## CE 377K: Homework 4

Due Thursday, May 7

Problem 1. Determine whether the following functions are convex or not. Prove your answer.

1. $f(x)=\log x, x \geq 1$
2. $f(x)=-x^{3}, x \leq 0$
3. $f(x)=\sin x$
4. $f(x)=e^{x^{2}-2 x-3}, x \in \mathbb{R}$

Problem 2. Determine whether the following sets are convex or not. Prove your answer.

1. $X=\left\{\left(x_{1}, x_{2}\right) \in \mathbb{R}^{2}: 4 x_{1}-3 x_{2} \leq 0\right\}$
2. $X=\left\{x \in \mathbb{R}: e^{x} \leq 4\right\}$
3. $X=\left\{\left(x_{1}, x_{2}, x_{3}\right) \in \mathbb{R}^{3}: x_{1}=0\right\}$
4. $X=\left\{\left(x_{1}, x_{2}, x_{3}\right) \in \mathbb{R}^{3}: x_{1} \neq 0\right\}$

Problem 3. You've been called upon to install a traffic signal at 46th \& Guadalupe. Assuming a simple two-phase cycle (where Guadalupe moves in phase 1, and 46th Street in phase 2), no lost time when the signal changes, and ignoring turning movements, the total delay at the intersection can be written as

$$
\frac{\lambda_{1}\left(c-g_{1}\right)^{2}}{2 c\left(1-\frac{\lambda_{1}}{\mu_{1}}\right)}+\frac{\lambda_{2}\left(c-g_{2}\right)^{2}}{2 c\left(1-\frac{\lambda_{2}}{\mu_{2}}\right)}
$$

where $g_{1}$ and $g_{2}$ are the effective green time allotted to Guadalupe and 46th Street, respectively; $c=g_{1}+g_{2}$ is the cycle length, $\lambda_{1}$ and $\mu_{1}$ are the arrival rate and saturation flow for Guadalupe, and $\lambda_{2}$ and $\mu_{2}$ are the arrival rate and saturation flow for 46th Street.

The cycle length must be 60 seconds to maintain progression with adjacent signals, and the arrival rate and saturation flow are $2200 \mathrm{veh} / \mathrm{hr}$ and $3600 \mathrm{veh} / \mathrm{hr}$ for Guadalupe, and $300 \mathrm{veh} / \mathrm{hr}$ and $1900 \mathrm{veh} / \mathrm{hr}$ for 46th Street. Furthermore, no queues can remain at the end of the green interval; this means that $\mu_{i} g_{i}$ must be at least as large as $\lambda_{i} c$ for each approach $i$.

1. Formulate a nonlinear program to minimize total delay.
2. Starting from the initial solution $g_{1}=g_{2}=30$, perform two iterations of the descent method using the conditional gradient rule and the method of successive averages.
3. Solve this problem exactly using the method of Lagrange multipliers, and compare your answer from the previous part.
