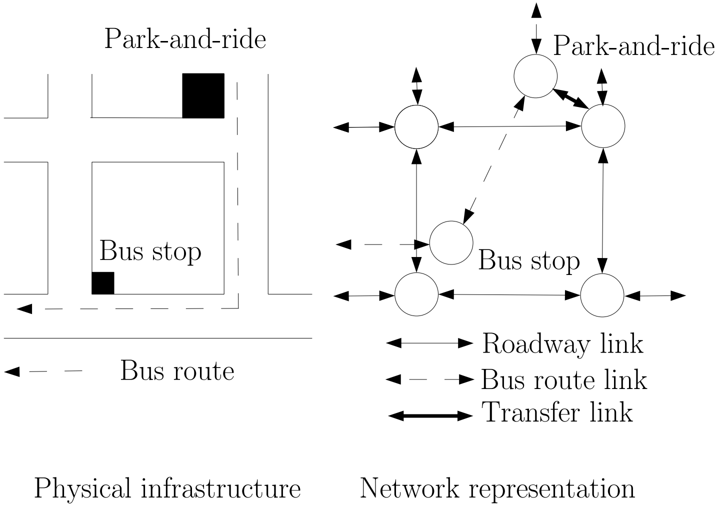
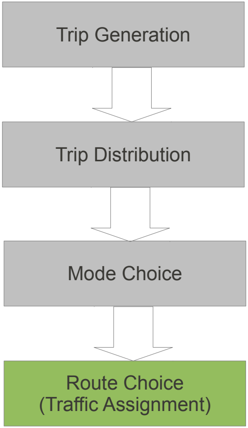
Networks are a simplification of the transportation system into *nodes* and *links*

- *Zones* are nodes where trips can start or end
- A *path* is a sequence of links connecting an *origin* node to a *destination*
- The *OD matrix* represents the number of travelers departing each origin for each destination.

Typically the network structure and O-D matrix are given, and we need to find the route choices and the associated congestion pattern.


Park-and-ride
Bus stop
Bus route
Park-and-ride
Bus stop
Roadway link
Bus route link
Transfer link
Physical infrastructure
Network representation

The four-step model is the traditional paradigm for transportation planning.



This model has shortcomings, and new paradigms have been proposed, but this is still the most commonly used in practice.

The four steps function as follows:

1. **Trip generation** predicts the total number of trips made to and from each zone ("one endpoint" trips).
2. **Trip distribution** creates the OD matrix by specifying both endpoints for all trips.
3. **Mode choice** predicts how many trips will be made driving, by bus, bike, etc., for each entry in the OD matrix
4. **Traffic assignment** predicts the routes that travelers will choose.

In this class, we'll primarily focus on the fourth step, and then look at how to integrate some of the earlier steps.

The output of traffic assignment (link flows) is used to predict mobility, congestion, emissions, safety, equity, etc.

Caveat!

The four-step framework assumes a certain sequentiality. Ideally there is "feedback" to ensure mutual consistency.

Preview: As we'll see later in the course, rather than just cycling through the models one at a time, it's often smarter to try to attack them all simultaneously.

NOTATION AND SPECIFIC TERMINOLOGY

A network $G = (N, A)$ has sets of nodes and links N and A

Z is the set of zones ($Z \subseteq N$)

The OD matrix D has entries d^{rs} for the number of trips from r to s, $(r, s) \in Z^2$.

A link (i, j) connects node i to node j. i is the "tail" or "upstream node", and j is the "head" or "downstream node."

x_{ij} is the *flow* on link (i, j).

t_{ij} is the *travel time* on link (i, j), and is a function of x_{ij}.

The function $t_{ij}(x_{ij})$ is often called the *link performance function*.

One commonly-used function is the BPR function

$$t_{ij}(x_{ij}) = t_{ij}^0 \left(1 + \alpha \left(\frac{x_{ij}}{u_{ij}} \right)^{\beta} \right)$$

where t_{ij}^0 is the free-flow time, u_{ij} is the "practical capacity", and α and β are calibration parameters.

What properties should link performance functions have?

Because the field of network analysis is rather young, different authors use different terms for the same (or very similar) concepts:

- **Link** = arc = edge
- **Node** = vertex
- **Zone** = centroid
- **Path** = route = walk
- **OD matrix** = trip table
- **Flow** = volume = link demand
- **Travel time** = link cost
- **Link performance function** = delay function = congestion function = BPR function = travel time function = impedance function = latency function

In the course, I will try (and hopefully succeed) at consistently using the boldfaced term in each group. If you read elsewhere, you may see other terms.

A path π is a sequence of n adjacent links connecting node i_0 to i_n, written either by specifying the links

$$\{(i_0, i_1), (i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)\}$$

or more compactly by specifying the nodes:

$$[i_0, i_1, i_2, i_3, \ldots, i_{k-1}, i_k]$$

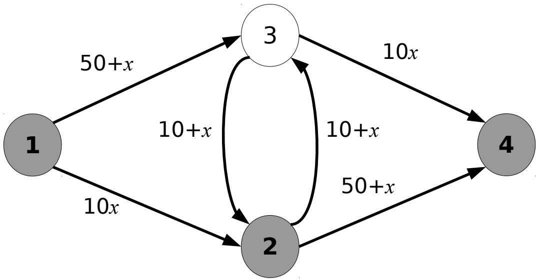
A path is a *cycle* if $i_0 = i_k$.

A path is *cyclic* if it contains a cycle, and *acyclic* if it does not.

A network is *acyclic* if there are no cyclic paths in the network.

Are the paths used in transportation networks usually acyclic?

Are transportation networks usually acyclic?



- [1, 3, 4] is a path
- [3, 2, 3] is a cycle (and a path)
- [1, 3, 2, 3, 4] is a cyclic path
- This network is not acyclic.

Let Π^{rs} denote the set of all acyclic paths connecting zone r to zone s.

Let $\Pi = \cup_{(r,s)\in Z^2}\Pi^{rs}$ be the set of all paths in the network.

Let h^π denote the number of travelers choosing path π (its *flow* or *demand*), and $\mathbf{h}$ the vector of all path flows.

$\mathbf{h}$ is a *feasible assignment* if the following conditions are satisfied:

1. $h^\pi \geq 0 \qquad \forall \pi \in \Pi$ (can't have a negative path flow)
2. $\sum_{\pi\in\Pi^{rs}} h^\pi = d^{rs} \qquad \forall (r,s) \in Z^2$ (no vehicle left behind)

Be sure you understand the notation in these slides, these types of formulas are very common in network analysis.

Link flows can be determined from path flows. Let δ_{ij}^{π} indicate the number of types path π uses link (i, j) (0 or 1 if π is acyclic).

($\delta_{ij}^{\pi} = 0$ if link (i, j) is not part of π; if π is an acyclic path, $\delta_{ij}^{\pi} = 1$ for each link (i, j) in π) Then

$$x_{ij} = \sum_{r \in Z} \sum_{s \in Z} \sum_{\pi \in \Pi^{rs}} \delta_{ij}^{\pi} h^{\pi}$$

or, in matrix notation

$$\mathbf{x} = \mathbf{\Delta h}$$

where $\mathbf{\Delta}$ is the *link-path adjacency matrix* whose rows and columns contain every value of δ_{ij}^{π}.

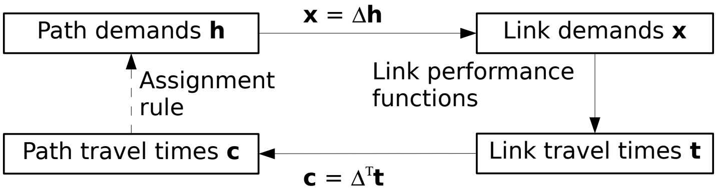
Let c^π denote the travel time on path π. Path travel times can be calculated from link travel times:

$$c^\pi = \sum_{(i,j) \in A} \delta_{ij}^\pi t_{ij}$$

or, in matrix form

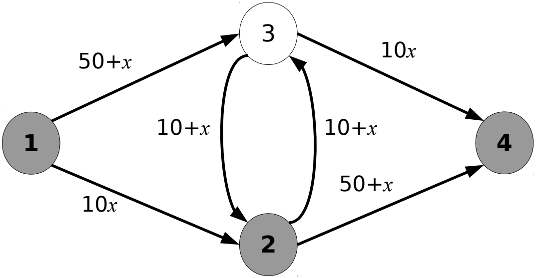
$$\mathbf{c} = \mathbf{\Delta}^T \mathbf{t}$$

There is a natural progression to these calculations:



The fundamental variables are the *path flows* (which must be a feasible assignment), which then determine *link flows*, which determine *link costs* (through delay functions), which determine *path costs*.

Example



Say $d^{14} = 5$, $h_{[1,3,4]} = 1.5$ and $h_{[1,2,3,4]} = 3.5$. What are the travel times on these two paths?

Note that **we do not require flow variables to be integers.** You should interpret them as an average rate of flow.