Name ________________________________________________

Instructions:

- **SHOW ALL WORK** unless instructed otherwise. No shown work means no partial credit!

- If you require additional space, you may use the back of each sheet and/or staple additional pages to the end of the exam.

- If you need to make any additional assumptions, state them clearly.

- Calculators are optional.

- The number of points associated with each part of each problem is indicated.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>100</strong></td>
<td></td>
</tr>
</tbody>
</table>
Problem 1. (25 points). A roadway link is 0.5 km long, with a free-flow speed of 90 kph, backward wave speed of 45 kph, jam density of 320 veh/km, and capacity of 3000 veh/hr. Applying the cell transmission model to this link, the link is divided into ten cells.

1. (5) What are $\Delta t$ and $\Delta x$?

2. (5) What is the maximum number of vehicles that can fit in a cell?

3. (5) Sketch the fundamental diagram corresponding to this link, providing enough labels that the diagram is unambiguous.

4. (5) Write the formula for the sending flow of a cell on this link if there are $n$ vehicles in it (substituting numerical values where possible).

5. (5) Write the formula for the receiving flow of a cell on this link if there are $n$ vehicles in it (substituting numerical values where possible).
Problem 2. (30 points). Consider a network with only one origin-destination pair connected by four paths, with three departure time intervals. At some point in the simplicial decomposition algorithm, \( \mathcal{H} \) contains the following three matrices:

\[
\begin{bmatrix}
20 & 0 & 0 & 0 \\
0 & 10 & 0 & 0 \\
0 & 0 & 30 & 0 \\
\end{bmatrix},
\begin{bmatrix}
0 & 20 & 0 & 0 \\
0 & 10 & 0 & 0 \\
0 & 30 & 0 & 0 \\
\end{bmatrix},
\begin{bmatrix}
20 & 0 & 0 & 0 \\
0 & 0 & 0 & 10 \\
0 & 0 & 30 & 0 \\
\end{bmatrix}
\]

and that the current path flow and travel time matrices are

\[
H = \begin{bmatrix} 14 & 6 & 0 & 0 \\ 0 & 8 & 0 & 2 \\ 0 & 9 & 21 & 0 \end{bmatrix}, \quad T(H) = \begin{bmatrix} 20 & 20 & 24 & 27 \\ 30 & 34 & 37 & 40 \\ 44 & 35 & 36 & 40 \end{bmatrix}
\]

1. (10) What is the (unrestricted) average excess cost of the current solution?

2. (20) What is the search direction \( \Delta H \) based on \( H, T(H), \) and \( \mathcal{H} \)?
Problem 3. (25 points.) Consider the following network, and a traveler leaving node 1 headed for node 4. The arcs are labeled with the time-dependent travel times.

1. (15) What is the shortest path when departing at $\tau = 5$? What time would a traveler arrive at the destination?

2. (10) What is the latest that a traveler can leave the origin to arrive at the destination by $t = 20$ (not necessarily using the path you found in part 1)?
Problem 4. (20 points.) Short answer.

1. (10) Develop a node model for a merge where priority is given to one approach (and this priority is always obeyed by drivers). That is, give a formula for the flow moving from each upstream link to the downstream link in terms of sending and receiving flows.

2. (10) Name one advantage and one disadvantage of incorporating departure time choice into a dynamic traffic assignment model, alongside route choice.