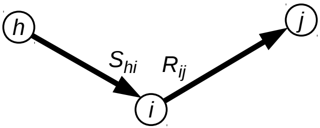
Some simple node models

CE 392D

OUTLINE

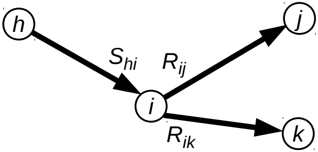
SOME SIMPLE NODE MODELS

The simplest node has just one incoming and outgoing link.

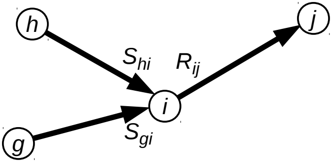


In this case, the number of vehicles moving between links is $y_{hij}(t) = \min\{S_{hi}(t), R_{ij}(t)\}$

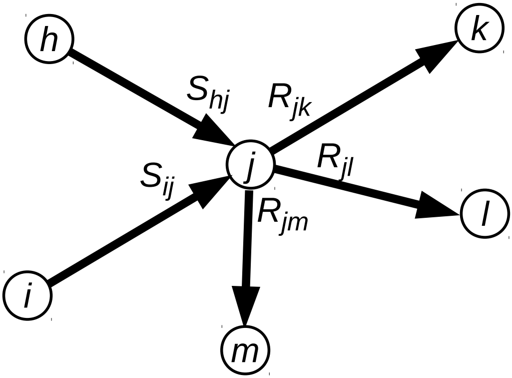
A diverge has one incoming link, and two (or more) outgoing links.



A merge has one outgoing link, and two (or more) incoming links.



A *general intersection* has more than one incoming link, and more than one outgoing link.



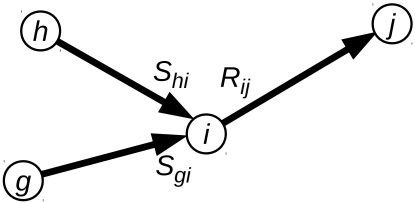
General intersection models vary based on whether they are signal-controlled, stop-controlled, roundabout-controlled, etc. These models are not as well standardized as diverge and merge models.

Node models should have the following properties:

1. Every y_{hij} value should be actively limited by some constraint. (No holding back.)
2. $y_{hij} \geq 0$ for all turning movements.
3. $y_{hij} > 0$ only for allowed turning movements ($[h, i, j] \in \Xi(i)$)
4. For each incoming link (h, i), the sum of y_{hij} values should not exceed S_{hi}
5. For each outgoing link (i, j) the sum of y_{hij} values should not exceed R_{ij}
6. Route choice must be respected. (Can't redirect vehicles to increase flow.)
7. FIFO principle must be respected. (Vehicles can't queue jump to increase flow.)
8. Invariance principle must be respected. (Will demonstrate with merges.)

MERGES

A merge node has more than one incoming link, and one outgoing link.



Let R_{ij} be the receiving flow downstream, and S_{gi} and S_{hi} the upstream sending flows; we must calcualte y_{gij} and y_{hij}.

There are three cases.

Case I : $S_{gi} + S_{hi} \leq R_{ij}$. This is the easy case: $y_{gij} = S_{gi}$ and $y_{hij} = S_{hi}$.

Case II : Flow from both upstream links is restricted by the merge. (This is the usual congested case.)

Case III : Only one of the upstream links is restricted by the merge; the other can still send all of its flow. (This happens when the flow is unbalanced.)

How does a merge behave when it is congested?

One assumption is that if there are vehicles in queue on both upstream links, the ratio $y_{gij} : y_{hij}$ equals the ratio $q_{max}^{gi} : q_{max}^{hi}$. That is, vehicles flow at a rate proportional to the links' capacities.

Case II

In this case, y_{gij} and y_{hij} satisfy $y_{gij} + y_{hij} = R_{ij}$ and $y_{gij}/y_{hij} = q^{gi}_{max}/q^{hi}_{max}$ so...

$$y_{gij} = \frac{q^{gi}_{max}}{q^{gi}_{max} + q^{hi}_{max}} R_{ij}$$

and

$$y_{hij} = \frac{q^{hi}_{max}}{q^{gi}_{max} + q^{hi}_{max}} R_{ij}$$

Case III

In this case, y_{gij} and y_{hij} satisfy $y_{gij} + y_{hij} = R_{ij}$, but for one of the links (say (g, i)), $S_{gi} < \frac{q^{gi}_{max}}{q^{gi}_{max} + q^{hi}_{max}} R_{ij}$. So...

$$y_{gij} = S_{gi}$$

and

$$y_{hij} = R_{ij} - S_{gi}$$

(with the formulas reversed if (g, i) is the link with the queue.)

In other words, one link's sending flow is less than its "fair share" of downstream capacity, so the other link can make use of this unused capacity.

We can combine Cases II and III into one equation.

$$y_{gij} = \text{med}\left\{S_{gi}, R_{ij} - S_{hi}, \frac{q_{max}^{gi}}{q_{max}^{gi} + q_{max}^{hi}} R_{ij}\right\}$$

Examples...

Assume that $q^{gi}_{max} = 2q^{hi}_{max}$.

1. $S_{gi} = 500$, $S_{hi} = 1000$, $R_{ij} = 300$
2. $S_{gi} = 500$, $S_{hi} = 1000$, $R_{ij} = 2000$
3. $S_{gi} = 100$, $S_{hi} = 1000$, $R_{ij} = 300$

Does the merge model satisfy the properties of node models?

1. Every y_{hij} value should be actively limited by some constraint.
2. $y_{hij} \geq 0$ for all turning movements.
3. $y_{hij} > 0$ only for allowed turning movements ($[h, i, j] \in \Xi(i)$)
4. For each incoming link (h, i), the sum of y_{hij} values should not exceed S_{hi}
5. For each outgoing link (i, j) the sum of y_{hij} values should not exceed R_{ij}
6. Route choice must be respected.
7. FIFO principle must be respected.
8. Invariance principle must be respected.

All except the last are easy to show.

Invariance principle

The first part of the invariance principle applies if $y_{gij} < S_{gi}$ (which means Case II or Case III).

In Case II, the formula for y_{gij} does not involve S_{gi} at all; and y_{gij} would not change if S_{gi} were increased.

The same is true for Case III (since (g, i) must be the link with the queue if $y_{gij} < S_{gi}$.)

Invariance principle

The second part of the invariance principle applies if $y_{gij} + y_{hij} < R_{ij}$ (which means Case I).

In Case I, the formula for y_{gij} and y_{hij} do not involve R_{ij} at all, and they would not change if R_{ij} were increased.

So, the merge model obeys the invariance principle too.

What if the merge model in Case II were changed to

$$y_{hij} = \frac{S_{gi}}{S_{gi} + S_{hi}} R_{ij}$$

(and similarly for $[h, i, j]$?

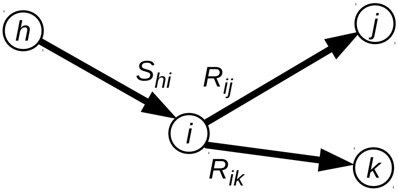
Now, in Case II, the formula for y_{gij} does involve S_{gi}, and if S_{gi} were increased then y_{gij} would increase.

Why is this a problem? Even if the inflow rate to (g, i) is steady, the first time instant after the queue forms the sending flow will increase to q^{gi}_{max}. This increases the amount of flow which can leave; if this discharges the queue completely, S_{gi} and y_{gij} would drop for the next time interval, so a queue would form again, etc.

(Will redo this example with specific numbers after studying shockwaves.)

DIVERGES

A diverge node has one incoming link, and more than one outgoing link.



Let p_{ij} and p_{ik} be the proportion of travelers turning onto links 1 and 2; S_{hi} the sending flow upstream, R_{ij} and R_{ik} the downstream receiving flow; y_{hij} and y_{hik} the number of vehicles moving from (h, i) to (i, j) and (i, k) in one time step.

There are two cases.

Case I : $p_{ij} S_{hi} \le R_{ij}$ and $p_{ik} S_{hi} \le R_{ik}$. This is the easy case: $y_{hij} = p_{ij} S_{hi}$ and $y_{hik} = p_{ik} S_{hi}$.

Case II : Otherwise, at least one of the links cannot accomodate all the flow that wants to enter.

For Case II, the common assumption is that flow waiting to enter a downstream link obstructs *all other* flow on the upstream link.

This implies that only some proportion ϕ of the sending flow can leave the upstream link, and that this same proportion applies to both downstream links.

That is, $y_{hij} = \phi p_{ij} S_{hi}$ and $y_{hik} = \phi p_{ik} S_{hi}$. ϕ is the same for both downstream links.

We want to make ϕ as large as possible (Property 1) while ensuring $\phi p_{ij} S_{hi} \leq R_{ij}$ and $\phi y_{hik} = \phi p_{ik} S_{hi} \leq R_{ik}$.

Therefore

$$\phi = \min \left\{ \frac{R_{ij}}{p_{ij} S_{hi}}, \frac{R_{ik}}{p_{ik} S_{hi}} \right\}$$

We can actually include Case I into this equation by having

$$\phi = \min\left\{\frac{R_{ij}}{p_{ij}S_{hi}}, \frac{R_{ik}}{p_{ik}S_{hi}}, 1\right\}$$

and $y_{hij} = \phi p_{ij} S_{hi}$ and $y_{hik} = \phi p_{ik} S_{hi}$

Examples...

Assume that $p_{ij} = 2/3$, $p_{ik} = 1/3$

1. $S_{hi} = 1200$, $R_{ij} = 800$, $R_{ik} = 300$
2. $S_{hi} = 1200$, $R_{ij} = 400$, $R_{ik} = 300$
3. $S_{hi} = 600$, $R_{ij} = 600$, $R_{ik} = 300$