Properties of dynamic user equilibrium

CE 392D

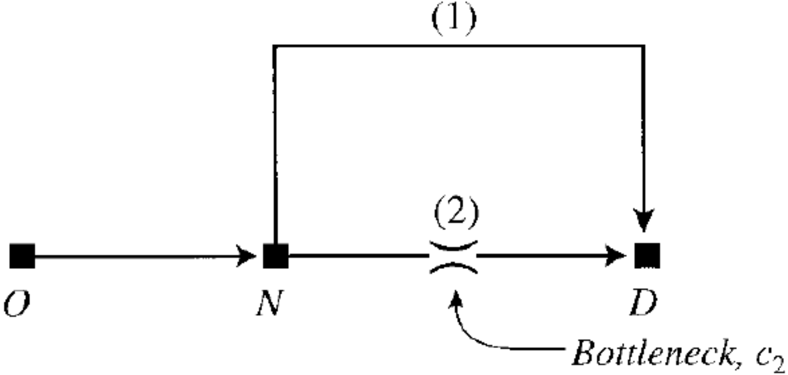
DAGANZO’S PARADOX

In some sense, this is a dynamic version of of the Braess paradox (which is often criticized for being a static model) which is even more surprising.

Increasing capacity *on the only bottleneck link* can worsen total system performance.

In Daganzo's model, this arises from the combination of two effects: (1) queue spillback and (2) shortest-path route choice

Setting



Link free-flow travel times are τ_1 and τ_2; we have $\tau_1 > \tau_2$. Link 1 is longer, but uncapacitated; link 2 is shorter, but has a bottleneck. The inflow rate is $Q > c_2$, and links have spatial queues.

What will happen in our model, with route choice?

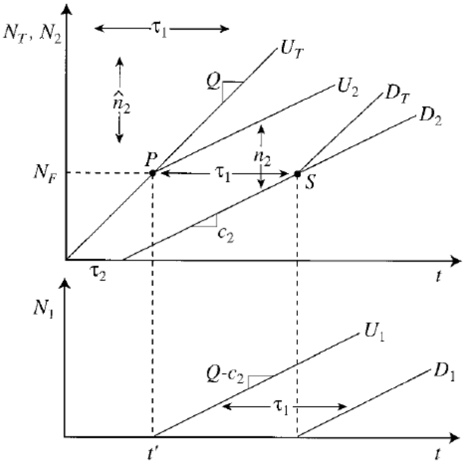
Initially, everyone will choose link 2, because it is faster and initially there is no queue.

A queue will start to build up on link 2, increasing the travel time on that link.

People will still choose link 2 *until the time spent waiting in queue is equal to* $\tau_1 - \tau_2$.

After that point, people will split between link 1 and link 2 *to ensure that the time vehicles spend in the queue is* $\tau_1 - \tau_2$.

Graphically...



The top graph shows upstream/downstream counts for link 2 (U_2 and D_2) and for both paths between N and D (U_T and D_T). The bottom graph shows upstream/downstream counts for link 1 (U_1 and D_1)

So, eventually we will settle on a stable solution, where the travel times on both links are equal:

$$\tau_1 = n_2/c_2$$

where n_2 is the number of vehicles on link 2, so $n_2 = c_2\tau_1$

As long as $c_2\tau_1 \leq \hat{n}_2$, the queue can fit on the link and everything is fine.

Assume that this is initially the case, and then we increase the capacity c_2.

n_2 will increase as well – in the stable solution, both links still have travel time τ_1. So if c_2 is higher, n_2 must also be higher to maintain travel time equality.

What happens if $n_2 > \hat{n}_2$?

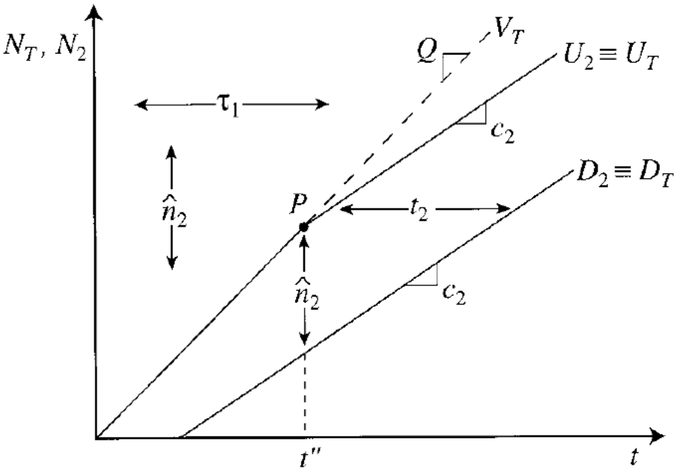
The queue spills back past the diverge point N.

So, now imagine you are a vehicle that has just arrived at the diverge point (after already having to wait in the queue for a little bit).

If you take link 1, your remaining travel time is τ_1. If you take link 2, your remaining travel time is $\hat{n}_2/c_2 < n_2/c_2 = \tau_1$.

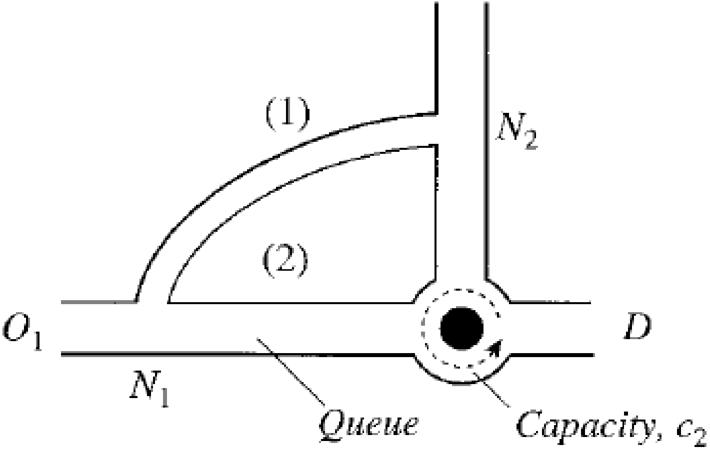
So, everyone will take link 2, and nobody will take link 1. Therefore, the effective throughput of the network has been reduced to c_2!

With the old capacity value, we were able to move vehicles through the network at the rate Q. **Increasing the capacity of link 2 reduced the throughput of the network.**



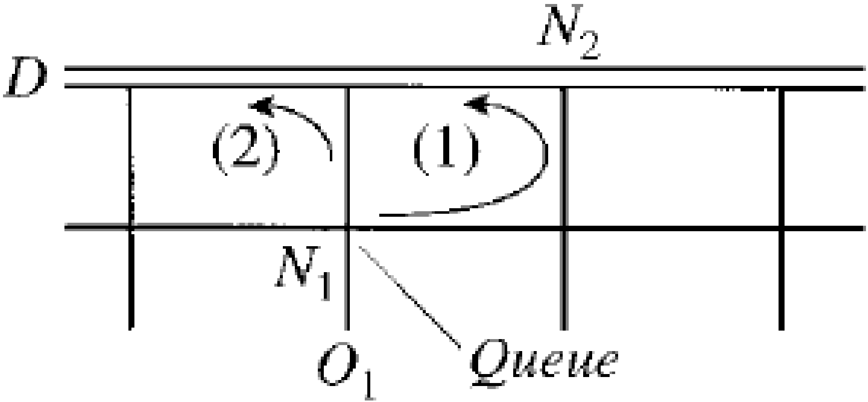
N_T, N_2
Q
V_T
$U_2 \equiv U_T$
τ_1
c_2
$D_2 \equiv D_T$
$\hat{n}_2$
P
t_2
$\hat{n}_2$
c_2
t''
t

When might this occur in practice?



Notice that c_2 depends on Q_1 in these examples.

When might this occur in practice?



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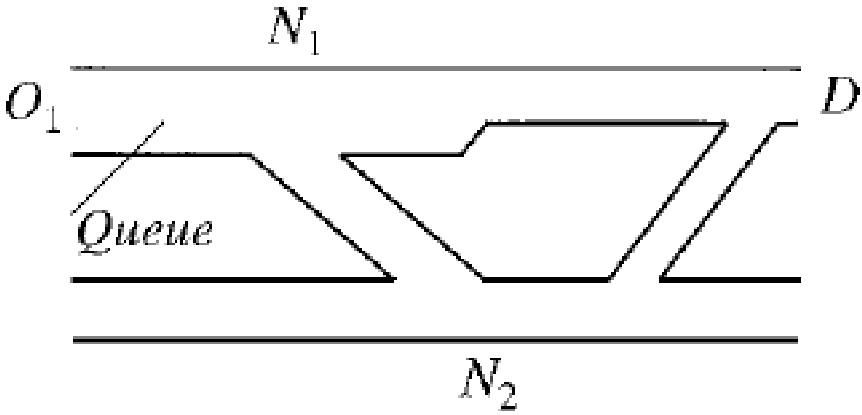
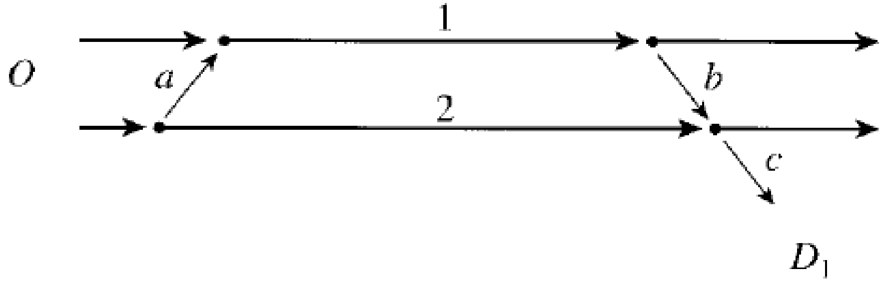
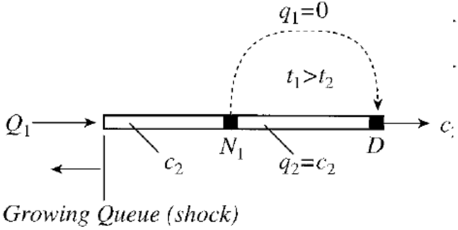
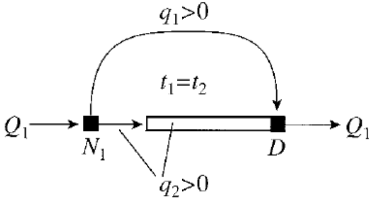


When might this occur in practice?



In such cases, there are two stable situations





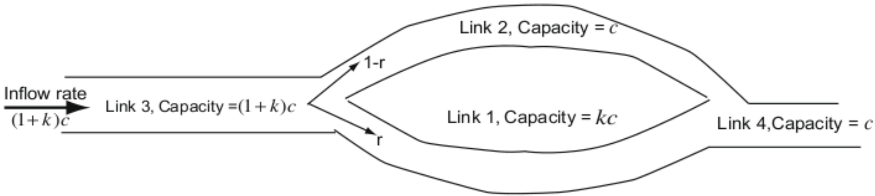
The *top* solution moves fewer vehicles than the *bottom* solution, but they are both equilibria. How can we get the right one?

The moral of the story...

- Network models can behave strangely. You actually need to do a proper analysis, don't just rely on engineering judgement or "obvious" solutions.
- Temporary capacity reductions or closures can improve the overall system capacity.
- Queue management can be more important than capacity.
- There can be more than one equilibrium solution.

NIE’S MERGE

In Daganzo's paradox, we saw that there could be two equilibrium solutions. In Nie's merge, there can be three.



One place this might arise is at a freeway lane drop:

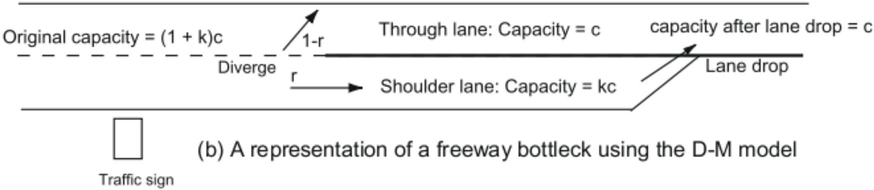
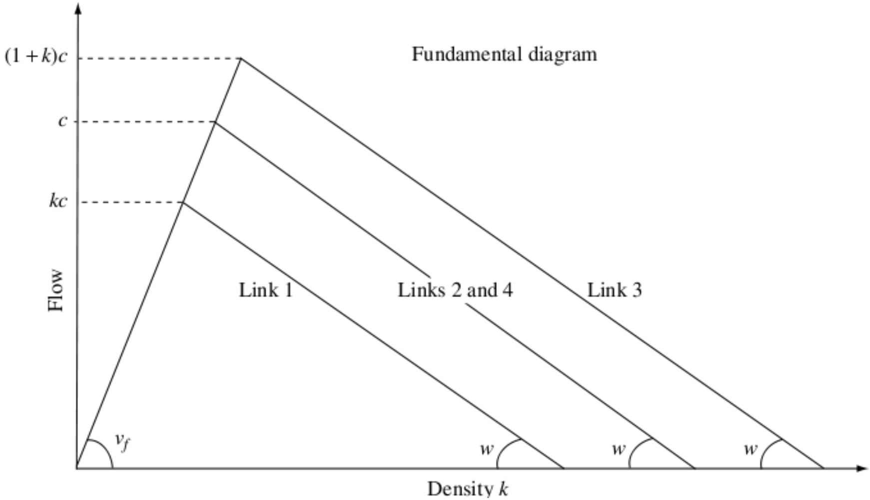


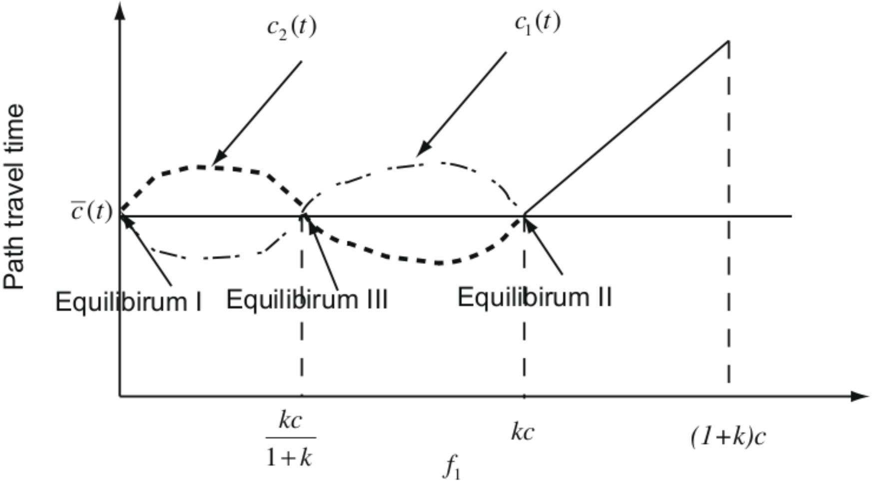
Fig. 1. A two-route network with a merge and a diverge.

This example uses the LWR model, with the following fundamental diagrams:



Note that all links have the same free-flow speed and backward wave speed, but different capacities.

As the flow of vehicles on path 1 f_1 varies from 0 to $(1+k)c$, the travel times on the two paths vary as follows:

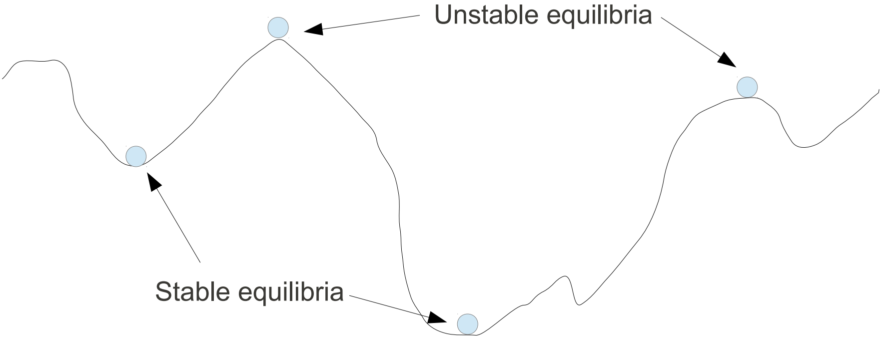


There are three equilibria! What do we think is most likely to occur? What will our models give us? What should planners plan for?

Demonstration

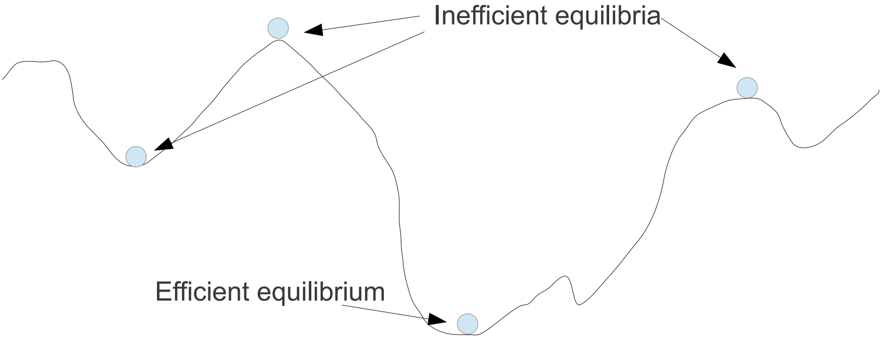
Nie proposes two criteria for distinguishing among these equilibria: *stability* and *efficiency*.

Stability: If we perturb the equilibrium slightly, will the path flows tend to adjust back towards the equilibrium, or move further away?



It is reasonable to think stable equilibria are likelier to occur.

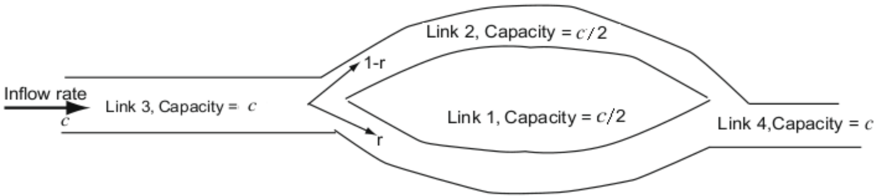
Nie proposes two criteria for distinguishing among these equilibria: *stability* and *efficiency*. **Efficiency:** Is there another equilibrium where we can reduce someone's travel time, without increasing anyone else's?



Efficiency is an "optimistic" criterion, perhaps better used as a goal for policy than a reliable projection of future conditions.

BDMR PARADOX

This paradox isolates one of the features of Nie's merge.



The "obvious" flow pattern has $f_1 = f_2 = c/2$, because no congestion will arise on the network.

This obvious pattern is indeed system optimal – everyone is at free flow, so the total travel time is minimal.

It is also a user equilibrium: both paths have equal and minimal travel time.

However, literally any other values of f_1 and f_2 satisfy the user equilibrium principle as well!

Demonstration

In other words, every feasible solution is a user equilibrium, even though the system optimal solution is unique.

Would this happen in real life?

What does this mean for equilibrium models?

In practice, average speeds would drop even when density is subcritical, something which triangular and trapezoidal fundamental diagrams don't capture.

One potential resolution involves adding more "pieces" to fundamental diagrams.

How would we change CTM or LTM if there were multiple "uncongested" pieces on the fundamental diagram?

NO EQUILIBRIUM

The matching pennies game

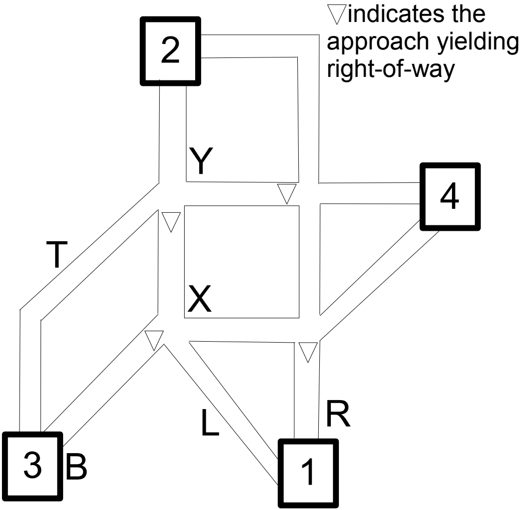
You and a friend each reveal a penny simultaneously (choosing heads or tails).

If both pennies are heads, or both are tails, you keep them both. If they are different, your friend keeps both.

This game has no equilibrium solution.

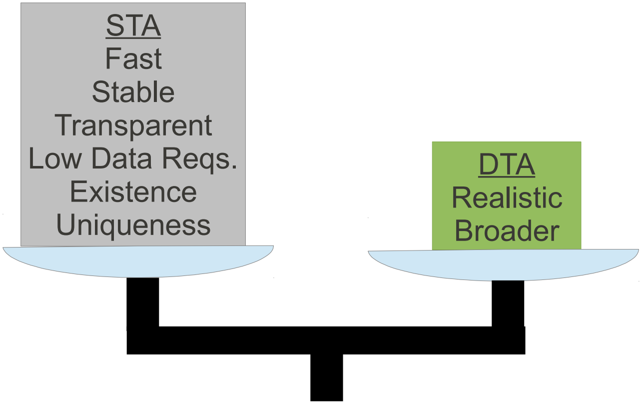
...or does it?

Matching pennies in traffic assignment



SUMMARY

Dynamic models are not universally better than static models.



Different tasks require different tools, and all models are wrong, but...