CE 377K: Homework 3
Due Thursday, April 16

Problem 1. Use the augmenting path algorithm to find the maximum possible flow between nodes 1 and 8 in the network in Figure 1. Capacities are provided in Table 1 (ignore costs and supply/demand). Now, let's say you have the money to increase the capacity on one arc. Name an arc where increased capacity would result in an increase in the maximum flow sent between 1 and 8.

Problem 2. Again using the network in Figure 1, find the minimum cost flow which satisfies supply, demand, and arc capacities. All problem data are found in Table 1.

Problem 3. Consider the feasible region $X \subset \mathbb{R}^4$ defined by the constraints

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 3x_1 + 2x_2 - x_4 &= 3 \\ 2x_1 + x_3 + x_4 &\le 5 \\ 2x_2 + x_3 + x_4 &\le 7 \\ x_1 &\ge 0 \\ x_2 &\ge 0 \end{aligned}$$

1. One extreme point of X is $(0, 0, 1, -3)$. List all of the extreme points which are adjacent to this one.
2. List all of the remaining extreme points of X.

Problem 4. This is a modified version of the toll-setting problem from the last exam. The morning peak period is **four** hours long and the toll can be switched every hour. The tolls can be **any nonnegative value** (they do not need to be capped at \$1), and between hours the toll cannot change by more than \$0.25. The number of people using the HOT lane in each of four hours is the "total demand" for that hour, minus the toll times the "toll sensitivity" for that hour, as shown in Table 2. Maximize the total number of travelers using the HOT lane during these four hours, without exceeding the lane capacity (2000 vehicles per hour) in any of the hours.

(a) Write a simple optimization problem representing this scenario.

(b) Place your simple problem into the standard form for linear programs.

(c) Write code to solve this problem using the simplex method.

(d) The capacity of the roadway may change from one hour to the next because of accidents, weather, etc. In each of the four hours, how much can the capacity change without the optimal basis changing? (Assume that the capacity only changes in one of the hours.)

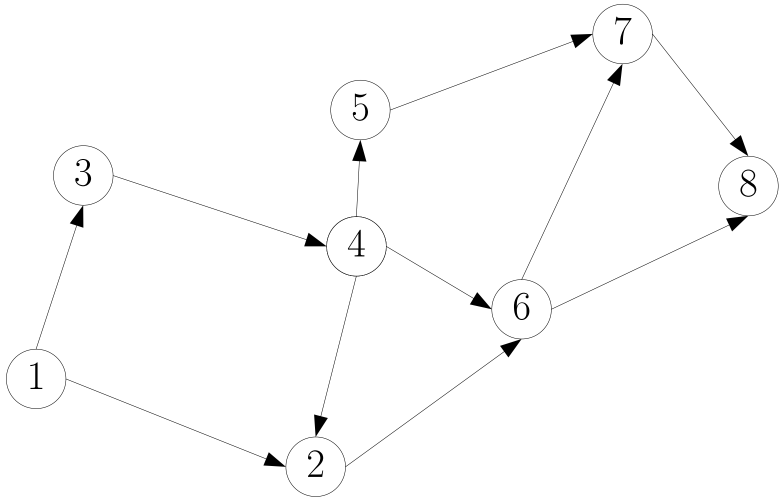

Figure 1: Graph for problems 1 and 2.

Table 1: Data for problems 1 and 2.

Arc	Cost	Capacity
(1,2)	3	2
(1,3)	8	3
(2,6)	9	5
(3,4)	4	5
(4,2)	2	6
(4,5)	10	5
(4,6)	5	1
(5,7)	7	8
(6,7)	1	3
(6,8)	6	2
(7,8)	1	2

Node	Supply/demand
1	+4
2	−4
3	+1
4	0
5	0
6	+3
7	−2
8	−2

Table 2: Data for Problem 4.

Hour	Total demand	Toll sensitivity
6–7 AM	2500	500
7–8 AM	3100	1000
8–9 AM	4000	1500
9–10 AM	3400	1000