CE 391F: Homework 2
Solutions

Problem 1. *Consider a roadway with a trapezoidal fundamental diagram, with free-flow speed 30 mph, capacity 1800 veh/hr, jam density 240 veh/mi, and backward wave speed 15 mph. The inflow rate has a triangular profile: the inflow rate is* $90t$ *veh/hr for* $0 \leq t \leq 15$ *(t measured in minutes),* $90(30-t)$ *veh/hr for* $15 \leq t \leq 30$*, and 0 afterwards. This inflow rate is measured 1 mile upstream of a traffic signal with a 60-second cycle length, starting at* $t = 0$ *with 15 seconds of effective green followed by 45 seconds of effective red; assume that the fundamental diagram is not affected by turning vehicles. For all parts of this problem, use the cell transmission model with a timestep of 15 seconds.*

(a) *What is the time at which the last vehicle passes the signal?*

(b) *Estimate the maximum queue length which will occur.*

(c) *What is the average travel time for the vehicles on this roadway, measuring travel time from 1 mile upstream of the signal to immediately downstream of the signal?*

(d) *Now, assume that the signal timing can be changed: the cycle length remains 60 s, but the effective green and red times can be changed (but must be multiples of 15 seconds). What is the minimum effective green time which would be needed to ensure that the queue* on this approach *is always cleared at the start of each red interval?*

See attached spreadsheet for detailed calculations.

(a) The last vehicle leaves 50 minutes and 15 seconds after the start of simulation.

(b) The queue reaches its maximum length at $t = 25.5$, at which point there are six cells with very high density (approximately 26 vehicles per cell), two cells of low density (approximately 1.8 vehicles per cell), and one of intermediate density (8.4 vehicles per cell). It is thus reasonable to believe that the queue extends roughly 27% of the way into this cell (because $0.27 \times 26 + 0.73 \times 1.8 \approx 8.4$), so the queue is approximately 6.27 cells long. With a cell length of 660 ft, the queue is thus 4138 ft long.

(c) The total time can be calculated by multiplying the number of vehicles on the roadway segment at each time interval by the length of the timestep, and summing over all time intervals (this is the area between the arrival and departure curves). As the spreadsheet shows, the total delay is thus 70.9 vehicle-hours. Since a total of 337.5 vehicles (found by integrating the inflow profile) approach the signal on this roadway, the average delay is 12.6 minutes per vehicle.

(d) By trial and error, with a 45 second green time, no queues persist at the end of the green interval.

Problem 2. *Continuing Problem 1, assume that this signal lies at the intersection of two one-way roads. One was described in Problem 1; the other has an identical fundamental diagram, but a slightly different inflow rate: a constant 1200 veh/hr for* $0 \leq t \leq 30$ *(t in minutes), again measured 1 mile upstream of the signal. The effective green time for each approach corresponds exactly to the effective red time to the other approach.*

(a) *Returning to the initial scenario in Problem 1 (the first approach has 15 seconds of green and 45 seconds of red), what is the average travel time if we account for vehicles on* both *roadways (again measuring travel times from 1 mile upstream of the signal to immediately downstream).*

(b) *Propose a revised signal timing for reducing average travel time. You may change either the cycle length or the allocation of green and red times, as long as all values are multiples of 15 seconds, and the signal turns green for the first approach at $t = 0$. (These values must remain constant over the modeling period.) What is the average travel time for your revised timing?* 10 extra credit points will be awarded to the best timing in the class, divided among the number of students who found that value.

See attached spreadsheet for detailed calculations.

(a) With the original timing, the total delay on Approach 2 is 23.4 hours, spread among 600 vehicles. Thus the total delay to all 937.5 vehicles from both approaches is 94.3 vehicle-hours, and thus the average delay is 6.04 minutes per vehicle.

(b) A 90-second cycle length, with 30 seconds of green allocated to Approach 1 and 60 seconds to Approach 2, gives a per-vehicle delay of 4.43 minutes.

Problem 3*. As discussed in class, a single loop detector can only measure flow and occupancy (the fraction of time a vehicle overlaps any part of the detector). Making some assumptions about vehicle length, we can relate this to other traffic variables. Assume that all vehicles have the same length L, and that the detector itself has a length of d.*

(a) *If a detector measures a vehicle on top of it for t seconds, what speed is the vehicle traveling?*

(b) *If we observe n vehicles passing the detector in a time interval T, find an equation relating the occupancy O to the space-mean speed $\overline{u}_s$ of the vehicles and the flow q; then use the fundamental relationship to find an equation relating density to occupancy.*

(a) The detector measures the vehicle from the moment the front bumper passes the upstream end of the detector, to the moment the rear bumper passes the downstream end of the detector. Thus the vehicle travels a distance of $L + d$ in time t, so its speed is $(L + d)/t$.

(b) Let t_i be the time that the i-th vehicle was on the detector, and u_i the speed of the i-th vehicle. Then

$$O = \frac{\sum_{i=1}^n t_i}{T} = (L+d)\frac{\sum_{i=1}^n (1/u_i)}{T} = (L+d)\frac{n}{T}\frac{1}{n/\sum_{i=1}^n (1/u_i)} = (L+d)\frac{q}{\overline{u}_s}$$

respectively using the definition of occupancy, the result from part (a), multiplying by a clever form of one and rearranging, and substituting $q = n/T$ and the formula for space-mean speed in terms of point observations. Finally substituting the fundamental relationship $q = \overline{u}_s k$, we obtain $O = (L + d)k$; density and occupancy are proportional (assuming vehicles do not change speed over the detector, which is reasonable if L is fairly small.)