Introduction to Course

CE 311S

January 20, 2015

COURSE OVERVIEW

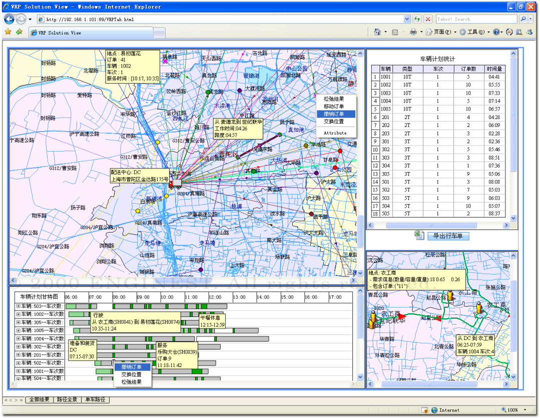
Introductions

1. What is your name?
2. Where are you from?
3. Something interesting about you...

Office Hours

Instructor: Steve Boyles, ECJ 6.204, T/Th 10:30–11:30 (priority), 9:30-10:30 (if nobody from other class is there)

Optimization is the study of how to *quantitatively* model the choices that should be made to best achieve some goal.

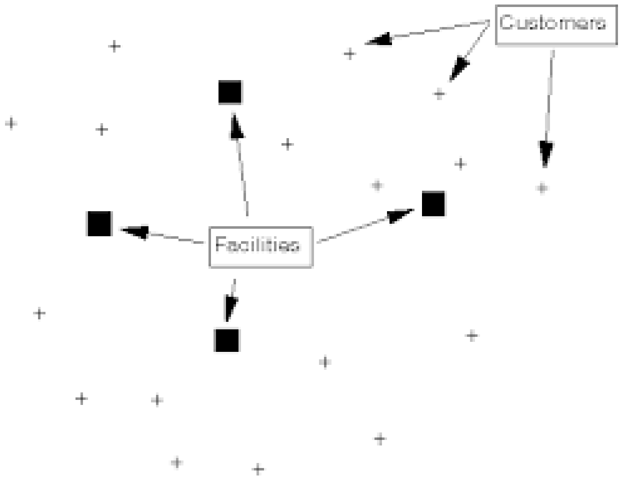


Engineering examples abound. Some examples:

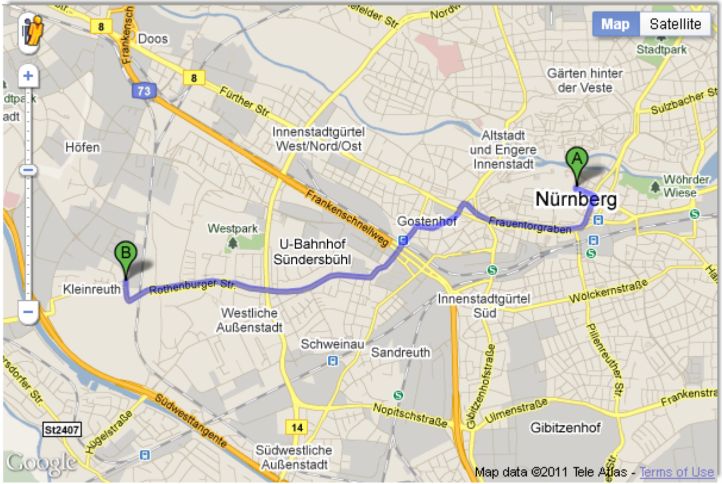
How many buses should be assigned to each route?

Where and when should roadway maintenance be scheduled?

Where should facilities be locatedd?



What is the fastest route between two points?



This course will teach you how to:

- Formulate optimization problems using mathematical language
- Solve different types of optimization problems
- Understand what kind of techniques to apply to which problems

Prerequisites

The only formal prerequisite is CE 321, since you have presumably taken the basic sequence courses already.

The mathematics of optimization involve a good deal of calculus, and solving optimization problems involves a good deal of computer programming.

If it has been a while since you have taken these courses or used the material, please review them.

This course is "language agnostic" and I will not be teaching you how a specific programming language works. You will be expected to implement some of the procedures in class in a language of your choice.

Course Materials

There is no textbook. I will also post lecture slides, but these cannot replace your own notes.

These notes (along with other course materials) will be posted on the course website (*not* Blackboard):
`http://tinyurl.com/ce377-sp15`

Grading

Category	Weight
Homeworks	30%
Midterm (March 12)	20%
Course project	25%
Final exam (May 18)	25%

$+/-$ grading will be used. If you need an extension, you must ask at least 48 hours in adavnce.

Miscellanea

- Consult catalog and departmental advisors for add/drop policy.
- Please coordinate with me and Services for Students with Disabilities if you have a disability requiring alternate accomodations.
- Academic dishonesty... don't do it.

COMPONENTS OF AN OPTIMIZATION PROBLEM

Every optimization problem has three parts:

Objective function: A single quantity to be either maximized or minimized (the goal)

Decision variables: Anything you can control or influence in order to achieve the goal

Constraints: Any restrictions on the decision variables you must obey

Examples of objective functions:

Minimize congestion; maximize safety; maximize accessibility; minimize cost; maximize pavement quality; minimize emissions; maximize revenue; etc.

In this class, we assume that there is only one objective. Multiobjective optimization is "fuzzier" and more complicated.

The generic notation for an objective function is f.

A decision variable can be something you either **directly control**, or something you can **indirectly influence** by choosing other decision variables.

Example: You are a toll road operator trying to maximize your revenue (toll times number of users). You can directly choose the toll, but not the number of users. Nevertheless, the number of users is influenced by the toll you choose. Therefore both the toll and the number of users are decision variables.

The generic notation for the decision variables is the vector $\mathbf{x}$.

A constraint represents any restriction on the decision variables. There are usually multiple constraints.

Constraints on the decision variables you directly control may represent resource or budget limits, level of service requirements, legal restrictions, etc.

You also need constraints for decision variables you indirectly influence, making sure they are consistent with what you can directly control.

For example, there may be a constraint giving number of users in terms of the toll.

Do not forget "obvious" constraints, like $x \geq 0$ if the decision variable x cannot be negative.

If the decision variables $\mathbf{x}$ satisfy *all* of the constraints, we say $\mathbf{x}$ is a **feasible solution** or simply **feasible**.

The generic notation for the set of all feasible solutions is X, called the **feasible set** or **feasible region**.

You have 60 feet of fence available, and wish to enclose the largest rectangular area possible. What dimensions should you choose for the fenced-off area?

The objective is to maximize the area of a rectangle.

The decision variables are the length and width of the rectangle.

The constraints are that you only have 60 feet of fence, *and* that the length and width are nonnegative.

Introducing mathematical notation, let L be the length and W the width.

$$
\begin{aligned}
\max_{L,W} \quad & LW \\
\text{s.t.} \quad & 2L + 2W && \leq 60 \\
& L && \geq 0 \\
& W && \geq 0
\end{aligned}
$$

Transit frequency setting

You must decide the frequency of service on each of the bus routes. The bus routes are known and cannot change, but you can change how the city's bus fleet is allocated to each of these routes (within given maximum and minimum levels). Knowing the ridership on each route, how should buses be allocated to routes to minimize the total waiting time?

The objective is to minimize waiting time, the decision variables are the number of buses assigned to each route, the constraints are the upper and lower limits on the number of buses assigned to a route.

Let n_r be the number of buses assigned to route r, L_r and U_r the upper and lower limits for route r. The constraints are $n_r \geq L_r$ and $n_r \leq U_r$.

How can we calculate the total waiting time?

Assume that passengers arrive uniformly regardless of the schedule, and that buses depart at uniform headways.

If n buses are assigned to a route which is T minutes long, a bus will pass by a stop every T/n minutes.

If passengers arrive uniformly, on average they will have to wait half this time, or $T/(2n)$.

So, if there are d_r passengers on route r, the total waiting time on this route is $(d_r T_r)/(2n_r)$.

$$
\begin{aligned}
\min_{\mathbf{n}} \quad & D(\mathbf{n}) = \sum_{r \in R} \frac{d_r T_r}{2 n_r} & \\
\text{s.t.} \quad & n_r \geq L_r & \forall r \in R \\
& n_r \leq U_r & \forall r \in R
\end{aligned}
$$

Maintenance scheduling

You must schedule routine maintenance on a set of pavement sections over the next 10 years. Each section can be described by a condition index from 0 to 100. Each section deteriorates at a known, constant rate, but if you perform maintenance in a given year, its condition will improve by a constant ammount. Given a budget for each year, when and where should you perform maintenance to maximize the average condition?

Let the decision variable $x_f^t = 1$ if maintenance is performed on facility f during year t, and 0 otherwise.

Let k_f be the cost of maintenance on facility f, and B^t the budget in year t.

There are 10 budget constraints, of the form $\sum_{f \in F} k_f x_f^t \leq B^t$

Let the condition of facility f at the end of year t be c_f^t, let d_f be the annual deterioration rate, and i_f the improvement if maintenance is performed. *(Which of these are decision variables?)*

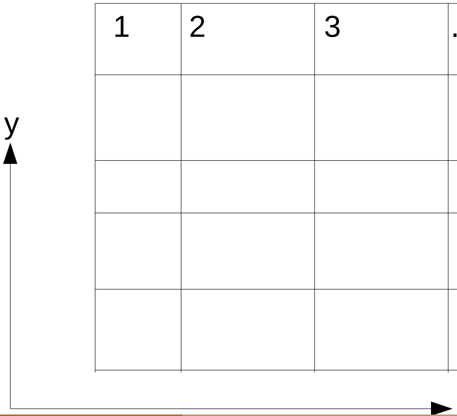
Then $c_f^t = c_f^{t-1} - d_f + x_f^t i_f$, plus the requirement that the condition be between 0 and 100.

What is the objective function?

$$
\begin{aligned}
\min_{\mathbf{x},\mathbf{c}} \quad & \frac{1}{10|F|}\sum_{f\in F}\sum_{t=1}^{10} c_f^t \\
\text{s.t.} \quad & \sum_{f\in F} k_f x_f^t \leq B^t && \forall t \in \{1,\dots 10\} \\
& c_f^t = \begin{cases} 100 & \text{if } c_f^{t-1} - d_f + x_f^t i_f > 100 \\ 0 & \text{if } c_f^{t-1} - d_f + x_f^t i_f < 0 \\ c_f^{t-1} - d_f + x_f^t i_f & \text{otherwise} \end{cases} && \forall f \in F, t \in \{1,2,\dots,10\} \\
& x_f^t \in \{0,1\} && \forall f \in F, t \in \{1,2,\dots,10\}
\end{aligned}
$$

Facility location

You must locate three bus terminals in a city with a grid network. You know the locations of customers throughout the city, and the cost of building a terminal at each intersection. Passengers walk from their home locations to the nearest terminal. Where should the terminals be located to minimize total walking distance and construction cost?



Let $x(i)$ and $y(i)$ be the coordinates of intersection i (out of I in total). Let H_p be the intersection corresponding to the home location of passenger p, and L_1, L_2, and L_3 the intersections where the three terminals are located. Let $C(i)$ be the cost of building a terminal at i.

The walking distance between intersections i and j is

$$d(i,j) = |x(i) - x(j)| + |y(i) - y(j)|$$

So, the walking distance for customer p is

$$D(p, L_1, L_2, L_3) = \min\{d(H_p, L_1), d(H_p, L_2), d(H_p, L_3)\}$$

How do we write the objective? There are two parts, total walking distance $\sum_{p \in P} D(p, L_1, L_2, L_3)$ and construction cost $C(L_1) + C(L_2) + C(L_3)$.

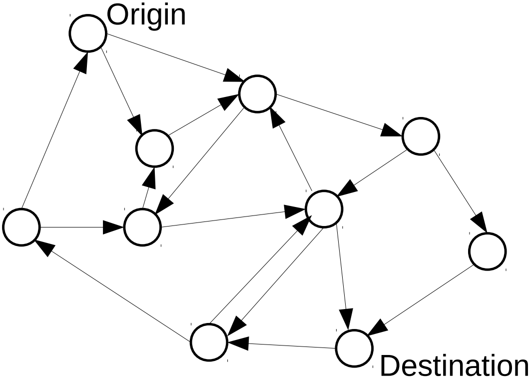
One "trick" is to add them together, using the weighting parameter $\Theta \in [0, 1]$ to show how important each objective is:

$$f(L_1, L_2, L_3) = \Theta[C(L_1) + C(L_2) + C(L_3)] + (1 - \Theta) \left[\sum_{p \in P} D(p, L_1, L_2, L_3) \right]$$

$$
\begin{aligned}
\min_{L_1,L_2,L_3} \quad & \Theta[C(L_1)+C(L_2)+C(L_3)] + (1-\Theta)\left[\sum_{p\in P} D(p, L_1, L_2, L_3)\right] \\
\text{s.t.} \quad & L_f \in \{1, 2, \ldots, I\} \qquad \forall f \in \{1, 2, 3\}
\end{aligned}
$$

Shortest path

Each roadway link in the network has a known travel time. What is the fastest route connecting the origin to the destination?



Number the roadway links from 1 to A and the intersections from 1 to I. Let $x_a = 1$ if link a is part of the route and 0 otherwise.

The travel time on the route is then $\sum_{a=1}^{A} t_a x_a$.

What are the constraints?

For the path to be valid, we need *flow conservation constraints* at each intersection.

Let $F(i)$ and $R(i)$ be the links leaving and entering intersection i.

For any intersection i, what can we say about how the x values for links in $F(i)$ and $R(i)$ are related?

$$
\begin{aligned}
\min_{\mathbf{x}} \quad & \sum_{a \in A} t_a x_a \\
\text{s.t.} \quad & \sum_{a \in F(i)} x_a - \sum_{a \in R(i)} x_a = \begin{cases} 1 & \text{if i = r} \\ -1 & \text{if i = s} \\ 0 & \text{otherwise} \end{cases} && \forall i \in \{1, \ldots, I\} \\
& x_a \in \{0, 1\} && \forall a \in \{1, \ldots, A\}
\end{aligned}
$$

MORE DEFINITIONS AND USEFUL FACTS

A feasible solution $\mathbf{x}^* \in X$ is a *global minimum* of f if $f(\mathbf{x}^*) \leq f(\mathbf{x})$ for all feasible $\mathbf{x}$, and a *global maximum* if $f(\mathbf{x}^*) \geq f(\mathbf{x})$ for all feasible $\mathbf{x}$

An *optimal* solution is a global minimum (for a minimization problem) or a global maximum (for a maximization problem).

It is easy to convert back and forth between maximization and minimization problems. If the feasible set is X, the feasible solution $\mathbf{x}^*$ is a global maximum of f iff it is a global minimum of $-f$.

As a result, we do not need to develop different techniques for maximization or minimization problems. **The default convention in this class is to work with minimization problems.**

Other useful facts:

- Constants can be added or subtracted to an objective function without changing the optimal solutions.
- You can multiply an objective function by a nonnegative constant without changing the optimal solutions.

INFORMAL ASSIGNMENT

Identify an optimization problem from your life. Define notation, the objective function, decision variables, and constraints. We'll discuss next class.