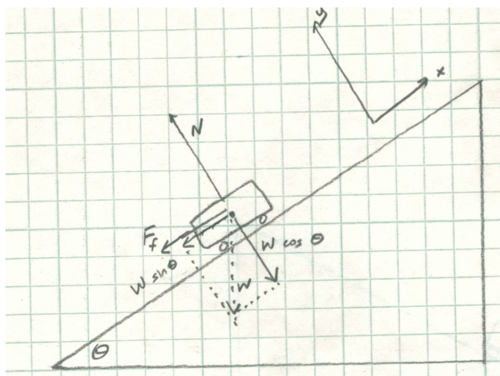
CE 3500: Homework 4
Due Monday, April 5

Problem 3. *Derive the braking-distance formula*

$$D_b = \frac{u_0^2}{2g(f+G)}$$

using a free-body diagram, where u_0 *is the initial velocity,* g *is acceleration due to gravity,* f *is the coefficient of braking friction, and* G *is the percent grade (uphill is positive). Assume that the vehicle's brakes are powerful enough that the limiting factor is friction between the tires and road. Remember that* D_b *is measured* horizontally*, not along the angle of the roadway (that is,* D_b *is the distance between the initial and stopping points as measured from an aerial photo), and that the grade* $G = \tan\theta$*, where* θ *is the angle between the horizontal and the roadway surface in the direction of travel.*

Begin by drawing a free body diagram:



Note that the coordinate system in the free-body diagram has been chosen so that the axes are parallel and perpendicular to the road surface; and that the vehicle is traveling uphill. The forces acting on the vehicle are its weight $W = mg$, with an x-component of $-mg\sin\theta$ and a y-component of $-mg\cos\theta$; the normal force N; and the frictional force F_f. The vehicle is not accelerating in the y-direction, so $N = mg\cos\theta$. The friction force is equal to the coefficient of friction times the normal force, so $F_f = -fmg\cos\theta$. Thus, the net force is $-mg(f\cos\theta + \sin\theta)$ in the x direction, so by $F = ma$ the net acceleration is $a = -g(f\cos\theta + \sin\theta)$. Under constant acceleration, the speed at any point in time is $u_0 + at$ and the distance traveled is $u_0 t + at^2/2$ by integration. Therefore, the vehicle comes to a stop when $u_0 + at = 0$, that is, at time $t = -u_0/a$; and during this time the vehicle travels a distance of $-u_0^2/a + u_0^2/2a = -u_0^2/2a$. Substituting $a = -g(f\cos\theta + \sin\theta)$ gives a distance of $u_0^2/2g(f\cos\theta + \sin\theta)$. However, this is the distance along the roadway surface. The braking distance D_b is the horizontal projection of this, or

$$D_b = \frac{u_0^2\cos\theta}{2g(f\cos\theta + \sin\theta)} = \frac{u_0^2}{2g(f+G)}$$

after dividing through by $\cos\theta$ and using $G = \tan\theta = \sin\theta/\cos\theta$.