Steady-state car following and continuum models

CE 391F

March 21, 2013

ANNOUNCEMENTS

- Homework 3 online (due Thursday, April 4)

REVIEW

The basic car following model

$$\ddot{x}_f(t) = \lambda(\dot{x}_\ell(t - T) - \dot{x}_f(t - T))$$

Local and asymptotic stability

How did λ and T affect these stability values?

What kind of λ values have been observed in experiments?

When we rescaled so the time delay was 1, where did the extra T factor come from?

$$\ddot{x}_f(t) = \lambda(\dot{x}_\ell(t - T) - \dot{x}_f(t - T))$$

$$\ddot{x}_f(t) = \lambda T(\dot{x}_\ell(t - 1) - \dot{x}_f(t - 1))$$

OUTLINE

1. Steady-state car-following scenarios
2. Connections between car-following and continuum models
3. More advanced car-following models

STEADY-STATE CAR FOLLOWING

Last class, we showed that a change in speed from u_1 to u_2 leads to a change in spacing of $(u_2 - u_1)/\lambda$

Assume that we have a large number of vehicles at uniform spacing, and the lead vehicle changes speed from u_1 to u_2.

If asymptotic stability holds, in the limit all vehicles will change their speed to u_2, and change the spacing by $(u_2 - u_1)/\lambda$

This suggests that a speed-spacing relationship is embedded in the car-following equations... at least under steady-state conditions.

Let k_1 and k_2 represent the density before and after the speed change.

Then $1/k_2 = 1/k_1 + (u_2 - u_1)/\lambda$

We need an initial value; say, $u_1 = 0$ and $k = k_j$, the jam density.

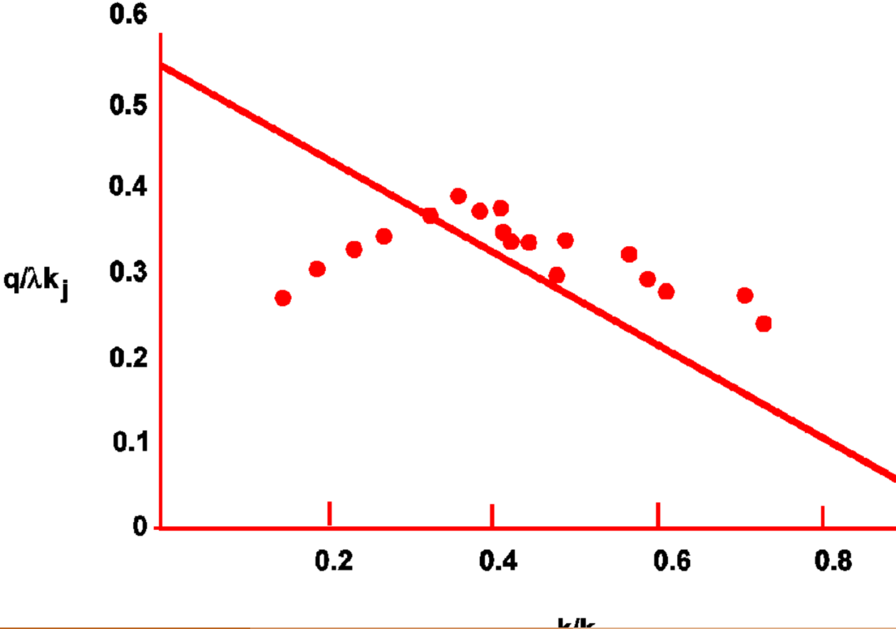
Then we have $1/k_2 = 1/k_j + u_2/\lambda$, or $u_2 = \lambda(1/k_2 - 1/k_j)$

Therefore, the implied fundamental diagram is $q = uk = \lambda(1 - k/k_j)$

What shape does this have?

What is the implied capacity value?

Is this a “reasonable” fundamental diagram?


0.6
0.5
0.4
0.3
0.2
0.1
0
q/λk_j
0.2
0.4
0.6
0.8
k/k

One key difference between the interpretation of this relationship, and the continuum flow model:

In the first-order continuum flow model, the fundamental diagram held almost everywhere. In car-following, the “fundamental diagram” refers only to *steady-state* flows which occur in the limit.

OTHER CAR-FOLLOWING MODELS

The basic car-following model $\ddot{x}_f(t+T) = \lambda(\dot{x}_\ell(t) - \dot{x}_f(t))$ has been criticized for being too simple:

- The response does not depend on the following distance
- The steady-state fundamental diagram is unrealistic

One strength of car-following models is that they can be made more sophisticated in a *behaviorally plausible* way.

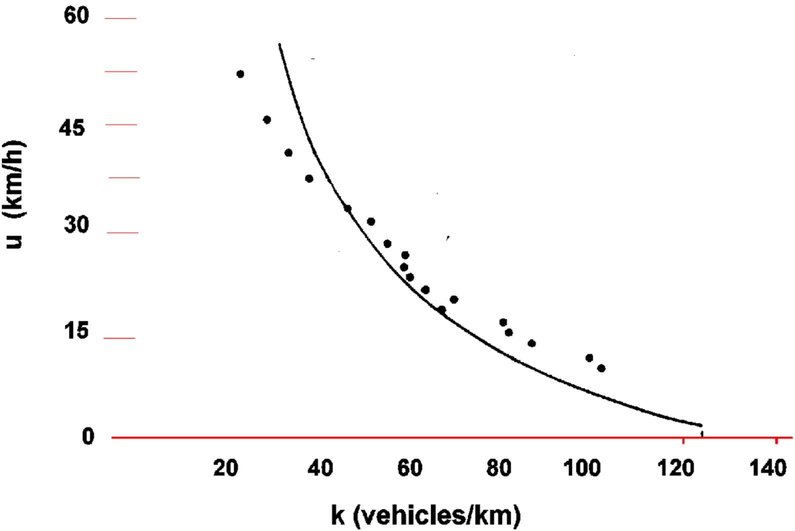
To incorporate following distance, we can divide the magnitude of the response by the following distance:

$$\ddot{x}_(t+T) = \frac{\lambda_1}{x_\ell(t) - x_f(t)}(\dot{x}_\ell(t) - \dot{x}_f(t))$$

(where λ_1 is a *different* scaling constant than λ.)

What is the steady-state flow model corresponding to this equation?

It turns out that this corresponds to the Greenberg model
$u = \lambda_1 \log(k_j/k)$ and $q = \lambda_1 k \log(k_j/k)$



One drawback of this model is that $u \to \infty$ as $k \to 0$ (equivalently, for low density values, $dq/dk \to \infty$).

One response: At low densities, car-following models are less appropriate anyway, since spacings are large and vehicle coupling is weak.

Another response: Nevertheless, can we try to patch this model too?

Edie's Model

Edie suggested adding still more terms to the car-following equation, dividing the response by the "time to collision"

$$\ddot{x}_(t+T) = \frac{\lambda_2}{x_\ell(t) - x_f(t)} \frac{\dot{x}_f(t+T)}{x_\ell(t) - x_f(t)} (\dot{x}_\ell(t) - \dot{x}_f(t))$$

or simply

$$\ddot{x}_(t+T) = \frac{\lambda_2 \dot{x}_f(t+T)}{[x_\ell(t) - x_f(t)]^2} (\dot{x}_\ell(t) - \dot{x}_f(t))$$

What is the steady-state flow model corresponding to this equation?

Can we go the other direction? Is there a car-following model which can replicate the Greenshields fundamental diagram $q = u_f(k - k^2/k_j)$?

Writing the spacing $S = 1/k$, we have $u = u_f(1 - 1/[k_j S])$.

Derivatizing with respect to t, we get $\dot{u} = (u_f/[k_j S^2])\dot{S}$

So, the car-following version of the Greenshields model is

$$\ddot{x}_(t + T) = \frac{u_f/k_j}{[x_\ell(t) - x_f(t)]^2}(\dot{x}_\ell(t) - \dot{x}_f(t))$$

All of the models discussed so far are special cases of the general family of equations

$$\ddot{x}_(t+T) = \frac{\lambda \dot{x}_f^m(t+T)}{[x_\ell(t) - x_f(t)]^k}(\dot{x}_\ell(t) - \dot{x}_f(t))$$

where m, k, and λ are nonnegative parameters.

In our basic car-following model we had $m = k = 0$.

Setting $m = 0$, $k = 1$ gives the Greenberg model.

Setting $m = 1$, $k = 2$ gives Edie's model.

Setting $m = 0$, $k = 2$ gives the Greenshields model.

Experimental data from the Eisenhower Expy in Chicago suggests $m = 0.8$, $k = 2.8$.

Another study on the same freeway suggests $m = 1$, $k = 3$.

Experimental data from the Gulf Fwy in Houston suggests $m = 0$, $k = 3/2$.

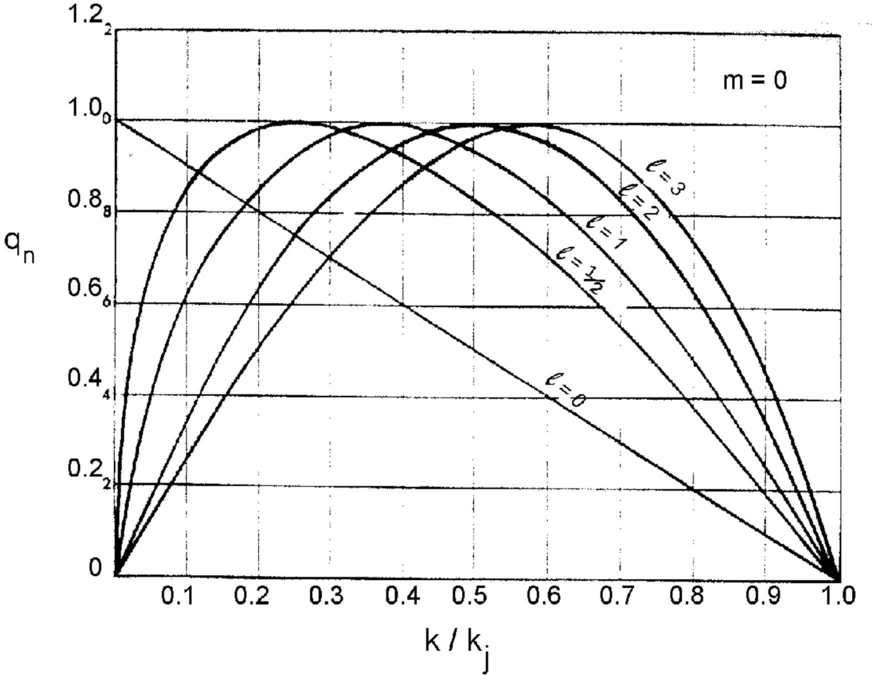
Can we determine a general form for the fundamental diagram corresponding to this equation?

Rearranging, we have

$$\frac{\dot{U}}{U^m} = \lambda \frac{\dot{S}}{S^k}$$

Integrating with respect to t, the left-hand side either becomes $\log U$ (if $m = 1$) or U^{1-m} (neglecting constants, which can be incorporated into λ)

The same holds with the right-hand side, and from here we can repeat as before.


m = 0
q_n
k / k_j
1.2
1.0
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
ℓ = 3
ℓ = 2
ℓ = 1
ℓ = 1/2
ℓ = 0

To wrap up, there *is* a connection between fluid models and steady-state car-following.

Keep in mind the “steady-state” part; first-order fluid models assume instant adjustment to the diagram.

Unlike higher-order fluid models, the car-following models trivially satisfy anisotropy, can be traced to behavioral concepts, and can accommodate driver and vehicle heterogeneity.