# A first car following model 

## CE 391F

March 21, 2013

## ANNOUNCEMENTS

- Homework 3 to be posted this weekend - Course project abstracts "due" Tuesday


## REVIEW

Driver characteristics...

- Reaction time and control time
- Visual ability and perception
- Differences among drivers


## OUTLINE

(1) Typical braking and acceleration rates
(2) A first car-following model

## BRAKING AND <br> ACCELERATION

Remember the difference between open-loop and closed-loop control?


Closed-loop


Open-loop

Open-loop braking applies to "panic stops" where the driver simply brakes as fast as possible until stopped.


## ABS off

Dry, $64.4 \mathrm{~km} / \mathrm{h}, \mathrm{T} 1-48$


The classic AASHTO formula for braking distance is $d=V^{2} /(257.9 f)$ where $V$ is speed in kph and $f$ is the coefficient of braking friction, roughly equal to deceleration in $g$ units.

Typical acceleration values for "open-loop" braking is 0.7 g on dry pavement, and 0.4 g on wet pavement.

Closed-loop braking is more common, where drivers braking rate is more measured and feedback-dependent.

Comfortable braking acceleration is around $0.2-0.4 \mathrm{~g}$.
"Unhurried" acceleration ( 0.1 g ) is roughly $65 \%$ of maximum acceleration.

## TOWARDS CAR-FOLLOWING

## Robert Herman (1914-1997)



## What is a "safe" following distance?

Assume that your speed is $V$, your reaction time is $T$, and your maximum deceleration rate is $a_{f}$, and...
(1) ...the vehicle in front of you stops instantaneously (a very conservative assumption)?
(2) ...the vehicle in front of you decelerates to a stop at a rate of $a_{l}$ ?

Drivers must make an assumption about the difference between their deceleration rate and that of the vehicle in front of them; let $\gamma=\left(\frac{1}{2 a_{f}}-\frac{1}{2 a_{l}}\right)$.

Interestingly, this formula implies a capacity rate. How?

With a vehicle length $L$, the space headways are $L+T V+\gamma V^{2}$

Thus, the density is $\left(L+T V+\gamma V^{2}\right)^{-1}$

Thus, the flow is $V /\left(L+T V+\gamma V^{2}\right)$.

What is the maximum value of this function?

Any car-following model must make assumptions about how users will behave. As a starting point, let's assume that drivers want to simultaneously:
(1) Keep up with the vehicle in front of them
(2) Avoid collisions

The simplest way to accommodate both factors is to base our model on relative speed $V_{\text {rel }}=V_{f}-V_{l}$
(Many variations and elaborations have been proposed. Can you think of some ways to extend this model?)

As our starting point, we'll use the linear stimulus-response equation

$$
R=\lambda S
$$

where we need to define $S$ and $R$ as the stimulus and response for car following.

The stimulus $S$ will be based on the relative speeds from the past $V_{\text {rel }}$, based on a weighting function $\sigma(t)$ :

$$
S=\int_{0}^{\infty} \sigma(\tau) V_{r e l}(\tau) d \tau
$$

where $\tau$ is going backward in time from the present.

$\sigma(t)$ defines the "memory" : how far back do drivers think when driving? What is the most recent information that can be used? and so forth.
$\sigma(t)$ must be a valid probability density function: it should be nonnegative, and integrate to 1 over the positive reals.

One simple option is to set $\sigma(t)=\delta(T)$, where $T$ is the reaction time and $\delta$ is the Dirac delta function

This means that the response is based entirely on the relative velocity $T$ seconds ago.

What should the response be?

The accelerator and brake pedals control the acceleration of the vehicle, so it is reasonable to define $R=\ddot{x}$

Thus, our first car-following model is simply

$$
\ddot{x}(t)=\lambda\left(\dot{x}(t-T)-V_{l}(t-T)\right)
$$

where $V_{l}$ is the speed of the lead car, and $\lambda$ indicates how strongly someone will react to the relative velocity between them and the lead car.

## Example

You are traveling at a constant speed of $90 \mathrm{ft} / \mathrm{s}$. A vehicle merges 1000 ft downstream of you, traveling at a constant $60 \mathrm{ft} / \mathrm{s}$. If your reaction time is 2 seconds, plot your trajectory.

Notice what's different about this model from all of our continuum models:

- It's directly based on a driver behavior model
- It is fully anisotropic
- Vehicles are discrete entities
- It is trivial to represent differences in drivers or vehicles by changing acceleration rates, $\lambda$, reaction times, etc.

