difference in cost is probably not enough to get me to fly, for instance). My route would almost certainly change, driving through Cheyenne if it were possible.

Problem 4. *Name three simplifying assumptions that are used in the mode choice and route choice steps. Pick one of them, and explain how you could change the procedure to relax this assumption and make the process more realistic.*

Again, multiple answers are possible. Potential answers are: utility theory is a valid way to predict human behavior, link performance functions exist and only depend on the flow on that roadway segment; people will choose a faster route, no matter how small the difference (e.g., even if it is only a 1 second difference, people care enough to switch). Picking the last one, we might create a new route choice model where all used routes are, say, within one minute of being the fastest travel time (rather than strictly equal and minimal).