Cell Transmission Model and Calibrating LWR models

CE 391F

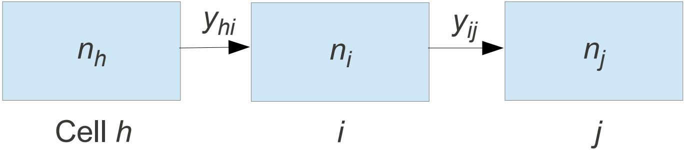
February 12, 2013

REVIEW

- Cell transmission model

CELL TRANSMISSION MODEL: FORMULAS

Remember, in CTM we iterate between calculating all of the y values using current n values, then updating the n values using the y values. Because these computations are separable, CTM is readily parallelized.



For the trapezoidal fundamental diagram, we derived the formula

$$y = \min\{n, q_{max}\Delta t, (w/v)(N - n)\}$$

But y is measured *between* the cells and n and N are measured *at* a cell. Which cells do we pick?

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Thus

$$y_{ij} = \min\{n_i, q^i_{max}\Delta t, q^j_{max}\Delta t, (w/v)(N_j - n_j)\}$$

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The *actual* flow is the minimum of the two.

The *sending flow* of a cell is the number of vehicles that would *leave* the cell if connected to an infinite reservoir downstream:

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$$S_i(t) = \begin{cases} Q_i(n_i(t)/\Delta x)\Delta t & n_i(t) \le k_c \Delta x \\ q^i_{max} & k_c > \Delta x \end{cases}$$

more generally, where k_c is the critical density.

The *receiving flow* of a cell is the number of vehicles that would *enter* the cell if connected to an infinite reservoir upstream:

$$R_i(t) = \min\{(w/v)(N_j - n_j(t), q^j_{max}\}\Delta t$$

for a trapezoidal fundamnetal diagram

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Thus, $y_{ij}(t) = \min\{S_i(t), R_j(t)\}$

EXAMPLE

ADVANTAGES AND DISADVANTAGES

Some major advantage of the cell transmission model:

- It is very easy to have nonhomogeneous fundamental diagrams, and many types of boundary conditions orf lows
- It works very well as part of a simulator
- It is relatively simple and transparent

One disadvantage is *shock spreading*.

COMPARISON OF LWR-BASED METHODS

Shockwaves

- Works well when there are large regions of constant density (steady inflow rates, homogeneous fundamental diagram, etc.)
- Can be tricky when there are multiple or complex boundary conditions.

Newell's method

- Works well when you only need output data at selected points, and when the fundamental diagram is triangular
- Can be tricky when the fundamnetal diagram is nonhomogeneous, and tedious if you need output data at many points

Daganzo's method

- Works well when you need output data at many points, and when the fundamental diagram is piecewise linear
- Can be harder to incorporate into a simulation model; graphical methods may be harder to automate

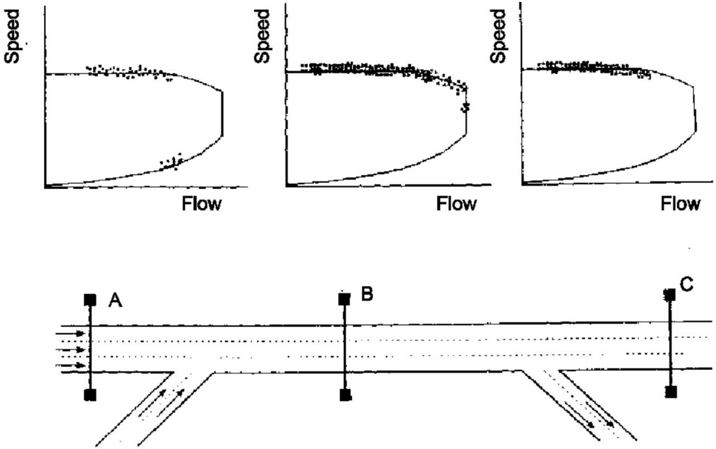
Cell transmission model

- Simple and transparent, can handle complex fundamental diagrams or boundary conditions
- "Shock spreading," no general conditions for sufficiency, can be difficult to model moving bottlenecks

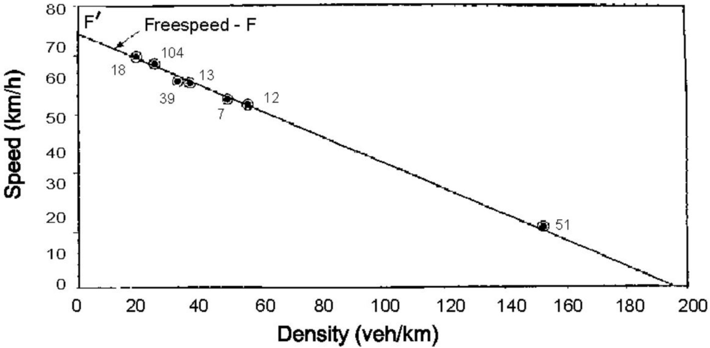
CALIBRATING LWR MODELS

How would you measure speed, flow, and density?

Be cautious where you collect data!



The (in)famous Greenshields relationship



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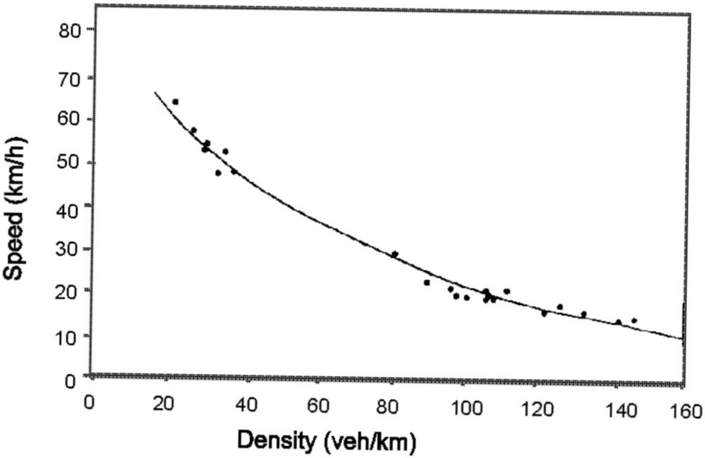
This work is seminal, but infamous. Why infamous?

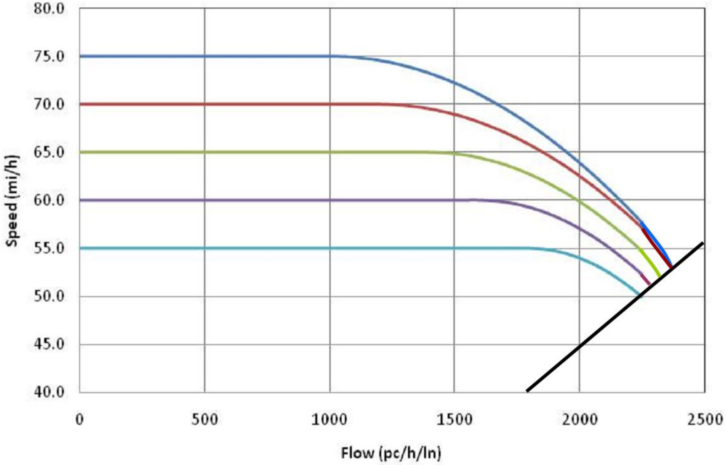
- Data collected on a one-lane road, not a freeway
- Vehicles were counted in “overlapping” groups of 100
- Some averaging occurred prior to regression
- The lone congested point was collected on a different roadway on a different day
- Model based on holiday traffic rather than regular commuters

The Greenberg model



Collected in the (one-lane!) Lincoln Tunnel in 1959, based on a logarithmic fit to data

Highway Capacity Manual



Notice constant speeds for low flow rates, and that the “congested” piece isn’t shown.