Stability in car following models

CE 391F

March 21, 2013

ANNOUNCEMENTS

- Homework 3 online (due Thursday, April 4)
- Course project abstracts “due” today

REVIEW

Driver characteristics...

- Reaction time and control time
- Visual ability and perception
- Differences among drivers

The basic car following model

$$\ddot{x}_f(t) = \lambda(\dot{x}_\ell(t - T) - \dot{x}_f(t - T))$$

OUTLINE

1. Local stability
2. Asymptotic stability
3. Real-world experiments

LOCAL STABILITY

Remember the Excel sheet we created last class?

As λ varied, we saw different types of behavior.

When λ was too large, there was “overreaction” and oscillation.

When λ was too small, there was “underreaction” which led to a collision.

Studying the impact of λ and T leads to *stability* analysis.

Local stability describes the behavior of a single vehicle following another.

The analysis is simplified if we choose time units such that $T = 1$. Then the model becomes

$$\ddot{x}_f(t) = \lambda T(\dot{x}_\ell(t-1) - \dot{x}_f(t-1))$$

and, letting $C = \lambda T$, we can analyze stability in terms of one parameter only.

Assume initially that the lead and following vehicles are traveling the same speed.

There is an exact solution, but it is rather ugly. However, it yields the following results:

1. If $C \leq 1/e \approx 0.368$, motion is non-oscillatory and exponentially damped (local stability)
2. If $1/e < C < \pi/2 = 1.57$, motion is oscillatory, but exponentially damped
3. If $C = \pi/2$, motion is oscillatory with constant amplitude
4. If $C > \pi/2$, motion is oscillatory with increasing amplitude

Demonstration

Assuming the solution is locally stable, if the lead vehicle changes velocity from U to V, the following vehicle will do so as well. What will be the change in spacing?

The change in spacing will be $\Delta x_{\ell f} = \int_0^\infty \dot{x}_f(t)\ dt - \int_0^\infty \dot{x}_\ell(t)\ dt$

$$\Delta x_{\ell f} = \int_0^\infty (\dot{x}_f(t) - \dot{x}_\ell(t))\ dt$$

$$\Delta x_{\ell f} = \frac{1}{\lambda} \int_0^\infty \ddot{x}(t + T)\ dt$$

$$\Delta x_{\ell f} = \frac{V - U}{\lambda}$$

In particular, if the lead vehicle comes to a stop, $V = 0$, and the initial spacing must be at least U/λ to avoid a collision.

What does this imply about λ and vehicle spacing?

What does this imply about T and vehicle spacing?

ASYMPTOTIC STABILITY

Asymptotic stability refers to how disturbances are propagated through an entire stream of vehicles (as opposed to *local* stability, which only concerns the stability of an individual vehicle's response).

Assume we have a line of N vehicles. Except for the lead vehicle, each uses the car-following equation

$$\ddot{x}_i(t) = \lambda(\dot{x}_{i-1}(t - T) - \dot{x}_i(t - T))$$

Note that we're assuming the same λ and T values for all drivers. When actually simulating traffic (or solving these differential equations numerically) it is easy to have different values for different drivers... but doing so makes a sensitivity analysis much more difficult.

Demonstration

Fourier analysis shows that asymptotic stability requires $\lambda T = C < 1/2$

Thus, local stability ($C < 1/e$) implies asymptotic stability, but we can have asymptotically stable situations which are not locally stable

Demonstration

NEXT-NEAREST NEIGHBOR COUPLING

It is not difficult to think of variations on the basic car-following model. One such variation is the *next-nearest neighbor* model where drivers' response depends on the two vehicles in front, not just the vehicle immediately in front:

$$\ddot{x}_{n+2}(t) = \lambda_1(\dot{x}_{n+1}(t-T) - \dot{x}_{n+2}(t-T)) + \lambda_2(\dot{x}_n(t-T) - \dot{x}_n(t-T))$$

Here, asymptotic stability requires $T(\lambda_1 + \lambda_2) < 1/2$.

(So, smaller λ values are needed for stability if drivers look further ahead.)

CAR-FOLLOWING EXPERIMENTS

Many experiments have been conducted to try to calibrate and validate different car-following models.

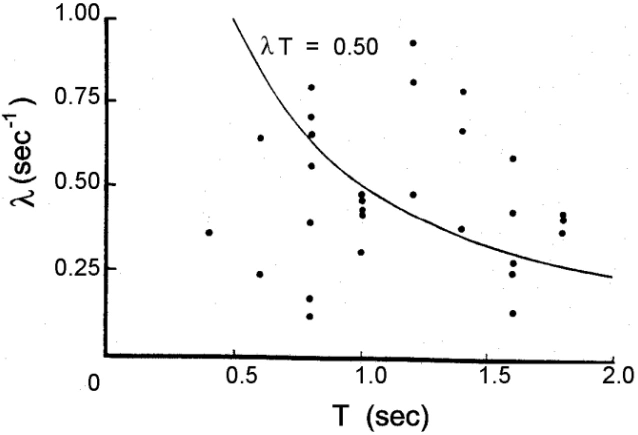
Chandler et al., 1958: eight male drivers on a one-mile test track facility

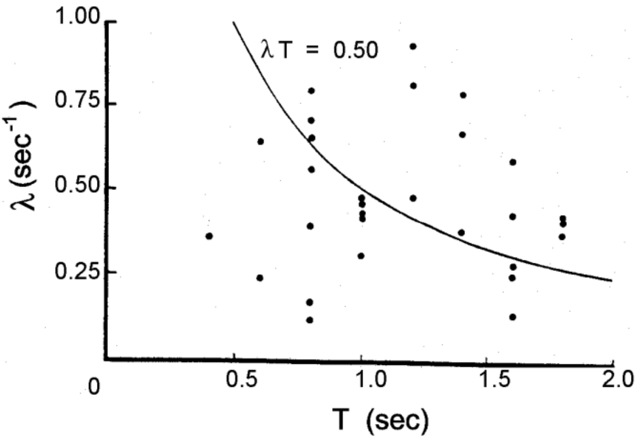
Driver	λ (Hz)	T (s)	C
1	0.74	1.41	1.04
2	0.44	1.00	0.44
3	0.34	4.47	1.52
4	0.32	1.50	0.48
5	0.38	1.71	0.65
6	0.17	1.12	0.19
7	0.32	2.25	0.72
8	0.23	2.04	0.47

Note an average C value close to $1/2$, the asymptotic stability limit.

Tunnel experiments

Lincoln, Holland, and Queens Midtown Tunnels, ten drivers in 30 test runs

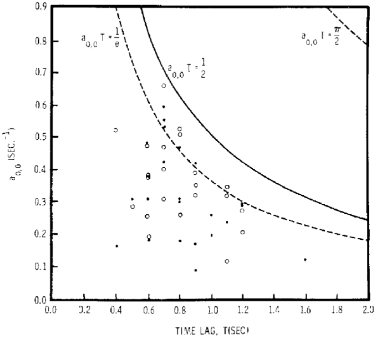




Some observations: note a number of drivers in the "unstable" region. These are drivers who have slower reaction times (and thus compensate by reacting more strongly to differences in speed). Statistically, such drivers are more likely to be involved in rear-end accidents.

Bus following experiments

How do drivers behave around other types of vehicles? 22 drivers were studied on a 4 km track.



Note that all of the drivers now fall into the asymptotically stable region.

Three-car experiments

Designed to test next-nearest neighbor coupling.

Adding next-nearest neighbor coupling does not significantly improve model fit.