Basic network loading

CE 392D

NETWORK LOADING PROBLEM

Network loading

In the network loading problem, we are given the paths and departure times of each vehicle, and must find the travel times along all paths for all possible departure times.

For dynamic traffic assignment, this essentially means tracking the trajectories of vehicles as they travel through the network.

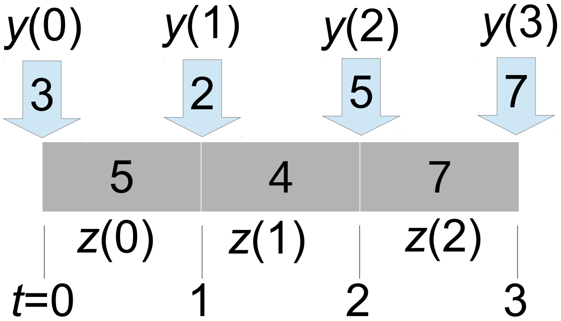
This complex problem involves both *link models* and *node models.*

- A *link* model focuses on what happens within a single roadway segment.
- A *node* model focuses at what happens at junctions, where links conicide.

By combining these together, we can solve large networks in a “modular” way (which is amenable to parallel computing).

We first study simple link models, then move to node models.

In this course, we will treat network loading in *discrete time*, where the timestep is Δt, and the time points are $\{0, 1, 2, \ldots, \bar{T}\}$.



Variables which occur *at* a point in time t are assumed to happen exactly at time t. Variables which occur *over* a period of time t happen between t and $t + 1$.

Network loading happens sequentially, in increasing order of time. We assume network conditions are empty at $t = 0$; then we calculate the state of the network at $t = 1$, $t = 2$, and so forth.

Essentially, we need to know how to compute the state of the network at $t + 1$, given its state at times $0, 1, \ldots, t$.

For each link, we can define the *sending flow* and *receiving flow*.

The sending flow $S(t)$ is the flow which would leave the link during the t-th interval if it were connected to a sink of infinite capacity.

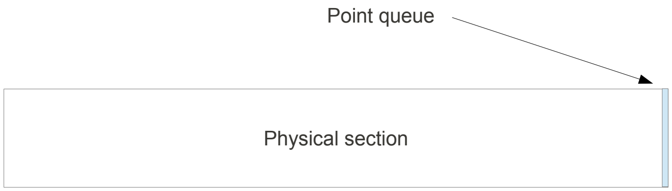
The receiving flow $R(t)$ is the flow which would enter the link if it were connected to a source of infinite capacity.

The purpose of sending and receiving flow is to see what can happen on a link *independently* of any other link. Node models will be used to see how many vehicles actually enter and exit a link based off of the interactions between different links' sending and receiving flows.

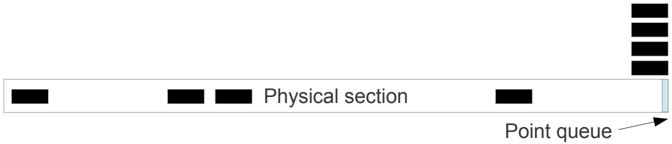
POINT QUEUE MOTIVATION

Divide a link into two sections:

1. An uncongestible *physical section* representing the time required to travel on the link with no congestion.
2. A *point queue* at the downstream end of the link, occupying no physical space but representing delay due to congestion.



Vehicles always cross the physical section in the same length of time.



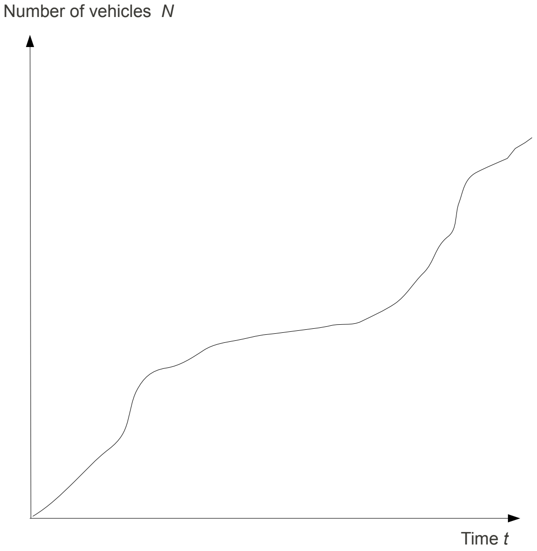
Further, when we study the queue, we don't have to worry about where vehicles are: they're all in the same physical location.

Both the physical section and point queue can be described by one parameter.

1. The relevant parameter for the physical section is the *free-flow time* t_f. (If we know the link length L and free-flow speed u_f, this is just L/u_f)
2. The relevant parameter for the point queue is the *capacity* $q^{\downarrow}_{max}$, the maximum rate at which vehicles can leave the link.

CUMULATIVE CURVES

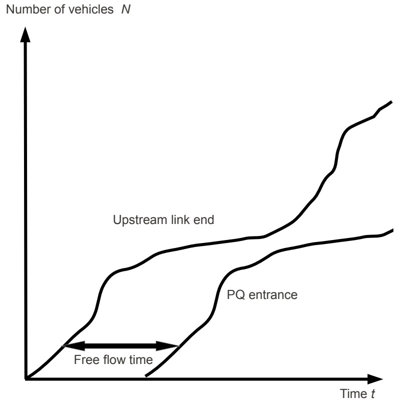
Let $N(t)$ denote the *total* number of vehicles which have passed some point at time t.



In particular, let...

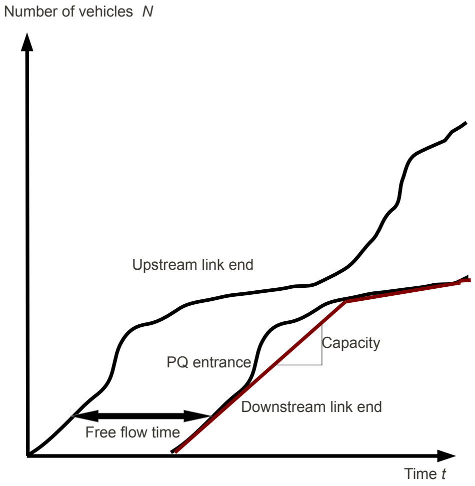
- $N^{\uparrow}(t)$ be the cumulative count at the *upstream* end of a link
- $N^{PQ}(t)$ be the cumulative count at the *entrance* to the point queue
- $N^{\downarrow}(t)$ be the cumulative count at the *downstream* end of the link (exit of the point queue).

How are $N^{\uparrow}(t)$ and $N^{PQ}(t)$ related?



$N^{PQ}(t) = N^{\uparrow}(t - t_f)$ because there is never any congestion delay on the physical section.

What about $N^{PQ}(t)$ and $N^{\downarrow}(t)$?



$N^{\downarrow}(t)$ takes the *highest possible value* given (1) $N^{\downarrow}(t) \leq N^{PQ}(t)$; (2) $N^{\downarrow}(t) - N^{\downarrow}(t - \Delta t) \leq q^{\downarrow}_{max} \Delta t$; and (3) any obstruction from

Since N^{PQ} is just $N^{\uparrow}$ shifted to the right, we can rewrite the formula solely in terms of $N^{\uparrow}$:

$$S(t) = \min\{N^{\uparrow}(t + \Delta t - L/u_f) - N^{\downarrow}(t), q^{\downarrow}_{max}\Delta t\}$$

One way to interpret how flow leaves the PQ

(in a discrete-ish setting)

1. We want to move as many vehicles as possible out of the PQ (no "holding back" flow that could move), however...
2. We can't have a negative number of vehicles in the PQ; and
3. We can't move more vehicles than the capacity of the link

The formula for receiving flow is simple, because the physical section is only constrained by the upstream capacity:

$$R(t) = q^{\uparrow}_{max} \Delta t$$

Example

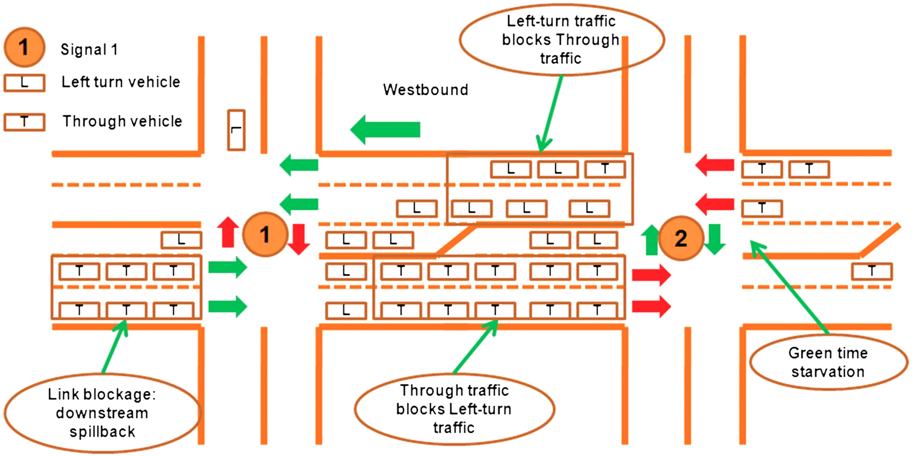
Assume that a link has $L/u_f = 3\Delta t$, $q^{\uparrow}_{max} = 10\Delta t$, and $q^{\downarrow}_{max} = 5\Delta t$, and that the $N^{\uparrow}(t)$ is given as a boundary condition.

t	$N^{\uparrow}$	$N^{\downarrow}$	R	S
0	0	0	10	0
1	1	0	10	0
2	5	0	10	0
3	10	0	10	1
4	17	1	10	4
5	27	5	10	5
6	30	10	10	5
7	30	15	10	5
8	30	20	10	5
9	30	25	10	5
10	30	30	10	0

When are there vehicles waiting in queue?

SPATIAL QUEUES

The major assumption of the point queue model was that the queue occupies no physical space.



1 Signal 1
L Left turn vehicle
T Through vehicle
Westbound
Left-turn traffic blocks Through traffic
Link blockage: downstream spillback
Through traffic blocks Left-turn traffic
Green time starvation
1
2

(Li, 2011)

In reality, queues that fill an entire link will block entry into the link and "spill back."

In the spatial queue model, we assume a jam density k_j, so that the maximum number of vehicles that can be on the link at any point in time is $k_j L$.

The sending flow formula is the same (nothing is different about how the queue discharges):

$$S(t) = \min\{N^\uparrow(t + \Delta t - L/u_f) - N^\downarrow(t), q^\downarrow_{max}\Delta t\}$$

The receiving flow is modified so that the number of entering vehicles cannot exceed the physical space on the link:

$$R(t) = \min\{k_j L - (N^\uparrow(t) - N^\downarrow(t)), q^\uparrow_{max}\Delta t\}$$

Repeating the same example with $k_j L = 20$ gives the following table:

t	$N^{\uparrow}$	$N^{\downarrow}$	R	S
0	0	0	10	0
1	1	0	10	0
2	5	0	10	0
3	10	0	10	1
4	17	1	4	4
5	21	5	4	5
6	25	10	5	5
7	30	15	5	5
8	30	20	10	5
9	30	25	10	5
10	30	30	10	0

It takes longer for all 30 vehicles to enter the link, because R drops as the link fills up. The other vehicles are "held back"

The spatial queue model has its limitations, too: it assumes that all vehicles move forward simultaneously on a link.