Daganzo's Method and Cell Transmission Model

CE 391F

February 7, 2013

REVIEW

- Newell's method
- Daganzo's method

EXAMPLES

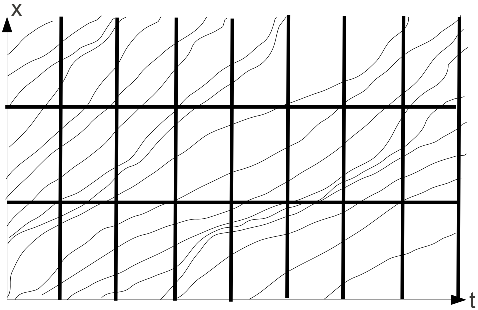
Consider a single-lane roadway with a triangular fundamental diagram, a free-flow speed of 60 mi/hr, a backward wave speed of 30 mi/hr, and a jam density of 180 veh/mi. Initially the roadway is empty; then for at each time $t \in [0, 30]$, vehicles enter at a rate of $2t$ vehicles per minute, after which vehicles enter at a constant rate of 60 veh/min.

15 miles downstream, the road is periodically closed and reopened at 15 minute intervals (say, for a work zone).

15 miles downstream, there is a work zone which only lets vehicles pass at a rate of 1200 veh/hr.

CELL TRANSMISSION MODEL

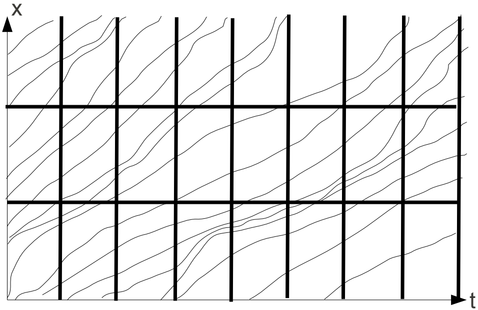
The cell transmission model is a discrete approximation to the LWR model



3	3	3	2	2	1	2	3
3	4	4	4	2	2	4	5
4	4	4	4	5	4	3	1

A roadway link is divided into "cells," and we track the number of vehicles in each cell at discrete points in time.

The key idea in CTM is that *the length of a cell is the speed a vehicle would travel at free flow*. Therefore, $\Delta x = u_f \Delta t$



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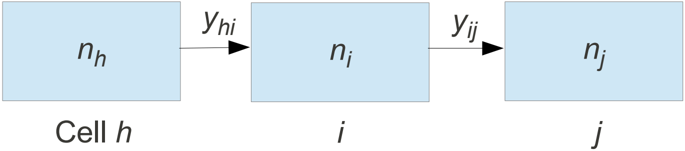
In CTM, we keep track of the number of vehicles in each cell i at each time t : $n_i(t)$



If $y_{ij}(t)$ is the number of vehicles moving from cell i to cell j during time interval t, then

$$n_i(t+1) = n_i(t) + y_{hi}(t) - y_{ij}(t)$$

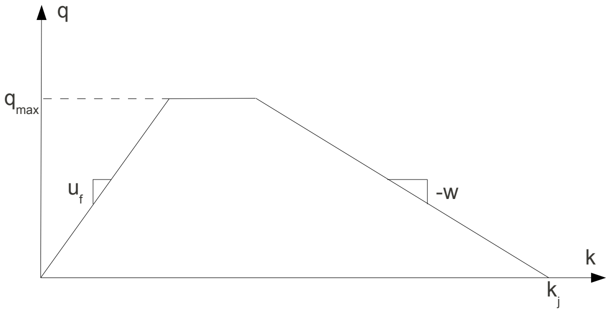
If we assume a uniform flow rate q between two cells during a time interval, and uniform density k in a cell during a time interval, then

$$y_{hi}(t) = q_{hi}\Delta t$$

and

$$n_i(t) = k\Delta x$$

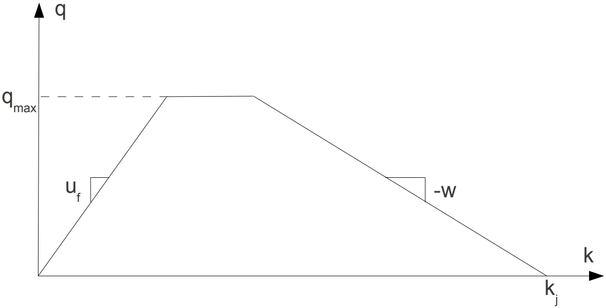
In the CTM, we typically assume a trapezoidal fundamental diagram.



This diagram is defined by four properties: the free-flow speed u_f, the capacity q_{max}, the jam density k_j, and the backward wave speed $-w$.

This fundamental diagram has the form

$$q = \min\{u_f k, q_{max}, w(k_j - k)\}$$



Therefore, the flow $y \approx q\Delta t$, so substituting the flow-density relation gives

$$y = \min\{u_f k\Delta t, q_{max}\Delta t, w(k_j - k)\Delta t\}$$

$$y = \min\left\{n_i(t), q_{max}\Delta t, \frac{w}{v}(N_i - n_i(t))\right\}$$

where $N_i = k_j\Delta x$ is the maximum number of vehicles which can fit into cell i

$$y = \min \left\{ n_i(t), q_{max}\Delta t, \frac{w}{v}(N_i - n_i(t)) \right\}$$

So, there are three possibilities:

1. $y = n_i(t)$. This is the *uncongested* case, so the wave propagates *downstream*. Every vehicle currently in the cell advances to the next.
2. $y = q_{max}\Delta t$. This is the *full-capacity* case. Vehicles leave the cell at capacity.
3. $y = w/v(N_i - n_i(t))$. This is the *congested* case, so the wave propagates *upstream*. The number of vehicles which can *enter* the cell is restricted by the number of vehicles which fit at jam density.

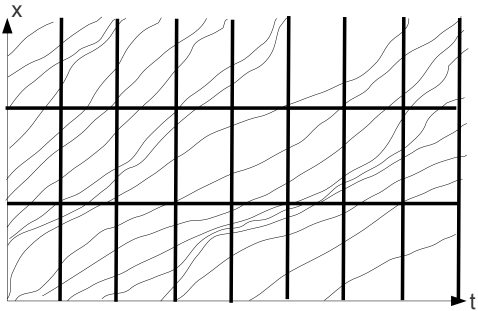
FANCIER FUNDAMENTAL DIAGRAMS

CTM can also be applied when the fundamental diagram has a more general shape:

$$y(t) \approx q\Delta t = Q(k)\Delta t \approx Q(n_i(t)/\Delta x)\Delta t$$

CONNECTION TO PDES

CTM essentially solves the partial differential equations of the LWR model through a finite difference scheme.



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Solving in increasing order of time from given initial conditions is called the *upwind method* or *Godunov scheme*.

One can show that this method is "stable" (that is, discretization errors do not propagate wildly) if the Courant-Friedrich-Lewy condition is satisfied:

$$\left| \frac{u \Delta t}{\Delta x} \right| \leq 1$$

where u is the fastest possible wave speed

In the LWR model, the fastest wave speed is u_f; since the CTM chooses $\Delta x = u_f \Delta t$, this condition is satisfied and the solution is stable.