CE 377K: Midterm

Thursday, March 12

12:30 – 1:45 PM

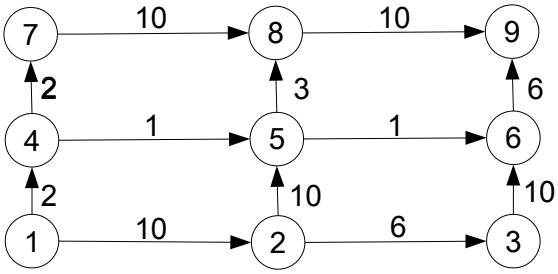
Name _____

Instructions:

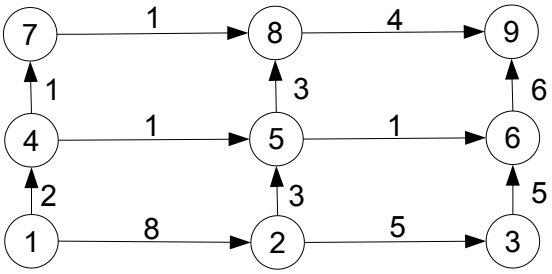
- **SHOW ALL WORK** unless instructed otherwise. No shown work means no partial credit!
- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.
- If you need to make any additional assumptions, state them clearly.
- You may use one regular-sized sheet of notes; please turn in the notes with your exam. No additional resources are permitted.
- The number of points associated with each part of each problem is indicated.

Problem	Points	Possible
1		25
2		25
3		20
4		30
TOTAL		100

Problem 1. (25 points). Consider a maximum flow problem on the network shown below, where the source node is 1, the sink node is 9, and the link capacities are shown in the **left** figure.



Link capacities



Current link flows

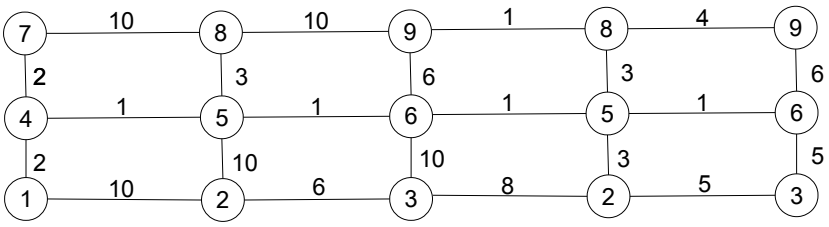
The **right** figure shows a feasible flow which ships 10 units from the source to the sink. Is this solution optimal? Justify your answer.

Problem 2. (25 points). Find the global *maxima* of the function

$$f(x, y, z) = x(5 - x) + y^2(8 - y^2)$$

explaining how you can be sure that your solution is globally optimal.

Problem 3. (25 points). Find the minimum spanning tree on the following network:



Problem 4. (30 points). Develop complete optimization formulations (clearly labeling decision variables, objective function, and constraints) for the following variants of the shortest path problem on a network with node and link sets N and A. Parts (a) and (b) are independent of each other: the additional concepts introduced in part (a) do not apply to part (b) and vice versa.

(a) (Constrained shortest path problem). Each link has a travel time t_{ij} and a cost c_{ij}. The travel time and cost of a path are the sums of the travel times and costs of the component links. You want to find the path between nodes r and s with the least total cost, but the path travel time cannot exceed some prespecified limit T.

(b) (Shortest path problem with relays). The path from origin r to destination s must pass through three pre-specified nodes a, b, and c in that order. Find such a path with minimum total cost.