CE 391F: Homework 3

Due Thursday, April 4

Problem 1. Consider a roadway with the following boundary conditions: for $x = 0$ and $t > 0$, $k(x,t) = 80$ vehicles per mile, $u(x,t) = 40$ miles per hour, and $w(x,t) = 0$. For $t = 0$ and $0 \leq x \leq 1$, $k(x,t) = 80 + 160x$ veh/mi, $u(x,t) = 40 - 40x$ mi/hr, and $w(x,t) = -40$ hr^{-1} (x measured in miles). The momentum equation on this road has $c_0^2 = 1.5$ mi^2/hr and $\tau = 6$ s.

Use an implicit method to solve the momentum and conservation equations on this roadway, with t ranging from 0 to 18 seconds and x ranging from 0 to 0.5 miles. Use a solution lattice with a 6 second time step and 0.1 mile spatial step, and ensure that the absolute values of the momentum and conservation equations are less than 10^{-4} at each lattice point (they would be zero in an exact solution). Report the k, u, and w values at the *midpoints* of the rectangles.

Problem 2. Your vehicle is initially traveling at a speed of 30 mph, the same initial speed as the car in front of you, with a 900-foot following distance. Over the next fifteen seconds, you accelerate at a constant rate to 40 mph while the car in front of you maintains its speed. For thirty seconds after that, you decelerate to 30 mph at a constant rate, a speed which you maintain afterwards.

(a) If the car in front of you is 6 feet high, plot the visual angle over time.

(b) At what time is the change in visual angle greatest?

Problem 3. Three cars are driving on a single-lane road, with the second car 500 ft behind the lead car, and the third car 500 ft behind the second. Initially all vehicles are driving at a steady speed of 80 ft/s, when the lead vehicle begins to brake. The lead vehicle's trajectory is described by $x_1(t) = 640/(1 + e^{-t/2})$, where t is measured in seconds and x_1 in feet. Use the basic car-following model $\ddot{x_f}(t) = \lambda(\dot{x_\ell}(t - T) - \dot{x_f})(t - T)$ (with $\lambda > 0$) to answer the following questions. The second car has a reaction time of 2 seconds; the third car has a reaction time of 1 second. What range of λ values do you believe provides reasonable behavior? Provide plots to support your answers.