CE 3500: Homework 2

Due Friday, February 25

Problem 1. *Three neighborhoods (Ash Grove, Beach Spring, and Charlestown) are connected by a one-way road as shown below, where the travel time on each roadway segment is five minutes if driving, and six minutes by bus. The AM Peak OD matrix is given as follows:*

	A	B	C
A	0	100	200
B	500	0	1000
C	600	600	0

You are given utility functions $U_{car} = 2 - T_{car}/10$ *and* $U_{bus} = -1 - T_{bus}/10$.

1. *What is the current total bus ridership?*
2. *How fast would the bus have to travel between Beach Spring and Charlestown to achieve a 5% share of commuters between these zones?*

1. Utilities for bus and car are calculated from the utility functions as follows:

U_{car}	A	B	C
A	2	1.5	1
B	1	2	1.5
C	1.5	1	2

U_{bus}	A	B	C
A	-1	-1.6	-2.2
B	-2.2	-1	-1.6
C	-1.6	-2.2	-1

Thus, the proportion of people choosing the bus for each OD pair is given by $\exp(U_{bus})/(\exp(U_{bus}) + \exp(U_{car}))$, and the total number of people riding the bus is given by multiplying this proportion by the value in the OD matrix:

P_{bus}	A	B	C
A	0.047	0.043	0.039
B	0.039	0.047	0.043
C	0.043	0.039	0.047

Busridership	A	B	C
A	0	4	8
B	20	0	43
C	26	23	0

Thus, the total bus ridership is 124.

2. For bus ridership to equal 5% of total demand, we need $\frac{\exp(U_{bus})}{\exp(U_{bus})+\exp(U_{car})} = 0.05$. Assuming car travel time is fixed, U_{car} is fixed at 1.5, so we can substitute 4.48 for $\exp(U_{car})$ in this equation. Thus we need $\exp(U_{bus}) = 0.114 \iff U_{bus} = -1.44 \iff T_{bus} = 4.4$ minutes.

Problem 2. *There are two routes connecting an origin and destination; the first has link performance function* $t_1 = 45 + \frac{3x}{1000}$ *and the second* $t_2 = 20 + \frac{4x^2}{2500}$. *If 2500 people are traveling between these zones, use the Frank-Wolfe method to find the equilibrium route flows and travel times.*

Following the steps in the notes, the first time through $i = 0$, the path travel times are 45 and 20, so path 2 is faster, so $\mathbf{x^0} = \mathbf{x}^* = \begin{bmatrix} 0 & 2500 \end{bmatrix}$. The second time through, $i = 1$, the travel times are $t_1 = 45 + \frac{0}{1000} = 45$ and $t_2 = 20 + \frac{4 \times 2500^2}{7500} = 3353$. The first path is shorter, so $\mathbf{x}^* = \begin{bmatrix} 2500 & 0 \end{bmatrix}$. We need to solve the equation

$$0 \times t_1(x_1^1) + 2500 \times t_2(x_2^1) = 2500 \times t_1(x_1^1) + 0 \times t_2(x_2^1)$$

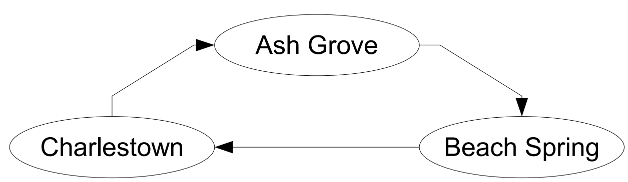
Simplifying and substituting $\mathbf{x^1} = \lambda \mathbf{x}^* + (1 - \lambda)\mathbf{x^0}$, we have

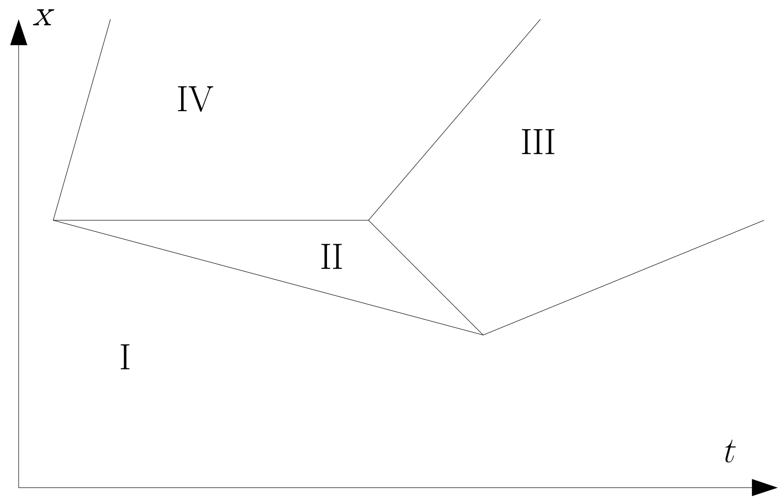
$$20 + \frac{4 \times (2500(1-\lambda))^2}{7500} = 45 + \frac{7500\lambda}{1000}$$

or $\lambda = 0.975$ so $\mathbf{x^1} = 0.975 \begin{bmatrix} 2500 & 0 \end{bmatrix} + 0.025 \begin{bmatrix} 0 & 2500 \end{bmatrix} = \begin{bmatrix} 2436 & 52 \end{bmatrix}$. Both paths have equal travel time (52.31 minutes), so we have found the equilibrium and we stop.

Problem 3*. Consider a roadway with capacity 2000 veh/hr and free-flow speed 40 mi/hr. Initially, the flow is 1000 veh/hr and uncongested. A traffic signal is red for 45 seconds, causing several shockwaves.*

1. *Sketch a time-space diagram, indicating all of the shockwaves which are formed.*
2. *Calculate the speed and direction of each shockwave from your diagram.*
3. *What is the minimum green time needed to ensure that no vehicle has to stop more than once before passing the intersection? (Neglect any yellow time, reaction time, etc. Assume that when the signal is green, people move immediately, and that when it is red, people stop immediately.)*





1.

2. The jam density is given by $k_f = 4q_{max}/u_f = 4 \times 2000/40 = 200$ veh/mi. In Region I, we have $q_1 = 1000$. Since flow is congested, $u_1 = u_f(1 + \sqrt{1 - q/q_{max}})/2 = 40(1 + \sqrt{1/2})/2 = 34.1$ mi/hr and $k_1 = k_j(1 - u/u_f) = 200(1 - 34.1/40) = 29.5$ veh/mi. In region II, $u_2 = 0$ so $k_2 = k_j = 200$ veh/mi and $q_2 = 0$ veh/hr. In Region III, $q_3 = 2000$ veh/mi so $k_3 = k_j/2 = 100$ veh/mi; and finally, in region IV, $q_4 = 0$ and $k_4 = 0$. Therefore, the speed of the shockwaves is as follows:

   I-II : $u_{I,II} = \frac{q_I - q_{II}}{k_I - k_{II}} = (1000 - 0)/(29.5 - 200) = -5.87$ mi/hr (negative, so moving upstream).

   I-III : $u_{I,III} = (1000 - 2000)/(29.5 - 100) = 14.2$ mi/hr downstream.

   I-IV : $u_{I,IV} = (1000 - 0)/(29.5 - 0) = 34.1$ mi/hr downstream.

   II-III : $u_{II,III} = (0 - 2000)/(200 - 100) = 20$ mi/hr upstream.

   II-IV : $u_{II,IV} = (0 - 0)/(200 - 0) = 0$ mi/hr (stationary).

   III-IV : $u_{III,IV} = (2000 - 0)/(100 - 0) = 20$ mi/hr downstream.

3. The last vehicle stopping at the traffic signal stops arrives at the rightmost point of region II on the space-time diagram, that is, where the shockwaves I-II and II-III intersect. t seconds after the light turns red, shockwave I-II is (5.87mi/hr)(ts)(1/3600hr/s)(5280ft/mi) $= 8.61t$ feet upstream of the signal. If $t > 45$, shockwave II-III is (20mi/hr)(($t - 45$)s)(1/3600hr/s)(5280ft/mi) $= 29.33(t - 45)$ feet upstream. These are equal if $t = 63.7$ seconds; that is, the last vehicle which has to stop at the signal does so 63.7 seconds after it turns red. Furthermore, this vehicle stops $8.61 \times 63.7 = 548$ feet upstream of the signal, then proceeds forward at 20 mi/hr (in Region III). Therefore, it reaches the signal in an additional (548ft)/(20mi/hr) $\times$ (1mi/hr/1.47ft/s) $= 18.6$ seconds, or $18.6 + 63.7 = 82.3$ seconds after the signal turns red. Therefore $82.3 - 45 = 37.3$ seconds of green time is needed.

Problem 4. *Extra credit: In class we used gap acceptance to calculate the "capacity" of a stop sign, assuming that no more than one car would turn in each gap. Extend this procedure to account for the possibility of multiple vehicles turning during one gap, if it is long enough.*

One possible solution: let q be the flow on the main road, and t_n be the minimum gap required for n vehicles to turn. Therefore, at capacity, n vehicles will turn if the gap is between t_n and t_{n+1}. The probability of the gap being this long is $\Pr(G > t_n) - \Pr(G > t_{n+1}) = e^{-qt_n} - e^{-qt_{n+1}}$. Therefore, the total number of vehicles which can turn is $q \sum_{n=1}^{\infty} n(e^{-qt_n} - e^{-qt_{n+1}})$.